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A multimode quantum optics approach to incoherent imaging

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Abstract

Recent works employing tools from quantum optics and quantum metrology proposed a new passive imaging technique that allows to resolve details far below the diffraction limit. This technique is based on replacing standard spatially-resolved intensity measurements, e.g. at each pixel of a camera, with spatial-mode demultiplexing (SpaDe) measurements that allow to acquire information in a more efficient way. In this contribution, we want to provide an intuitive explanation of why such a SpaDe approach is so effective, and illustrate how we used these idea to discriminate one point source from two, and to estimate the separation between two incoherent sources.

Introduction

Modes of the electromagnetic fields have always been a useful tool for the theoretical understanding of physical phenomena and for the development of optical technologies [1]. For example, in optical communication the search for additional degrees of freedom to encode information into a single light beam led to demultiplexing into a vast variety of optical modes ranging from temporal modes, through frequency modes, to spatial modes.

If we consider a light field where multiple optical modes are populated, and we take into account the quantum nature of light, we enter into the realm of *multimode* quantum optics [2]. The latter provides, not only the tools to characterize the properties, e.g. the entanglement, of quantum states of light occupying multiple optical modes, but also those to individuate which optical modes to measure to optimally extract information from such quantum states.

From a quantum information point of view, imaging corresponds to extracting information from the spatial distribution of an optical field. Following this approach, we can use multimode quantum optics to identify the optimal spatial modes to detect in order extract the most information. Recently, these ideas have been vastly applied to the analysis of incoherent optical sources, leading to development of a new *quantum-inspired* super-resolution technique, i.e. an imaging approach that allows to resolve details beyond the diffraction limit [3, 4, 5, 6]. The main idea beyond this innovative technique consists in replacing spatially-resolved intensity detection, e.g. at each pixel of a camera, with spatial mode demultiplexing (SpaDe) into an optimally selected set of spatial modes.

Arguably, the most interesting feature of SpaDe-based imaging is that it allows to surpass the diffraction limit only optimizing the measurement stage. Accordingly, this results in a completely passive technique which is particularly appealing for all those scenarios where one cannot control the sources emission, e.g. in astronomy and in certain non-invasive biomedical applications.

The scope of this contribution is to provide an intuitive account of the theory behind this quantum-inspired, approach to super-resolution imaging. Then, building on such an introduction, present our theoretical results on optimal discrimination between point sources [7], and our latest experiment where we obtained record sensitivity in estimating the separation between two point sources via SpaDe measurements [5].

Optical modes and images

We call optical modes an orthonormal set of transverse solutions of Maxwell's equations in vacuum, i.e. any set of complex functions $u_i(\mathbf{r},t)$ satisfying the following equations

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right) u_i(\mathbf{r},t) = 0, \quad \nabla \cdot u_i(\mathbf{r},t) = 0 \tag{1a}$$

$$\int u_j^*(\mathbf{r},t) u_k(\mathbf{r},t) d\mathbf{r} = \delta_{jk}, \tag{1b}$$

where * denotes complex conjugation and δ_{jk} denotes the Kronecker delta. Since, we are dealing with imaging applications, we are are only interested here in the spatial part of the optical

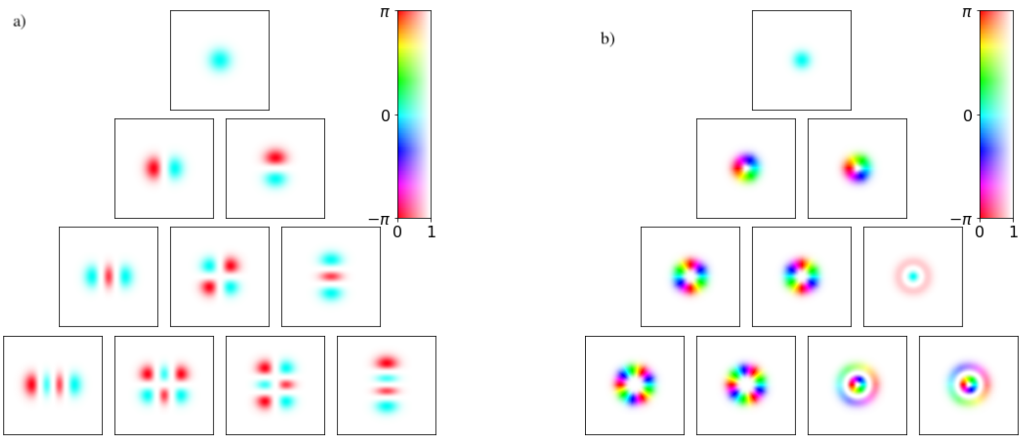

Figure 1. *The first ten Hermite-Gaussian (HG) modes a) and Laguerre-Gaussian (LG) modes b). To achieve a two-dimensional depiction of complex fields, we used colors to denote phase values, and opacity to represent the field's intensity. We can see that HG modes are real (the phase is either* 0 *or* π*) while LG modes present a complex phase structure.*

