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Additive vs Multiplicative luminosity-color decomposition

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Abstract

The visual system decomposes light entering the eye into an achromatic and a chromatic signal. Knowing whether this decomposition is additive or multiplicative is still a current research question. Luminance has been found to be additive when measured physiologically but multiplicative in appearance. Demosaicing multispectral images (shoot through a color or spectral filter array), show how additive decomposition is a linear solution to the inverse problem of mosaicing. But for reflectance estimation, a multiplicative decomposition would be preferred. Both decomposition imply two different geometries that share their vector spaces but not their metric.

Introduction

Luminance is the value assigned to a light by multiplying its spectral power distribution (expressed as a real function of wavelength) by the spectral efficiency function of the eye, $V(\lambda)$ and integrating over wavelength. Luminance estimation is linear because of the integration. The luminance of the composition of two lights is the sum of their luminance. But, the sensation of luminous intensity (sometime called brightness) is rather expressed by a log function of the energy. This log function is the support for the so-called Weber-Fechner's law [1] applying in light intensity [2].

Whether luminance is more appropriate to describe the physiology of the eye as measured by heterochromatic flicker photometry and the luminous intensity the sensation given by a light stimulus, is likely. From the retina, visual signal encoded by the cone-photoreceptors is known to project quite directly to the brain visual area (V1, V2, V4) but also to the motion regions of the brain (MT/V5) [3]. The signal carries by this so-called magnocellular pathway is known as achromatic. This is what made the visual system so efficient for motion perception. But there is no color variation inside, at least for a fixed condition of observation. If there exists a brain function that is fixed for fixed condition of observation, it must be estimated easily. This is what have been done with photometry measurement by hetero-chromatic flicker photometry [4, 5]. The average result of the experiment is normalized by the CIE as the spectral efficiency function $V(\lambda)$. It is a fixed function of wavelength because it is supposed fixed for fixed conditions.

How this function behaves when the condition change is still a not solved problem. There are evidences that the function change with overall level of light, from dim light to dazzling light. Also spectral efficiency of the eye is a different function in mesopic vision [6]. But the way this function change and if the change enables the so-called color constancy phenomenon is still under debate [7]. Here also, the function should be fixed for fixed condition of observation but the way the function change, adapt to the chromatic environment, is guided by a multiplicative decomposition [8].

Both cases can be glued together if one considers a fourth dimension intensity. Suppose the visual system is able to encode three-color mechanisms plus an intensity mechanism. Intensity is not a fourth independent variable because it depends on the three others. But it can correspond to different functions depending (1) on the light level (2) on the considered physiological pathway (3) on the chromatic adaptation state of the observer. The three mechanisms are the LMS fundamental that represents the spectral rate of photon absorbed by a cone's cell [9]. The intensity is a relative measure of the strength of the light by applying a sensitivity function (by point-wise spectral multiplication and integration) onto the spectral power distribution of a light. Using then the rules of projective geometry [10 - 13] these two spaces (additive and multiplicative) may lies.

Color space

Let suppose a vector space of dimension three called color vision space. To any vector corresponds a spectral efficiency function, that we consider present in the visual system. The three absorption spectra of the cone-photoreceptors $\varphi_i, i = 1..3$ form a basis for the color vision space. On this basis one can construct a coordinate system in which any light stimuli correspond to a point. The coordinate of this point is given by the scalar product between the spectral efficiency function and the spectral power distribution of the light stimulus (which is again the integral of the point-wise product of the two functions). This writes:

$$c = \langle \varphi | c \rangle = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} \int \varphi_1(\lambda)c(\lambda)d\lambda \\ \int \varphi_2(\lambda)c(\lambda)d\lambda \\ \int \varphi_3(\lambda)c(\lambda)d\lambda \end{bmatrix}$$

Where c is the coordinates of the color having $c(\lambda)$ as a spectrum. Function $\varphi_i(\lambda)$ is the ith spectral efficiency function for the three color mechanisms.

Additive decomposition

On this color vision space, the visual system operates a decomposition between the intensity and the color. This decomposition allows for perceiving a constant hue and saturation from object's reflectance despite a large variation of intensity along the object's surface. Suppose the decomposition is driven by a measure of intensity that is the average of coordinates of the color vector. Called $v = [1 \quad 1 \quad 1]^t/3$ the luminosity vector the decomposition write:

$$L = v^t c,$$

$$c = \mathcal{L} + \mathcal{C} = Lv + (c - Lv) = \frac{1}{3}\begin{bmatrix} L \\ L \\ L \end{bmatrix} + \begin{bmatrix} c_1 - L/3 \\ c_2 - L/3 \\ c_3 - L/3 \end{bmatrix}$$

$$= \frac{1}{3}\left(\begin{bmatrix} L \\ L \\ L \end{bmatrix} + \begin{bmatrix} 2c_1 - c_2 - c_3 \\ -c_1 + 2c_2 - c_3 \\ -c_1 - c_2 + 2c_3 \end{bmatrix}\right)$$

This decomposition is often called luminance-chrominance decomposition and was largely used for the development of color television. As shown on Figure 1 this decomposition can be interpreted as a cutting of the three dimensional vector space into parallel planes. Notice that if another vector v is chosen the decomposition is still possible. For Figure 1 we choose the vector v equal to V_λ corresponding to the coordinate of the spectral efficiency function $V(\lambda)$ in the coordinate system of a particular display. Each plane is the surface for which L is constant. All the light stimulus that have same luminance are confined into a plane orthogonal to v at a given level L.

