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Point and Line to Surface: The Geometric Elements of Display Color Modeling

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Abstract

Color appearance is multidimensional, and color space has been a useful geometric representation for display modeling and optimization. However, the three fundamental attributes of color, i.e., brightness, saturation, and hue, have not found their singly corresponding physical correlates. Changes along one physical dimension interfere with other color attributes, which has been a deficiency of the existing color spaces, particularly prevalent for high-dynamic-range and wide-color-gamut displays. This paper describes how we set out to develop independent color scales for each attribute. Based on both psychophysical experiments and computational modeling, the surfaces/lines of equal brightness/saturation, as well as the boundaries between surface versus illumination color modes, have been characterized. Furthermore, the independent relations between those new scales have been quantitatively evaluated. Those results promise a new color representation that is more intuitive and efficient for color controls in displays.

Introduction

To quantify the perception of color rendered on a display, color space has been a useful geometric representation for both color difference and color appearance modeling that are heavily used in display metrology and processing. However, the ideal color space is still not yet successfully derived to be a uniform Euclidean space; thus the utilization of existing color spaces at best only provides an approximation to different levels of satisfaction. The difficulties of deriving such an ideal color space come from various fundamental and practical issues of how color perception works. This paper reviews those issues and in particular highlights our current efforts to understand the internal structure of color spaces and the feasibility of representing color as multiple independent scales, which promise a new color representation that is more intuitive and efficient for color controls in displays.

The main title is inspired by Wassily Kandinsky's classical book, *Point and Line to Plane*, where he elaborates in a kaleidoscopic way on the composability of geometric points and lines and their interactions on a painting plane. Here I primarily use the framework to think of color in a low dimensional way with points and lines in color spaces without expanding on the associations between color and form as well as other abstract concepts. The word "surface" is an intended pun that means both any surface structure in a color space and the screen surface, neither of which is necessarily flat, although the idiosyncrasies of different (flat or curved) display technologies that may impact color metrology and perception are not thoroughly considered.

The Quest For A Uniform Euclidean Space

A color space to a color scientist or a display engineer is like a map to a cartographer or a traveler. How to make/use this map reflects the understanding of color in general and the relations between different (anchor) points and between different color attributes as spatial dimensions. A comprehensive and historical review of different color spaces from 1-D to 3-D can be found in [1]. Among those color order systems, Munsell and Natural Colour System (NCS) are two representative examples that embody the relations either between hue, chroma, and lightness (as Munsell value), or between hue, blackness, and whiteness (or chromaticness as the residual). A set of more standardized color attributes, i.e., hue, lightness, brightness, saturation, chroma, and colorfulness, can be found on CIE's International Lighting Vocabulary (`https://cie.co.at/e-ilv`). For both unrelated colors and related colors, the three fundamental attributes of hue, brightness, and saturation can be arranged/visualized in a 3-D space, where the dimensionality sometimes alludes to the trichromatic nature of normal color vision.

To understand the intrinsic properties of the (ideal) color space, previous work suggested that color similarity is judged not with a Euclidean but a city-block metric [2]. More recently, Ennis and Zaidi used midpoints between color pairs to gauge the structure of color space and concluded that a less strict, in contrast with Euclidean, affine structure is valid, and suggested the underlying neural computations (affine geometry implies comparisons based on ratios) [3]. CIELAB as the most popular color space to date has a rectangular (L^*, a^*, b^*) or polar-cylindrical (L^*, C^*_{ab}, h_{ab}) structure [4]. The proposed uniform color space based on the color appearance model CAM16, i.e., CAM16-UCS [5], has a similar rectangular structure and assumes uniformity in a Euclidean space when calculating the color difference. In another study, based on the visual results on color pairs with large color differences, a hybrid metric (city-block for lightness and Euclidean for chromatic components in CIELAB) shows the best correlations [6]. An alternative of hyperbolic geometry was also suggested [7].

