https://doi.org/10.2352/issn.2694-118X.2020.LIM-20

Texture Stationarity Evaluation with Local Wavelet Spectrum

Michele Conni; [1]Barbieri Electronic; Brixen, Italy; [2]Department of Computer Science, Norwegian University of Science and Technology (NTNU); Gjøvik, Norway
Hilda Deborah; [2]Department of Computer Science, Norwegian University of Science and Technology (NTNU); Gjøvik, Norway

Abstract

In texture analysis, stationarity is a fundamental property. There are various ways to evaluate if a texture image is stationary or not. One of the most recent and effective of these is a standard test based on non-decimated stationary wavelet transform. This method permits to evaluate how stationary is an image depending on the scale considered. We propose to use this feature to characterize an image and we discuss the implication of such approach.

Introduction

In texture analysis, a *stationary texture image* is defined as an image containing only one type of texture [1]. From this definition, we can see that the *stationarity* of texture has two connotations. The first one is related to perception, where an image is said to have one type of texture when it is perceived to be homogeneous. To apply this definition to texture analysis is, however, challenging. To this day, the boundaries of what a human perceive as homogeneous texture are extremely fuzzy as it depends on a vast variety of factors [2]. On the other hand, stationarity has been precisely defined from a mathematical point of view. A statistically stationary texture image has the same local properties everywhere in it [1].

The concept of stationary processes is widely used in various fields, and it is mainly related to time series. In texture analysis, stationarity is a fundamental assumption for the application of global texture models, i.e. Markov random fields, the Fourier transform and the autocorrelation function. However, these techniques face challenges when applied to non-stationary images. Ideally, this problem can be easily solved with a segmentation algorithm, which autonomously partitions a digital image into multiple homogeneous sub-regions. As discussed in [1], in order to decide if the segmentation is necessary, we would need to know if the image is stationary.

Numerous stationarity evaluation methods have been proposed. One of the most promising ones is based on locally stationary wavelet fields [3] [4], addressing the stationarity at multiple scales. Nonetheless, there, the average of the wavelet field over multiple scales is eventually used, without discussing their relationship with the nature of the texture. Being the dependence of texture parameters on the scale a fundamental problem for texture analysis, this coincides to neglecting elements which could be fundamental for human appearance modeling. In fact, different scales could influence in distinct ways the perception of the homogeneity of a textured image, i.e., the *perceived* stationarity.

Taylor, *et al.* [4] proposed a wavelet-based stationarity test, developed strictly in the mathematical sense. Even though its nature is strongly multiscale, as discussed in the paper, it does not explicitly provide stationarity measures for individual scales. However, it is known that the human visual system takes both local and global spatial information [5]. With the end goal of linking mathematical stationarity to its perceptual context, in this pilot study, we modify the test in order to allow the numerical analysis of individual scales. We start by discussing in detail the foundations of the texture stationarity test. Then, we describe its proposed modification, in which we assess stationarity for each scale of an image instead of providing a single global index. We then apply the proposed test to a subset of the Amsterdam Library of Textures (ALOT) texture database [6].

Definitions
Stationarity

A discrete time series $X_t \in \mathbb{Z}$ is defined as *strictly stationary* if the joint distribution of $(X_{t_1}, \ldots, X_{t_n})$ is identical to $(X_{t_1+\tau}, \ldots, X_{t_n+\tau})$ $\forall t_i, n, \tau \in \mathbb{N}$. However, this definition is usually too strict for applications, therefore in signal processing the concept of *weak stationarity* is generally used, instead: a time series is weak-sense (or second-order) stationary if it has constant mean and covariance between two time coordinates t_1 and t_2 function only of their difference ($K_{XX}(t_1,t_2) = K_{XX}(\tau, 0), \tau = t_2 - t_1, \forall t_1, t_2 \in \mathbb{N}$). These formalizations mean that the joint probability distribution generating the series is constant in time. They both can be easily expanded to image processing by introducing the bidimensional pixel coordinates (u, v). This paper focuses on the weak-sense stationarity, since it is a common assumption in many texture analysis methods.

