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Detection and correction of errors in psychophysical color difference Munsell Re-renotation dataset

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Abstract

The Munsell dataset holds a prominent position in the field of color science. This dataset describes large color differences covering a wide color gamut, making it highly valuable for the development of color models. Currently, the widely used version is the Munsell Renotation, which is the second version of the dataset. In this paper, we analyze the third version, known as the Munsell Re-renotation, identify significant errors within it, and provide corrections for obvious typos. We propose a novel method for detecting nonuniformities, utilizing the L_1*-STRESS measure and the proLab uniform color space (UCS). Our findings demonstrate that the revised version of the Munsell Re-renotation dataset achieves significantly better consistency with established UCSs compared to the original Munsell Re-renotation data. Additionally, we discuss modifications of the STRESS measure for data with unknown scales. Unlike previous modifications, the proposed measure,* $STRESS_{group}$*, is identical to the classic STRESS measure when the scales are the same.*

Introduction

TV and smartphone manufacturers strive to achieve the most accurate color reproduction. Evaluating the quality of the displayed image is one of the most crucial tasks in the development of visualization algorithms. The displayed image quality assessment aims to define a measure of reproduction accuracy. The distances in the utilized color space (CS) should correctly reflect color differences (CDs) perceived by humans. In a uniform color space (UCS), Euclidean color distance exhibit linear dependence on the perceived CD.

To develop a UCS, sets of color pairs and their corresponding estimated differences are used. Such sets are known as psychophysical datasets. However, employing these datasets is challenging: the collected data come from different laboratories. To combine data across laboratories, modifications [1] of the $STRESS$ measure [2] are used.

Most existing UCSs have been derived from data within the sRGB gamut [3] which represents a small range of colors. However, with the growing popularity of wide color gamut (WCG) displays, there is a need to develop UCSs that can accurately handle colors within this expanded range. UCSs like $\mathrm{IC_TC_p}$ [4] and $\mathrm{J_za_zb_z}$ [5] have been specifically designed for WCG colors. Nevertheless, there is a scarcity of reliable experimental data [6] available to assess the uniformity of these UCSs, and such data are not readily accessible to researchers.

Thus, all available datasets, particularly the Munsell dataset, are highly valuable. Furthermore, the Munsell dataset currently stands as the sole dataset encompassing WCG by describing large CDs. Its extensive utilization [5, 7, 8, 9, 10, 11, 12] reinforces the need to validate the correctness and consistency of the dataset. In this paper, we demonstrate the presence of errors, including misprints, in the published Munsell "Re-renotation" dataset. These substantial errors manifest as outliers, adversely impacting the optimization of UCSs. Given the labor-intensive nature of manually verifying all data in psychophysical datasets, it is preferable to employ automatic verification methods. Here, we propose a semi-automatic method for detecting errors in experimental CD data by optimizing a UCS model using a STRESS-based loss function.

Munsell color system

In 1905, A. Munsell, an American scientist and artist, introduced the Munsell color system, which is physically represented by the Munsell Book of Color (1929). The Munsell color system defines a color body with an irregular shape in three dimensions: Value (V), Chroma (C), and Hue (h). Each dimension preserves equal perceptual increments, meaning that adjacent colors on a scale (with fixed values of the other dimensions) are perceived as equally distant without any established relationship between the units of the three axes. Consequently, the Munsell color dataset consists of chains of equidistant colors, and the CDs in different chains may have different scales.

Munsell Renotation (Re) dataset

Usually, the Munsell system is represented by "Munsell Renotation" dataset (hereinafter referred to as Re). The Re dataset was introduced in 1943 after the Optical Society of America (OSA) Colorimetry Committee revised the Munsell Book of Color [13]. This revision involved meticulous verification of hundreds of visual experiment results and interval equality estimations to ensure uniformity. Chroma loci were extrapolated from the Munsell sample bounds to theoretical colorants [14], and Hue loci were extended. The Re dataset can be accessed on the Rochester Institute of Technology (RIT) website [15].

The Re dataset has been extensively used in various studies [5, 8, 9, 11, 16, 17]. However, despite efforts to achieve perfect visual uniformity, deviations from uniformity still persist, as noted by several authors [18, 19, 20].