While the ideal uniform color space considering both appearance correlates and color difference modeling is still under development, its utilities have a lot of application demands in display science. From the perspective of metrology, color accuracy in display calibration and characterization can be better predicted with a color difference formula in such space, where the uniformity causes fewer biases across the color space, and more accurate appearance correlates provides more insights about the appearance shift directions. Another application is the quantification of color gamut volume, where currently CIELAB has been a reasonable option [8]. In addition to those involving the color space's local and global properties, gamut mapping is a more general case where color manipulations can find controls closer to the intentional mapping preservation and shifts [9].

Anchor Points in Color Spaces

Across the color space, there exist a few salient points, not only because they correspond to some prototypical color categories but also because they help shape the color space structure as important anchor points. The white point has been an essential component in both color appearance models and engineering specifications, both of which have been extensively investigated [10, 11]. Similarly, the black point can also affect the perceived image qualities [12]. While white and black can be associated with physical reflectances of 90% and 3%, respectively [13], for displays capable of rendering a high dynamic range of luminance, the (implicit) constraints in physically reflective objects might be violated, and thus a more general characterization of lightness scales is needed. The preferences and the corresponding dynamic range requirements for different appearance modes such as diffuse and emissive whites were suggested [14]. More recently, our work based on Ralph Evans's concept of brilliance can be useful for generalizing to all the chromaticities at different levels of luminance [15]. For a constant chromaticity, varying luminances have different perceived grayness levels that correspond to different appearance modes from black, reflective colors, to self-luminous colors. The zero grayness point, or more intuitively the glowing threshold, has a special location as the boundary condition between surface vs. illumination appearance modes [16]. Figure 1 shows the G_0 luminance results averaged from 12 observers for 16 different chromaticities, which were verified to be equally bright using a paired comparison experiment. Those G_0 points, therefore, have the same (or likely higher) lightness of L^* 100 as the diffuse white. More interestingly, those G_0 luminance results were found to be correlated with the MacAdam optimal colors, which provides convenient computational handles for all the chromaticities that were not tested in the G_0 psychophysical experiments. A set of MacAdam optimal luminances are plotted in Fig. 2 [17]. This equally bright surface was further used in our experiments.

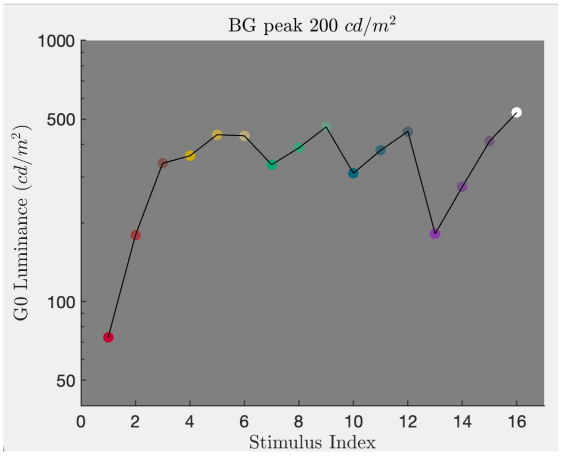

Figure 1. *G_0 luminances under 200 cd/m^2 peak background from individual observer's averaged result.*

Those points have their own (inner) possibilities (or tensions as how Kandinsky described the geometric points), meaning a fixed physical specification may not account for the variations in observers' individual differences and/or viewing conditions. The surprisingly large individual difference in our G_0 results might resemble the individual difference in estimating the illumination's chromaticity [18, 19], but in the luminance dimension instead. The changes across viewing conditions may correspond to different states of chromatic adaptation [10] and have implications in ambient light adaptive displays [20]. Other than the grayscale, chromatic anchor points such as unique hues (similarly the full colors in NCS system) and the Munsell principal hues [21] are also of significant importance in specifying the chromaticness and the hue composition (H). And the uniqueness of some hues has become a debatable issue [22]. While spectral matches between physically reflective and self-luminous cannot be distinguished, any spectral mismatches across the color space may lead to different magnitudes of observer disagreements [23].