In particular, we want to verify if an image is stationary or not. In order to achieve that, various stationarity testing techniques have been recently developed. Some of them are not suitable to our purposes, e.g., one was introduced [7] and tested [8] aiming to detect acoustic sources in shallow water. To do that, the fact that the matrix form of the second order spatial cumulant spectrum is diagonal in case of weak-stationarity was used, i.e. the Fourier transform of the cross-correlation function of the. The problem with this method is that it requires multiple realizations of the same process, and it is therefore not applicable to single texture images. Different but analogous issues also appear in other proposed methods [9], [10], [11].

Locally stationary wavelet fields

An approach that satisfies the requirements of our application is the one introduced in [4]. It analyses the stationarity of textured images through the wavelet transform, which has been proven to be more appropriate than Fourier-based ones in the study of potentially non-stationary signals [3]. Such a performance is justified by the localization and multiscale nature of the wavelet functions [12].

First of all, a wavelet $\psi(x)$ is a compact support function with oscillatory characteristics, $x \in \mathbb{R}$. By shifting and scaling $\psi(x)$, it is possible to generate a complete functional basis $\{\psi_{j,k}, \phi_k\}$, which can then be used to decompose any function $f(x) \in L^2(\mathbb{R})$; in this case, $\psi(x)$ is referred to as *mother wavelet*, and j and k are the scaling and shifting indices, respectively. ϕ_ks are functions generated by scaling the *father wavelet* $\phi(x)$, which covers the region of frequency space around the origin. Hence, we can define the *wavelet trans-*

form of $f(x)$ as $f(x) = \sum_k c_k\phi_k(x) + \sum_{j\leq J}\sum_k d_{j,k}\psi_{j,k}(x), k \in \mathbb{Z}$, where $c_k = \int_{-\infty}^{\infty}\phi_k^*(x)f(x)dx$, $d_{j,k} = \int_{-\infty}^{\infty}\psi_{j,k}^*(x)f(x)dx$ and J is the total number of scales taken into account. According to Ref. [13], this process can be then discretized by starting from a mother wavelet ψ with the low-/high-pass filter pair $\{h_k, g_k\}$ associated. Once defined N_h as $\#\{h_k\} \neq 0$ and $L_j = (2^j-1)(N_h-1)+1$, a discrete wavelet at scale $j \in \mathbb{Z}^+$ is a vector $\psi_j = (\psi_{j,0},\ldots,\psi_{j,L_j-1})$, where $\psi_{-1,n} = g_n$ and $\psi_{j-1,n} = \sum_k h_{n-2k}\psi_{j,k}$, $\forall n \in [0,\ldots,L_{j-1}-1]$. It is possible to expand this framework to a 2D description: given $\mathbf{k} = (k_1,k_2)$, with $k_{1,2} \in \mathbb{Z}$, a two-dimensional discrete wavelet filter at scale j and direction l can be defined as the $L_j \times L_j$ matrix:

$$\psi_j^l = \begin{bmatrix} \psi_{j,(0,0)}^l & \cdots & \psi_{j,(0,L_j-1)}^l \\ \vdots & \ddots & \vdots \\ \psi_{j,(L_j-1,0)}^l & \cdots & \psi_{j,(L_j-1,L_j-1)}^l \end{bmatrix}. \tag{1}$$

The direction index l is used to refer to horizontal h, vertical v and diagonal d 2D fundamental wavelets, which can be defined as $\psi_{j,\mathbf{k}}^h = \phi_{j,k_1}\psi_{j,k_2}$, $\psi_{j,\mathbf{k}}^v = \psi_{j,k_1}\phi_{j,k_2}$ and $\psi_{j,\mathbf{k}}^d = \psi_{j,k_1}\psi_{j,k_2}$.