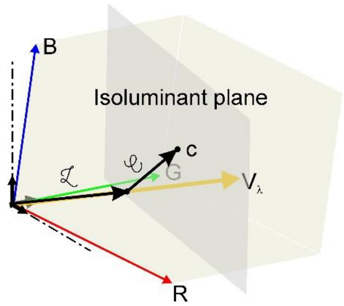

Figure 1. A color space such as the color cube produce by a computer display can be decomposed into luminance $\mathcal{L}$ and chrominance $\mathcal{C}$ vectors. Here is shown additive decomposition with the vector V_λ corresponding to the spectral efficiency function $V(\lambda)$ represented in a RGB cube of a particular display. The locus of light stimuli that share the same luminance as c are located in the plane orthogonal to V_λ passing through c.

Surprisingly this decomposition corresponds to the luminance and chrominance signals implied in the frequency spectrum of a mosaic image (raw camera image sampled through a color /spectral filter array), allowing for a frequency selection demosaicing using linear convolution filters [14]. A generalization for a higher dimensional vector space including as well a local neighborhood of the mosaic and using the minimum square error for learning the linear solution over a database, shows a level of performance comparable to more complicated and time consuming methods [15,16]. In all these developments the principle of additivity between luminance and chrominance is respected [17].

Multiplicative decomposition

In the color vision space, only an opened positive convex cone pointing to zero contains physical light. Any spectral distribution can be decomposed on the basis of Dirac's delta distribution [18]. The locus of the delta distribution in the color vision space, called spectrum locus, forms the limit of the cone, its envelop [8]. It was found that the spectrum locus; measured on observers by color matching experiment using mono-chromatic lights as Dirac δ; made a circular shape like a cone cut by the purple plane. The locus of monochromatic lights is a path indexed with variable t, and writes:

$$\ell(t) = \begin{bmatrix} \int \varphi_1(t)\delta(t)dt \\ \int \varphi_2(t)\delta(t)dt \\ \int \varphi_3(t)\delta(t)dt \end{bmatrix} = \begin{bmatrix} \varphi_1(t) \\ \varphi_2(t) \\ \varphi_3(t) \end{bmatrix}$$

Thus the shape of the spectral efficiency functions of the cone-photoreceptors fundamental determined the spectrum locus in the vector space. Because those function are not triangular, the spectrum locus is not a triangle. Rather a circular shape is observed.

The circular shape corresponds to a cone (here considered perfectly circular) and the equation of its envelop δC is formed by the isotope vectors of a quadratic form:

$$\delta C = \{x \in \mathbb{R}^3, x^t \mathcal{J}_1 x = 0\} \text{ with}$$
$$\mathcal{J}_1 = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

To express the multiplicative decomposition one need to defined a change of variable. This change of variable is made such that one axis in the new system is collinear to the sum of the three primaries axis. Otherwise said collinear to the vector v defined above. The matrix of passage P is also normed to 1, $P^{-1} = P^t$. The change of variable writes:

$$y = Px \text{ with}$$
$$P = \frac{1}{\sqrt{3}}\begin{bmatrix} -1/\sqrt{2} & \sqrt{2} & -1/\sqrt{2} \\ -\sqrt{6}/2 & 0 & \sqrt{6}/2 \\ 1 & 1 & 1 \end{bmatrix}$$

In this new coordinate system y, the envelop of the cone becomes:

$$y^t \mathcal{J} y = 0 \text{ with}$$
$$\mathcal{J} = P\mathcal{J}_1 P^t = \frac{1}{2}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

In the new coordinate system y, $\mathcal{J}$ is a diagonal matrix of Minkowski's kind. The quadratic form can be used to define a norm for vectors in the vector space, $||y|| = \sqrt{y^t \mathcal{J} y}$. If one associates this norm with the intensity of the light stimulus, then the iso-intensity surface becomes the hyperboloid of equation $y^t \mathcal{J} y = k^2$ rather than a plane. By consequence, the intensity of a light stimulus is given by the level k of the hyperboloid that passes through the corresponding point (Figure 2).