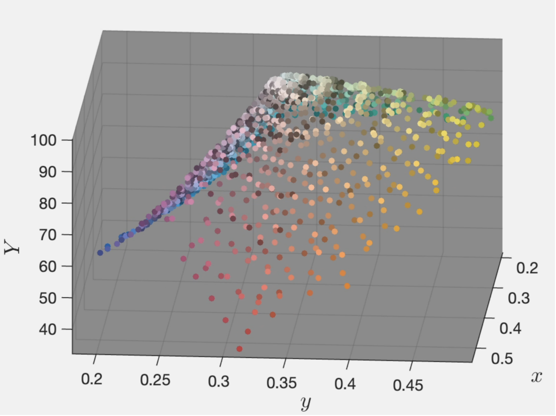

Figure 2. *The MacAdam optimal luminance for different chromaticities corresponding to the Munsell reflectances under D65 and 2-degree observer.*

Proposing Independent Color Scales

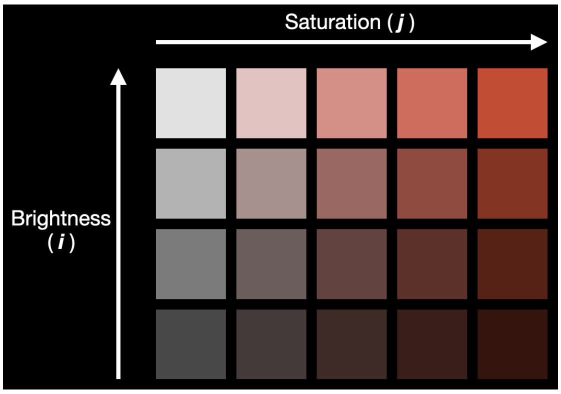

Figure 3. *The brilliance and saturation array derived from the partition scaling experiments. They served as the stimuli in the maximum likelihood conjoint measurement experiment for verifying their independence.*

Back to the question of why color space cannot be a 3-D uniform Euclidean space, a general answer would be color vision is still not fully understood and cannot be modeled with simple equations [24]. Instead, can color be deconstructed and represented as multiple independent scales? This question was first asked by Fairchild and Heckaman [25, 26], who provided a conceptual framework with basic implementations. Following this, the G_0 equally bright anchors plus a black point were used as references in a partition scaling experiment to derive uniform brightness scales for each chromaticity. Then, using the maximum-saturation hue and the neutral point, the uniform saturation scale was similarly derived as a function of excitation purity for a given hue [27]. The stimuli were adjusted along the equally bright surface previously mentioned. The saturation scales have been linearly scaled to different levels of brightness. The end results can be visualized in Fig. 3, where one dimension corresponds to brightness or brilliance following Evans's term and the other dimension corresponds to saturation. The underlying physical dimensions are (scaled G_0) luminance and purity, respectively.

The framework of independent scales assumes that constant chromaticity should remain constant saturation under a certain level of brightness (probably G_0), and that the scaled G_0 surfaces can keep brightness equal across chromaticities. So, the next question would be probing the interactions between dimensions. It has been widely known that colorimetric attributes are not independent as revealed by different color appearance phenomena, for example, Helmholtz-Kohlrausch (H-K) effect, Bezold-Brück effect and Abney effect [10, 28]. With a recent and reliable psychophysical approach, maximum likelihood conjoint measurement (MLCM) [29, 30], the interaction between CIELUV lightness and chroma metrics was evaluated [31]. However, the physical dimension L^* is expected to be deficient in incorporating the H-K effect. Following the MLCM, the interactions between the brilliance scale which automatically incorporates the H-K effect and the saturation scale which is closer to object properties than chroma were investigated [32]. One representative result of the Munsell 5R hue is presented in Fig. 4, where the perceptual saturation is plotted against luminance and purity, respectively. Purity linearly contributes to the saturation scale, which is consistent with the partition scaling results. On the other hand, luminance hardly contributes to perceptual saturation, which verifies that constant chromaticity has constant saturation. Similar results were found for perceptual brightness, with a relatively small amount of H-K effect, which in turn highlights the deficiency of luminance or L^*. In other words, when there were co-variations in both brightness and saturation in the stimulus array, the observers on average were able to focus on one attribute without much interference from the other dimension. The grid arrangement in Fig. 3 thus reflects the orthogonality of the two dimensions, both physically and perceptually. For display color modeling, navigation along one of those scales would only change the corresponding perceptual attribute without iterations and complex compensations on the others.