The 2D discrete wavelet matrices so defined can be used to calculate the wavelet coefficients of an image by applying them on the picture in various pixel positions. While for a compact description of the image, e.g., in JPEG2000 compression [14], these positions are chosen so that the minimum number of filters possible is used, in our case the most complete description can be achieved by applying each wavelet on every pixel of the image, thus calculating the so-called *stationary wavelet transform*. In [3], this bidimensional wavelet basis was used to define the a random field modeling framework, referred to as *locally stationary two-dimensional wavelet fields* (LS2W). An LS2W process can be defined as:

$$X_{\mathbf{r}} = \sum_l \sum_{j=1}^{\infty} \sum_{\mathbf{u}} w_{j,\mathbf{u}}^l \psi_{j,\mathbf{u}}^l(\mathbf{r})\xi_{j,\mathbf{u}}^l \,, \tag{2}$$

where l indicates the direction of the wavelet, j its scale and $\mathbf{u}$ its shifting. $\{w_{j,\mathbf{u}}^l\}$ is a set of constant wavelet coefficients, $\{\psi_{j,\mathbf{u}}^l(\mathbf{r})\}$ of 2D discrete non-decimated wavelets and $\{\xi_{j,\mathbf{u}}^l\}$ is a zero-mean random orthonormal increment sequence. It is therefore possible to interpret the definition of $X_{\mathbf{r}}$ as a wavelet transform with coefficients $w_{j,\mathbf{u}}^l\xi_{j,\mathbf{u}}^l$, where the ξs represent the stochastic component of the field. $\mathbf{u},\mathbf{r} \in [0,\ldots,R] \times [0,\ldots,S]$ are coordinates of the generated $R \times S$ image. From this, it is possible to define, in analogy with the Power Spectral Density of a discrete Fourier transform, an evolutionary wavelet spectrum (EWS) (sometimes also referred to as local wavelet spectrum (LWS)) $S_j^l(\mathbf{u})$ at scale j and with direction l.

To apply such a model to an existing image means to evaluate $S_j^l(\mathbf{u})$. In [3], it was demonstrated that the local wavelet periodogram (LWP):

$$\mathbf{I}(\mathbf{u}) = \{I_{j,\mathbf{u}}^l\} = \{|d_{j,\mathbf{u}}^l|^2\} = \left\{\left(\sum_{\mathbf{r}} X_{\mathbf{r}}\psi_{j,\mathbf{u}}^l(\mathbf{r})\right)^2\right\}, \tag{3}$$

which, in fact, is a biased estimator of the LWS. The correction of this bias is performed by applying a correction matrix A, which, as widely discussed in [15], is a $J \times J$ array obtained from the autocorrelations of the wavelet functions used. The corresponding calculation is $\hat{S}(\mathbf{u}) = A^{-1} \cdot \mathbf{I}(\mathbf{u})$.

Wavelet-based stationarity tests

In the following section, we first describe in detail the stationarity test introduced in [4], then we propose a novel variation of this, which gives additional insights on the characteristics of the image under study.

The standard wavelet stationarity test

The model described in (2) attributes to the random sequence $\xi_{j,\mathbf{u}}^l$ the spatial variation of the image. Therefore, a picture is stationary in this framework if and only if its EWS S is constant on the spatial coordinate $\mathbf{u}$, $\forall l \in \{h,v,d\} \wedge \forall j \in \mathbb{Z}^+$. On this basis, Ref. [4] built a novel stationarity test:

H_0: $S_j^l(\mathbf{u})$ is a constant function of $\mathbf{u}$ $\forall l \in \{h,v,d\} \wedge \forall j \in \mathbb{Z}^+$

H_1: $S_j^l(\mathbf{u})$ is not a constant function of $\mathbf{u}$ $\forall l \in \{h,v,d\} \wedge \forall j \in \mathbb{Z}^+$.