Munsell Re-renotation (Re-Re) dataset

The development of "Munsell Re-renotation" (hereinafter referred to as Re-Re) [21] in 1967 was prompted by the evidence of local nonuniformities in Re. To improve the uniformity of the Munsell system, extensive visual evaluations were conducted, resulting in a dataset comprising 2946 colors with standardized colorimetric coordinates (x, y, Y) and Munsell system coordinates. However, Re-Re was not published as a printed atlas and has received limited attention in the literature. A scanned report[21] containing tables of color coordinates is available on the RIT website [22]. It is not machine-readable, so the utility of these tables for studying uniformity or developing UCSs is limited. Nevertheless, Re-Re offers potential advantages over the Re dataset in terms of improved uniformity. Thus, we have digitized the Re-Re dataset and examined the uniformity of color values within its color chains.

Method for errors correction in CD datasets

Since the Re-Re dataset is only available in scanned pages of the report, we initially converted it into a machine-readable format by manually inputting the table values. Subsequently, we employed a semi-automatic procedure to identify nonuniformities in the resulting dataset. The proposed method involves optimizing the UCS model using a uniformity score as a loss function. The nonuniformities were categorized into three types: (1) typos made by us during the retyping process, (2) typos present in the original table, and (3) nonuniformities that cannot be easily corrected and require psychophysical experiments; we provide a separate list of such colors and suggest excluding them from the dataset when high uniformity is desired.

Retyping into a machine-readable form

To minimize errors during the conversion, several experts independently performed the typing procedure. The procedure was carried out as follows:

1. Each page was processed by two experts.
2. The results were compared by a third expert.
3. For each character with a contradicting final representation, the correct version was selected.

This procedure identified 37 typos made by the experts. They have been corrected.

Concept of nonuniformities detection in CD dataset

The analyzed datasets consist of numerical values that lack an intuitive interpretation, making them susceptible to difficult-to-detect errors. Some of these errors may have been present in the original tables. In this paper, we propose a method for detecting such errors. While an ideal UCS is not yet established, it is known that the true metric of a color space exhibits gradual and smooth changes between points. Furthermore, there are UCS models available that offer adjustable parameters. It can be expected that by fitting a UCS model robustly to the dataset, the portion of the dataset that is least consistent with the resulting UCS likely contains errors rather than genuine deviations in human perception properties from the model. We will explore the process of fitting the UCS model to the dataset and robust techniques to achieve this.

STRESS as a measure of the discrepancy between experimental data and model predictions

The uniformity of a CS is determined by its ability to accurately represent perceived CDs through the Euclidean metric. The STRESS measure is commonly employed to quantify the similarity between computed and psychophysical CDs [23]. Let $\vec{x} \in \mathbb{R}^n$ represent a dataset of psychophysical CDs for n color pairs and $\vec{y} \in \mathbb{R}^n$ represent the vector of CDs for the same color pairs in a given UCS. The $STRESS(\vec{x}, \vec{y})$ can be expressed as follows:

$$STRESS(\vec{x}, \vec{y}) = \frac{\|k^*\vec{x} - \vec{y}\|_2}{\|\vec{y}\|_2} = \frac{\sqrt{\sum\limits_{i=1}^{n} (k^* x_i - y_i)^2}}{\sqrt{\sum\limits_{i=1}^{n} y_i^2}}, \quad (1)$$

where k^* is the factor that is used for scale normalization: $k^* = \operatorname{argmin}_k \|k\vec{x} - \vec{y}\|_2 = (\vec{x}^T\vec{y})/(\vec{x}^T\vec{x})$. It should be noted that the STRESS value is independent of vector lengths, as indicated by the equations above. Consequently, a STRESS value of $STRESS(\vec{x}, \vec{y}) = 0$ suggests that the CDs in $\vec{x}$ differ from those in $\vec{y}$ by a constant multiplier.

STRESS for datasets with varying CDs scales

In a single dataset, the scales of CDs are usually the same. However, in cases such as the Munsell data or when dealing with multiple datasets, the scales may vary, and their relationship is unknown beforehand. Therefore, the original form of the $STRESS$ measure cannot be directly applied.