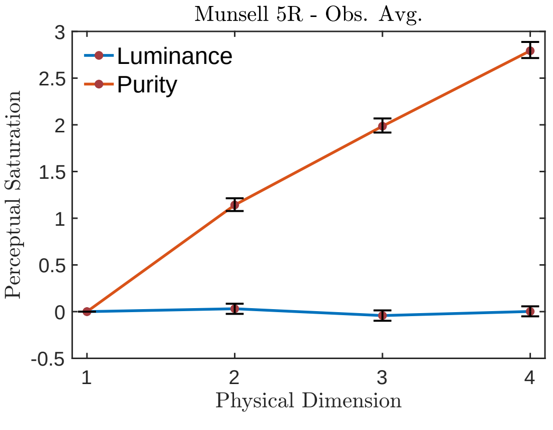

Figure 4. *The contributions to perceptual saturation from physical luminance (blue) and purity (red) for the Munsell 5R hue. The physical dimensions correspond to the rows and columns in Fig. 3. Error bars show 95% confidence intervals via bootstrap with 1000 times.*

More Than Three-dimensional

While a 3-D color space is usually adequate, for different viewing conditions, the absolute color attributes of brightness and colorfulness and the relative (more invariant from an object color perspective) color attributes of saturation, chroma, and hue can provide different kinds of information. As suggested in [25], four of the six are adequate (hue, saturation, brightness and lightness) with the other two easily derived. Nayatani and Sakai [33] well explained the distinction between attributes using a brightness-lightness (similarly colorfulness-chroma) inversion example, where depending on the adapting (il)luminance (not measured but inferred in an image scene) one image location can have low lightness but high brightness and vice versa. They pointed out this distinction would be useful especially in high dynamic range (HDR) imaging. Using MDS, a 3-D structure was identified by comparing neutral samples with cast and attached shadows [34] (2-D for cast shadow only, which may be linked to brightness and lightness). Similarly, material and lighting color can be simultaneously perceived [35]. Those “new” dimensions may not necessarily require additional primaries than RGB. Recently there were novel displays set up in the lab to independently simulate various photoreceptors for vision research [36, 37]. Those aspects may further complicate the quest for ideal color space and the representation of independent scales.

Discussion and Conclusion

In this paper, different issues in developing a uniform Euclidean color space are reviewed. Instead, a new representation of independent color scales including hue, brilliance, and saturation was proposed, and the preliminary results of those scales are presented. The surfaces/lines of equal brightness/saturation, as well as the boundaries between surface vs. illumination color modes (G_0), have been characterized. Furthermore, the independent relations between brightness and saturation scales have been quantitatively verified. The approaches adopted in this work include how to take advantage of the existing colorimetric space to start with (luminance plus chromaticity), deriving uniform color scales along those physical dimensions, and evaluating the independent relations between those scales, via psychophysics including partition scaling and the method of MLCM.

The concept of color space has been ingrained among color scientists and display engineers, probably because Euclidean geometry [38] is a useful and sensible way to host all the colors and visualize their relations, such as CIELAB. This work, representing color as multiple independent scales, does not necessarily aim to develop a new color space. Instead, we set out to characterize the complete and better set of attributes to describe color and to improve and evaluate the independence between those attributes. Those results promise more intuitive and efficient color controls in displays as independent knobs. Practically, the G_0 results or the MacAdam optimal colors, can be implemented as a 2D look-up table for maintaining brightness when changing the chromaticity for different saturation and hue. And changing brightness while maintaining saturation should keep the chromaticity constant for constant saturation as shadow series, which is a convenient connection to colorimetry.

There remain further investigations on other relations such as hue versus saturation (the Abney effect), using potentially better metrics than dominant wavelength and excitation purity, as well as on how to calculate the total color difference across those attributes, i.e., the isotropy across dimensions if it exists [39, 1]. Kandinsky once said, “each color lives by its mysterious life.” While the mysteriousness might not be homogeneous across the color space, representing color as independent scales can help demystify and provide more useful tools for display engineering.

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