To implement this, a test statistic based on the average variation of the complete EWS $\mathbf{S} = \{S_j^l(\mathbf{u})\}$ in (4) was defined.

$$T_{ave}\{\mathbf{S}\} = (3J)^{-1}\sum_l \sum_{j=1}^{J} \mathrm{var}_{\mathbf{u}}(\hat{S}(\mathbf{u})) \tag{4}$$

Given the lack of knowledge on the original distribution of $T_{ave}\{\mathbf{S}\}$, one can perform a parametric bootstrap procedure, based on the evaluated EWS $\mathbf{S}$ and on the assumed model of statistical innovations ξ. The p-value of the test is eventually obtained by looping over this process B times, and then calculating $p = \frac{1+\#\{T_{ave}^{obs} \leq T_{ave}^{(i)}\}}{B+1}$. Here, B is the number of loops performed by the bootstrap, which is arbitrary, while *obs* and (i) indicate the sample and ith bootstrap synthesis, respectively.

The p-value p is representative of the distribution of the test statistic under the null hypothesis H_0. In its theoretical significance, p represents the probability obtaining the observed results in the test, assuming that the null hypothesis is correct In this case, this is achieved by comparing the observed T_{ave}^{obs} with the corresponding $T_{ave}^{(i)}$s obtained by assuming a stationary spectral structure based on the observed process. Hence, as noted in [4], the p-value can also be interpreted as a measure of how non-stationary the observed process is.

The p-values' vector

The previously described test has proven to be a powerful tool to assess the stationarity of an image [4] [16]. Although this method is based on stationary wavelets at multiple scales, none of the references has employed it to assess the variation of stationarity of textured scenes with the scale. To this end, it is useful to introduce a parameter which evaluates the departure from constancy at each scale:

$$T\{\mathbf{S}\} = \{T_j^l\} = \mathrm{var}_{\mathbf{u}}(\hat{S}(\mathbf{u})). \tag{5}$$

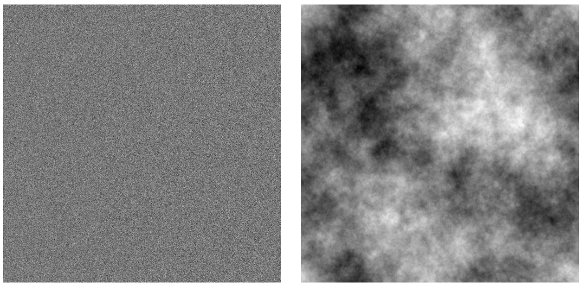
Note that $\hat{S}(\mathbf{u})$ is a matrix that contains the local stationary wavelet coefficients at each scale j and direction l. This has been used in [3] for classification purposes, and it was the base of a second multi-hypothesis bootstrap test proposed in [16]. However, the properties of such a function have never been discussed, and it has never been applied to traditional texture databases. In particular, from $T\{\mathbf{S}\}$ is possible to derive a matrix $\mathbf{p}$ composed by the p-values at each scale and direction: $\mathbf{p} = \{p_j^l\}$, where $p_j^l = \frac{1+\#\{T_j^{l\,obs} \leq T_j^{l\,(i)}\}}{B+1}$. $\mathbf{p}$ can be used as a estimator for stationarity at each dyadic distance 2^j $\forall j \in \mathbb{Z}^+$.

Results

To test the two methods described in the previous section, we have chosen a set of images, stationary and not, for which we derived both p and $\mathbf{p}$. The whole work was developed in Python, building on a dedicated library PyWavelet [17]. The next subsections discuss the results obtained with different typologies of grayscale images. In the final work, we also intend to assess the results of these tests for artificial non-stationary images.