Safdar et al. [5, 24] estimated the uniformity of the UCS on the COMBVD [25] meta-dataset (a union of several datasets) of small CDs using the classical $STRESS$ applied to concatenated sub-datasets. This estimation method is valid only if the scales of CDs in the meta-dataset are known to be the same. Li et al. [16] calculated $STRESS$ individually for each group and defined the integral score for the meta-dataset as an average. However, this method does not take into account the structure of a meta-dataset: groups of different sizes may make a different contribution to the $STRESS$ value. To address the issue of group size, Melgosa et al. [1] used a weighted-normalized $STRESS$ to estimate the uniformity of CD formulas on the COMBVD dataset. This approach equalizes the contributions of each group using weights that depend on group sizes.

To account for the different scales among the dataset, we propose a modification of $STRESS$ that we denote as $STRESS_{group}$. We refer to each sub-dataset with the same scale of CDs as a group. Let us consider a j-th group of target $\vec{x}_j$ and predicted $\vec{y}_j$ CDs: $\vec{x}_j = (x_{j,1}, \ldots, x_{j,n_j})^T, \vec{y}_j = (y_{j,1}, \ldots, y_{j,n_j})^T \in \mathbb{R}^{n_j}$. The ground truth and predicted CDs for meta-dataset consisting of m groups will be regarded as tuples of target X and predicted Y vectors: $X = \langle \vec{x}_1, \ldots, \vec{x}_m \rangle$, $Y = \langle \vec{y}_1, \ldots, \vec{y}_m \rangle$. The size of the meta-dataset is $n = \sum_{j=1}^{m} n_j$. Thus, we propose calculating the $STRESS_{group}$ as follows:

$$STRESS_{group}(X, Y) = \frac{\sqrt{\sum\limits_{j=1}^{m} \sum\limits_{i=1}^{n_j} (k_j^* x_{ji} - y_{ji})^2}}{\sqrt{\sum\limits_{j=1}^{m} \sum\limits_{i=1}^{n_j} y_{ji}^2}}, \quad (2)$$

where $k_j^* = \left(\sum_{i=1}^{n_j} x_{ji} y_{ji}\right) / \left(\sum_{i=1}^{n_j} x_{ji}^2\right)$. Unlike Melgosa et al. [1] we do not introduce weights w_j for the groups, as we assume all color chains are of equal importance. Among all modifications of $STRESS$ that take into account the variability of CD scales, $STRESS_{group}$ is the only modification that is equal to the classic $STRESS$ when the CD scales are the same: $k_1 = \cdots = k_m \Longrightarrow STRESS_{group}(X, Y) = STRESS(X, Y)$.

L_1-STRESS

Errors in the training dataset can introduce bias into the predictions of the trained model, and these erroneous data points are commonly referred to as outliers. To identify outliers in the dataset, it is crucial to train the UCS model in a manner that prevents the predictions from being heavily influenced by them, i.e., the model should be robust to outliers.

In order to mitigate the influence of outliers on the optimized UCS parameters, we suggest a modification to the $STRESS$ measure that is less susceptible to the presence of outliers. The proposed modification involves replacing the L_2-norm in equation (1) with an L_1-norm:

$$STRESS^{L_1}(\vec{x}, \vec{y}) = \frac{\|k_{L_1}^* \vec{x} - \vec{y}\|_1}{\|\vec{y}\|_1} = \frac{\sum\limits_{i=1}^{n} |k_{L_1}^* x_i - y_i|}{\sum\limits_{i=1}^{n} |y_i|}, \quad (3)$$

where $k^*_{L_1} = \text{argmin}_k \|k\vec{x} - \vec{y}\|_1 = \text{argmin}_k \sum_i |kx_i - y_i|$. When employing $STRESS^{L_1}$ for optimization, outliers in the dataset will have a reduced impact compared to using the original $STRESS$.