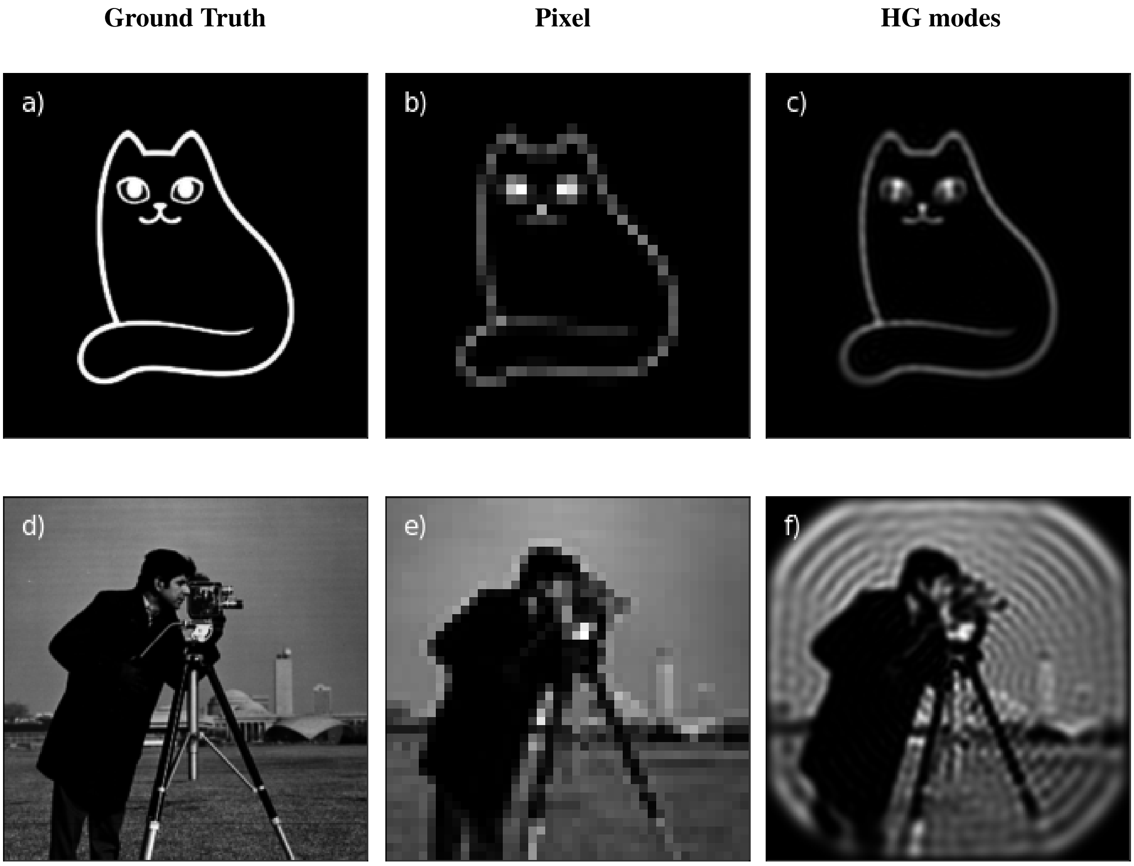

Figure 2. *A binary image a) and a grey-scale image d) reconstructed using a* $40 \times 40 = 1600$ *pixel grid [b) and e)] and the same number of HG modes [c) and f)].*

modes. Therefore, in the following, we will simply write $u_i(\mathbf{r})$, and drop the temporal dependence.

Equations (1) admit several solutions, however there are two families of spatial modes $u_i(\mathbf{r})$ which are most commonly considered in both classical and quantum optics: the Hermite-Gaussian (HG) and the Laguerre-Gaussian (LG) modes. For both mode families, the fundamental mode $u_0(\mathbf{r})$ is a Gaussian mode. HG modes have Cartesian symmetry and can be constructed from successive Cartesian derivatives of $u_0(\mathbf{r})$. On the other hand, LG modes have radial symmetry and can be constructed from radial derivatives of $u_0(\mathbf{r})$. The first ten modes of both families are presented in Fig. 1.