Let consider the change of variable:

$$k = \sqrt{c^t \mathcal{J} c}$$
$$\xi = \tan^{-1} c_2/c_1$$
$$s = \tanh^{-1}(c_1^2 + c_2^2)^{1/2} / (\sqrt{2}c_3)$$

Then the coordinate of the color point can be expressed as:

$$c = k\begin{bmatrix} \sqrt{2}\cos 2\pi\xi \sinh s \\ \sqrt{2}\sin 2\pi\xi \sinh s \\ \cosh s \end{bmatrix}$$

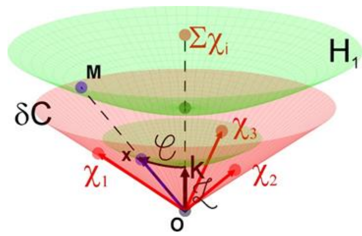

Figure 2. Multiplicative decomposition of the color space. The point x corresponds to a light stimulus can be decomposed as a vertical vector $\mathcal{L}$ from the origin to the affix k plus a curved vector $\mathcal{C}$ along the hyperboloid H_k of level k passing through x. Once k is known, the saturability s and hue ξ can be computed in the unitary hyperboloid H_1.

Notice that the three variable k, s and ξ have physiological meaning. k is the perceived intensity that may correspond to brightness compared to luminance L expressed with the vector $V(\lambda)$ or to lightness that is $L^{1/3}$. $s \in [0, 1]$ is the saturability, the arc tangent hyperbolic of the saturation and $\xi \in [-1/2, 1/2]$ is the hue.

This formulation allows a multiplicative decomposition of the color space. Any point c corresponding to a color stimulus writes $c = kH_1(s, \xi)$ where k is the hyperbolic norm of the light stimulus, and s and ξ the cylindrical coordinates of the point projected onto the unitary hyperboloid H_1. Given a particular illuminant as a single light source, and placing the corresponding vector as the axis of symmetry of the cone allows to obtain the reflectance factor of objects as k and the reflectance function parametrized by the hue ξ and saturability s. This decomposition would certainly participate to the color constancy phenomenon if one considers that the axis of symmetry of the cone is align with the illuminant [8].

Conclusion

There is not a preferred geometry per se as said Poincaré [19]. Geometry is a tool for lying everyday objects expressed as point, line and surface with rules. Additive and multiplicative decomposition of color space can be lied with projective geometry. One supposes a fourth variable construct on the three others either with a linear $L = v^t x$ or a quadratic form $k^2 = y^t \mathcal{J} y$. This variable then serves as a factor for projecting the points onto a unitary plane $L = 1$ or a unitary hyperboloid $k = 1$. As shown above the two models can be expressed together through a change of coordinate system. In Figure 3 the vectors χ_i are the basis vectors for the linear model. Those vectors correspond to the LMS color mechanism. For the hyperbolic model the basis is made with vectors ψ_i with the relation $\psi = P\chi$.

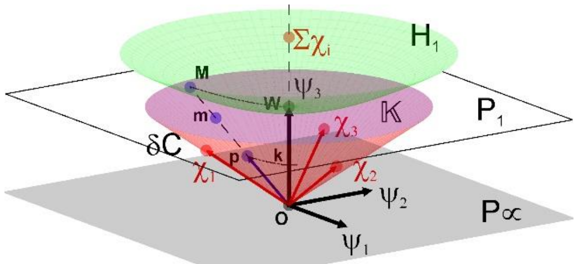

Figure 3. Additive and multiplicative models can by lied together in projective geometry. One defines an intensity measure on any point p, corresponding to a light stimulus, as a fourth variable either linear as $L = v^t p$ or hyperbolic $k = \sqrt{p^t \mathcal{J} p}$. The projection on the corresponding iso-intensity surface is given by dividing the three coordinates by this supplementary variable.

In both linear and hyperbolic projective geometry, the rule of additive composition of light is respected. The spectrum of an additive composition of two light stimuli is the sum of their two spectra. Thus the vector corresponding to the mixing of two lights is the sum of the two vectors. A system of three primaries forms a double opposite pyramids with vertices composing a parallelepiped, the color cube (Figure 1). The projection on the isoluminance plane forms a triangle called color gamut. With the construction shown in Figure 2, the position of the projection does not change with geometry. This is given by assigning the axis of symmetry of the hyperbolic model the same vector v as in the linear model. For hyperbolic geometry the point m is considered belonging to the Klein disk $\mathbb{K}$ rather than to the projection plane P_1. The point M is the projection onto the unitary hyperboloid H_1. Remark that with hyperbolic geometry, as far as the saturation increase the energy of the light should be increased to leave the perceived intensity constant. Because the hyperboloid is inscribed into the cone, as far as we want to reach the envelop of the cone, the distance from the origin is increased meaning an increase of energy.

The prediction given by the linear projective model for the demosaicing problem is in favor of the additive decomposition of the color vision space or at least an additive decomposition of spatio-chromatic vector space. But for the perception or sensation of colors the hyperbolic geometry is preferred. It is likely that these two representations of colors into their achromatic vs chromatic part are present in the brain and communicate each other for given us our sense of vision.

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