In the case of multiple datasets, the $STRESS_{group}$ formula is also modified by utilizing the L_1-norm:

$$STRESS^{L_1}_{group}(X,Y) = \frac{\sum_{j=1}^{m} \sum_{i=1}^{n_j} |k^*_j x_{j,i} - y_{j,i}|}{\sum_{j=1}^{m} \sum_{i=1}^{n_j} |y_{j,i}|}. \quad (4)$$

The introduced $STRESS^{L_1}_{group}$ measure enables the optimization of the UCS model on datasets with varying scales, while leveraging the use of the L_1-norm, which is less sensitive to outliers.

Semi-automatic detection of errors in a dataset

We propose an approach for detecting errors in experimental CD data, which is based on the optimization of UCS model with the $STRESS^{L_1}_{group}$ as the loss function. The UCS model selected for this study is proLab [26]. The proLab color space is derived by the 3D projective transformation of the CIE XYZ CS and is described by 8 independent parameters. Instead of relying on the original parameter estimates given in the proLab paper [26], we optimize the parameters of the proLab model on the given dataset using the BFGS algorithm [27].

The procedure for detecting and correcting errors in the dataset requires the initial version of the dataset, D_0, and the starting amount of worst chains which we viewed (with the highest $STRESS$ values), N_0, along with an increment value s. At each i-th iteration, N_i worst chains are viewed, and N^*_i chains with outliers among N_i are identified. By ΔN_i we denote the number of additional viewed worst chains. Two independent tests are performed to identify chains with outliers: (1) visual inspection of colors, and (2) analysis of the consistency of CD predicted by different models. If a color stands out from the general trend in both cases, it is classified as an outlier. The procedure stops when the following condition is met: $\Delta N_i \geq \Delta N_{max}$ (where ΔN_{max} is the maximum acceptable additional number of viewed chains). A detailed procedure for detecting and correcting errors is provided below.

The iteration process begins by initializing the counter i to zero. The following steps are then executed iteratively until the stopping condition is met:

1. Optimize proLab using the $STRESS^{L_1}_{group}$ loss function on the current version of the dataset D_i.
2. Calculate the $STRESS$ value for each chain and sort chains in descending order according to their $STRESS$ value.
3. Visualize a list of N_i color chains with the highest $STRESS$ and manually inspect each chain for outliers.
4. If errors are detected ($N^*_i > 0$), correct them in D_i, increment the iteration counter by 1 with $N_{i+1} = N_i$, and return to step 1.
   Else set $\Delta N_i = 0$ and repeat the following steps until $\Delta N_i \leq \Delta N_{max} - s$:
   (a) $\Delta N_i += s$.
   (b) Increase the number of viewed chains: $N_i += \Delta N_i$.
   (c) If errors are found, correct them in D_i, increment counter $i += 1$ with $N_{i+1} = N_i$, return to step 1.
   (d) If $\Delta N_i > \Delta N_{max} - s$, return the dataset D_i.

Method validation on the Re dataset

In addition to the Re-Re dataset, we also tested the effectiveness of the proposed method on the Re dataset. As is well known, the Re dataset is not perfectly perceptually uniform, but no significant outliers have been found in it [18, 19, 20]. This allows us to evaluate the specificity of the method. During the first iteration of the proposed method, no outliers were found among the 80 worst color chains in the Re dataset, indicating that it does not contain obvious errors. However, it is worth noting that several color chains with less apparent outliers were found in the Re dataset, but their correction cannot be suggested based on the adjacent sections of the data table. As it turned out, these nonuniformities in the Re dataset correspond to the two most significant anomalies of the Munsell system found in experiments conducted by Indow [18, 28]. Therefore, we conclude that the proposed method exhibits high specificity.

The revised version of the Re-Re dataset

We implemented the proposed method to validate and correct the Re-Re dataset. The initial number of chains N_0 was set to 20 and the value of s parameter was set to 10. With $\Delta N_{max} = 30$ we executed 11 iterations, the procedure terminated when the amount of viewed chains reached $N_{10} = 110$, resulting in a single chain detected as erroneous.

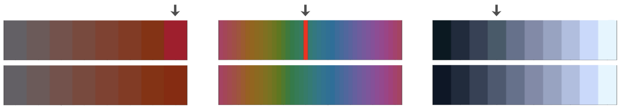

Figure 1. The visualization of equidistant color chains, with wrong colors indicated by arrows before correction (upper row). The corresponding result after correction is illustrated in the lower row.