From a physicist's point of view reconstructing an image corresponds to detect the spatial distribution of an optical field. This is usually done by locally detecting the intensity of the field in a series of non-overlapping spatial functions, e.g. defined by the locations and the extent of the pixels of a camera. However, an optical field $E(\mathbf{r})$ can be expressed as an expansion in an arbitrary mode basis

$$E(\mathbf{r}) = \mathscr{E} \sum_j a_j u_j(\mathbf{r}), \tag{2}$$

where $\mathscr{E}$ is a dimensional constant proportional to the total field intensity $I = \int |E(\mathbf{r})|^2 d\mathbf{r}$, while the complex coefficients a_j can be computed as $a_j = \int u_j^*(\mathbf{r},t) u_k(\mathbf{r},t) d\mathbf{r}$. Accordingly, it is also possible to reconstruct an image by detecting the field's intensity in each mode of an arbitrary spatial basis. As an example, in Fig. 2 we compare the reconstruction of a binary image (where pixels are either off or on) and a grey-scale image (first column) in terms of pixels (second column) and the same number of HG modes (last column).

The most obvious difference between the second and the third columns in Fig. 2 is that image reconstructed using HG modes (which are delocalized) are smoother that those reconstructed using pixels (which are tightly localized). If we look at Fig. 1, increasing the mode order, we obtain modes that on the one hand occupy a larger area. Accordingly, when reconstructing an image, adding higher order modes allows to reconstruct regions further away from the center of the image. This is clearly evident in Fig. 2 f), where the limited number of HG modes used for the reconstruction did not allow us to access the corners of the image. A way to increase the covered area would be to increase the width of the fundamental Gaussian mode $u_0(\mathbf{r})$. However, this would also imply the use of modes with a less fine spatial structure, and therefore it would result in a loss of spatial resolution. The only way to increase the reconstructed area, and also increase the resolution is using a larger number of modes.

In this section, we have showed how an image can be decomposed into various mode bases, and compared the use of localized pixel modes and delocalized spatial modes such as the HG modes. We did our comparison assuming that we have access to the image directly. However, this is never the case in practice, where diffraction and the finite size of the collecting aperture of our imaging system cause the acquired image to smear. In the following section, we will see how quantum metrology and SpaDe can help us in this case.

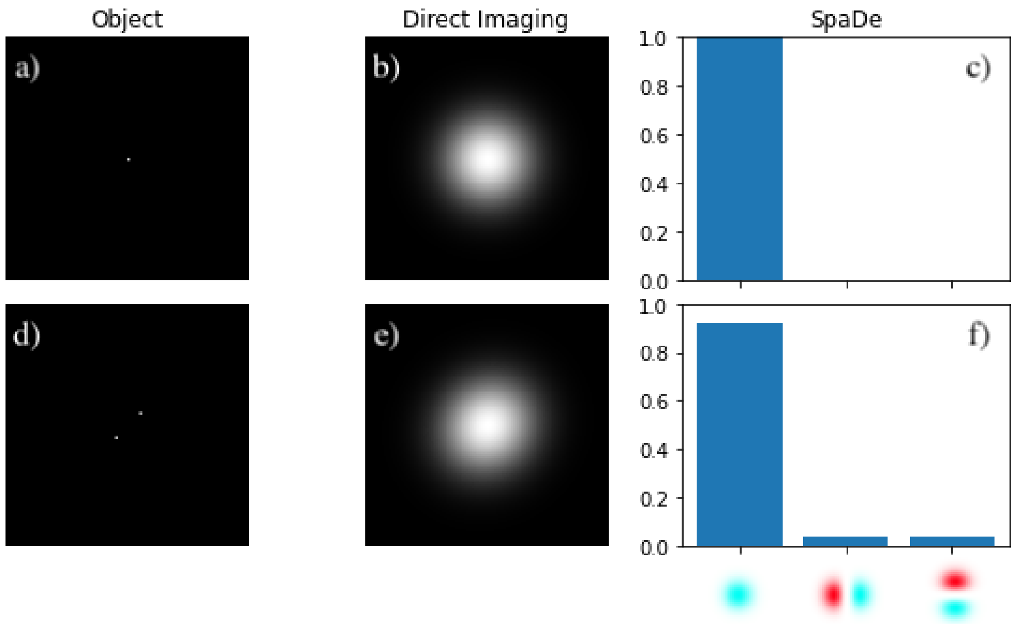

Figure 3. *Imaging point sources with a diffraction limited system. The first column shows the arrangement of point sources in the object plane, the second and the third column show, respectively, the corresponding direct imaging and SpaDe (for demultiplexing in the first 3 HG modes, compare Fig. 1) distributions in the image plane. Having one source in the object plane (a) or two (d) results in indistinguishable direct imaging distributions (b and e), but very distinct SpaDe histograms (c and f).*

Beyond diffraction with quantum metrology

Historical resolution criteria, like that of Abbe [8] and Rayleigh [9] tell us that objects whose separation is smaller than the width of the point spread function (PSF) of the imaging system are impossible to resolve. These criteria are based on the visual inspection of intensity distributions and the resolution of the human eye. We can experience the rationale behind these criteria by observing how a point source produces an image which is indistinguishable from that of two closely separated ones (compare Fig. 3 b) and e)).