	2.5P			5.0P			7.5P			10.0P		
V/C	Y	x	y	Y	x	y	Y	x	y	Y	x	y
6/20								–	–		–	–
18	18.3	0.250*	0.150*		–	–	18.0	0.290_5	0.149	17.7	0.322	0.150
16	18.9	.255	.165	18.8	0.271	0.166	18.7	$.292_5$	$.165_5$	18.4	.321	.168
14	19.8	.261	.183	19.7	.275	$.182_5$	19.7	.295	.183	19.4	.320	.185
12	20.6	.266	.199	20.4	.279	.198	20.3	.297	$.197_5$	20.2	$.319_5$	.200
10	21.5	.272	.215	21.5	.283	$.215_5$	21.3	.299	.216	21.3	$.318_5$	.217
8	22.6	$.277_5$	.231	22.4	.287	$.230_5$	22.4	.301	.231	22.3	.317	$.238_5$
6	23.9	.284	.249	23.9	$.292_5$	$.248_5$	23.6	$.302_5$	.249	23.6	.317	$.251_5$
4	25.2	.292	$.269_5$	25.2	.297	.269	25.2	.205	.269	25.2	$.214_5$	$.270_5$
2	27.3	.301	.292	27.3	$.303_5$	.292	27.3	$.207_5$	.292	27.3	.313	.293

Figure 2. The fragment of the table displays errors in a chain of equidistant colors with h=7.5P (Purple) and V=6. The two "x" coordinates of the colors with saturation C=2 and C=4, highlighted with underlines, exhibit significant differences from the "x" coordinates of the other colors in the chain.

As a result, the proposed method revealed a total of 52 incorrect values, 34 of which were errors in the original dataset and 18 were mistakes made during the dataset digitization process that we missed during the corresponding stages. Most corrections were associated with a single coordinate from xyY, except for 7 tricky cases where more than one coordinate required correction. Figure 1 provides examples of color chains before and after correction. Note that the middle chain in the figure has a longer length (42 colors) and all colors in it are visualized as narrower bands. The left chain shows an easily detectable wrong color in the 8th position, while in the right chain, the 4th color deviates slightly from the general trend, and the correction results in a more visually uniform chain.

In all chains with identified outliers, the dynamics of a coordinate along one of the xyY axes were clearly disrupted. Figure 2 illustrates an example of a fragment of the Re-Re table where one of the coordinates varies significantly from the other coordinate values within a chain. After correcting the errors, the dynamics of coordinate changes within the chain were restored and the chains appeared visually more uniform. Figure 3 illustrates the

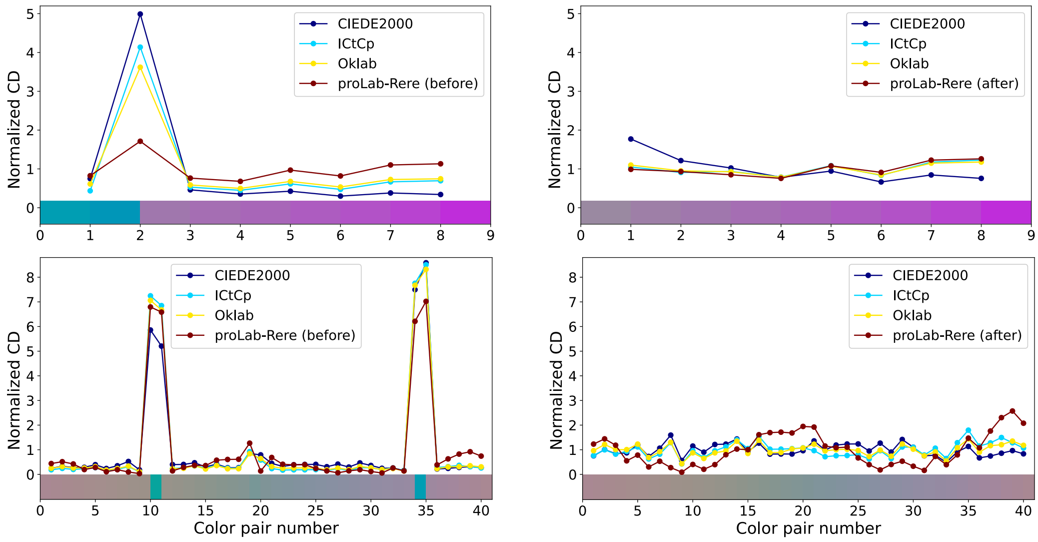

Figure 3. Difference values for equidistant color chains from the Re-Re dataset after Mean Normalization: before correction (left) and after (right).

contrast between the CD value dynamics before and after corrections. As depicted, the CD values underwent significant changes.

The revised version of the "Munsell Re-renotation" dataset is available at github.com/iitpvisionlab/mrr-revised. In addition to 34 detected misprints, we have also revealed 2 duplicate colors and 4 significant nonuniformities that are not obvious typos. The version of the dataset excluding these ambiguous outlying colors and a duplicate is also provided.

UCS evaluation on the revised Re-Re dataset

To assess the effectiveness of error correction, we examined the consistency of several UCSs and one CD formula with both the original and corrected Re-Re datasets. The considered UCSs were CIELAB, IC_TC_P [4], CAM16-UCS and CAM16-LCD [16], $J_za_zb_z$ [5], Oklab [29], proLab [26], and as CD formula we considered CIEDE2000 [25]. For the CIEDE2000 and UCSs parameterized by illuminant, we specified illuminant C. For CAM16-based UCS, the luminance of adaptive field L_A=64, luminous factor Y_b=20 and "average" surround were specified according to [5] for Munsell Renotation data. We optimized proLab UCS on two versions of the dataset and evaluated $STRESS_{group}$ values ($STRESS_{group}$ was also used as a loss function for optimization). For other UCS and CD formulas, we evaluated their consistency with the dataset using the $STRESS_{group}$ as a measure. The results for the original and revised datasets are demonstrated in Table 1.

The correction of 34 colors out of 2858 unique colors, which corresponds to approximately 1.2% of the data, resulted in a significant improvement in the consistency between models and the Re-Re dataset. Notably, in the case of the CAM16-UCS and IC_TC_Pmodels, it was not even possible to calculate $STRESS_{group}$ for the original data, as the conversion of xyY coordinates with misprints produced NaN (not a number) values for the coordinates in these UCS. Although the number of typos is relatively small, they greatly distorted the uniformity of the data, rendering the Re-Re dataset unusable. Now, after the proposed corrections, the revised version is much more uniform and can finally be used.

UCS or CD formula	The original Re-Re data	The revised Re-Re data
CIELAB	0.928	0.566
CAM16-LCD	NaN	0.364
CAM16-UCS	NaN	0.360
IC_TC_P	NaN	0.341
CIEDE2000	0.480	0.466
proLab*	0.464	0.256
$J_za_zb_z$	0.877	0.179
Oklab	0.482	0.144

Table 1. $STRESS_{group}$ values for various UCSs and CIEDE2000. A lower $STRESS_{group}$ value indicates better consistency. The symbol "*" indicates that the model was optimized using the corresponding dataset version.

Conclusion

In this article, we have presented the "Munsell Re-renotation" dataset in a digital format, with corrected misprints. We have proposed a semi-automatic method for detecting errors in datasets containing chains of equidistant colors. This method allowed us to identify 34 wrong colors (≈ 1.2% of the total amount of colors) in original data. We have proposed corrections for these errors to restore the uniformity of the color chains these colors belong to. Comparison of the original and corrected versions of the dataset in terms of consistency with various uniform color spaces' predictions shows that the misprints had a critical impact on the data.

As the major contribution of the study we provide two revised versions of the "Munsell Re-renotation" dataset in machine-readable format: (1) with corrected erroneous colors and (2) with removed ambiguous and duplicate colors that we have also found. The datasets are available at github.com/iitpvisionlab/mrr-revised. As well as the original "Munsell Re-renotation" data that we converted from page scans into digital format is provided.

Additionally, we proposed a new measure, $STRESS_{group}$, for estimating the consistency between a uniform color space and color difference meta-dataset with various scales.

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