It is possible to go beyond these visual resolution criteria by performing a full statistical analysis of the measured intensity distribution. Furthermore, we can allows ourselves to go beyond spatially resolved intensity measurements, i.e. to go beyond direct imaging (which is the kind of measurement performed by a camera). If we do that, we can ask ourselves which is the measurement that allows us to achieve the best possible resolution. We can answer this question by employing a quantum optical description of the imaging problem, and using the tools of quantum metrology to perform an optimization over all possible measurements allowed by quantum mechanics [10]. This approach revealed that SpaDe is the optimal measurement for a large variety of tasks, such as estimating the displacement of a light source [11], the separation between two incoherent sources [3], or distinguishing one point source from two [12].

Why SpaDe works so well for these problems is simple to understand. Let us consider a point source at position $\mathbf{r}_0$ in the object plane observed through an imaging system with a Gaussian PSF $u_0(\mathbf{r})$. In this case, if we assume that $|\mathbf{r}_0| \ll w$, with w the width of the Gaussian PSF, the optical field in the image plane can be written as

$$\begin{aligned} u_0(\mathbf{r}-\mathbf{r}_0) &\sim u_0(\mathbf{r}) + (\mathbf{r}-\mathbf{r}_0)\cdot\nabla u_0(\mathbf{r}) \\ &= u_0(\mathbf{r}) - \frac{1}{w}(\mathbf{r}-\mathbf{r}_0)\cdot(u_1(\mathbf{r}), u_2(\mathbf{r})), \end{aligned} \tag{3}$$

where in the second line we have used that the HG modes can be constructed from derivatives of the fundamental Gaussian mode. From this expansion, we can see that a small shift of the position of a point source induces populations in the first two HG modes. Accordingly, detecting these populations allows to reach very high sensitivities in estimating the position of a single source, or the separation between two incoherent one, but also to discriminate two incoherent sources from a single one with minimal error probability. How the SpaDe distribution for a single source is notably different from that of two closely separated ones can be observed by comparing Fig. 3 c) and f).

Following this approach, we designed a statistical test that allows the discrimination of one incoherent source from two by monitoring the population of a single HG mode, and that achieves optimal performances even in presence of imperfect demultiplexing [7]. Furthermore, we performed an an experiment where we achieved record sensitivity in the estimation of the separation between two incoherent sources through SpaDe [5]. In this experiment, we investigated both the low photon-flux and the high photon-flux regimes. While in the former, we achieved a sensitivity beyond that of an ideal camera (with no noise and infinitely many and infinitely small pixels), in the latter we reached a sensitivity five order of magnitudes beyond the diffraction limit.

Conclusion

In this contribution, we gave an intuitive introduction to the use of SpaDe measurements in imaging applications. We discussed how an arbitrary transverse field distribution can be reconstructed not using localized pixels, but also in terms of different delocalized spatial mode bases. Furthermore, we showed how while reconstructing a full image requires to acquire information in a large number of modes, small variations of an object (far below the diffraction limit) can be revealed by monitoring a very small number of higher order spatial modes.

Finally, we briefly recollected some of the major results that we achieved using this technique. Our experimental results, in particular, were possible thanks to the use of a high-quality SpaDe system constructed by the french company Cailab using a technique known as multiplane light conversion [13]. This technique is establishing itself as the gold standard for the design and construction of spatial mode demultiplexers: very efficient design algorithms are available [14], leading to the simultaneous demultiplexing of hundreds of spatial modes in laboratory environment [15] and commercial devices controlling around 50 modes [16]. The fast expansion of this technology leading to high quality devices which are able to control more and more modes is extremely promising for their future applications in imaging problems going beyond the characterizations of few point sources.

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Author Biography

Giacomo Sorelli received his Master degree in physical and astrophysical sciences from the University of Florence (Italy, 2015) and his PhD in physics from the University of Freiburg (Germany, 2019). From 2019 till 2022 he worked at the Kastler Brossel laboratory (France) and is currently an external scientist at Fraunhofer IOSB. His work focuses on quantum sensing and atmospheric propagation. He is a member of Optica and SPIE.