Stationary artificial images

As a first performance evaluation, we want to check the test against theoretical images, for which we know what to expect. The chosen two are, first, an uncorrelated Gaussian white noise process generated with a simple normal distribution $N(0, 1)$ (Fig. 1a) and, second, a 2D correlated and scale-invariant Gaussian random field (Fig. 1b). For the latter, we used a Python implementation found in [18], for which the correlations are due to a scale-free spectrum $P(k) \sim 1/|k|^{\beta/2}$ of the image (with $\beta = 3$). Both images had a size of 512×512.



(a) *Gaussian white noise* (b) *Gaussian random field*

Figure 1: Two stationary processes to test p and $\mathbf{p}$.

We determined the mean stationarity p-value of the images with the test described in the previous section. We used $b_{sim} = 100$ bootstrap simulations. Following [4], it is reasonable to fix the significance level at 5%. The resulting p for Fig. 1a is 0.34, while for Fig. 1b is 0.99. This difference between results can be connected to the lack of correlation in the first process, which affects the second order stationarity of the single realization.

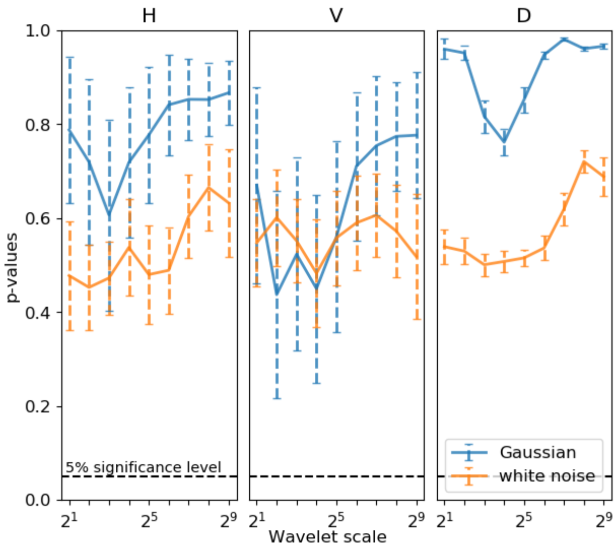
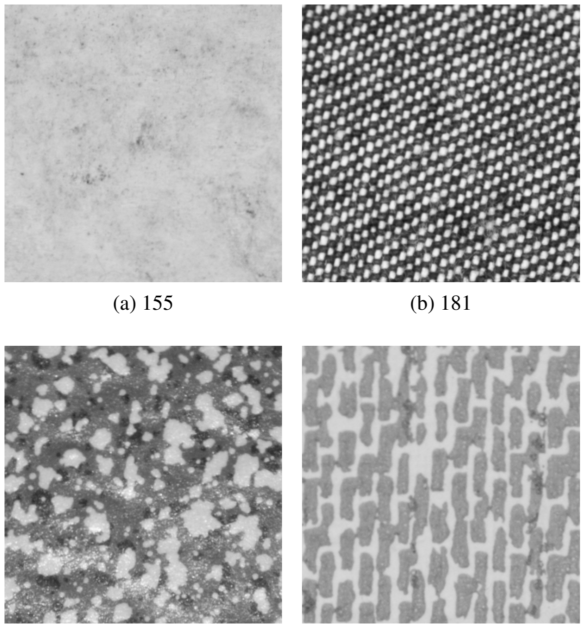


Figure 2: p-value of the stationary test for images in Fig. 1, shown for all directions l: horizontal (H), vertical (V), and diagonal (D). The axis of abscissa represents finer to coarser dyadic scales.



(a) 155 (b) 181

(c) 182 (d) 185

Figure 3: Subset of images from the Amsterdam Library of Textures (ALOT) texture database.

This conclusions are consistent with what can be obtained with the second approach. The $\mathbf{p}$ vectors recovered with this are shown in Fig. 2. The x axis of the plot accounts for the scale j of the p_j, and it goes from finer to coarser. From this figure, it is clear that the p_js of the white noise process are generally smaller than those of the Gaussian random field one. In the latter case, the high number of $p_j = 1$ ensures a strong stationarity at multiple scales, as can be expected by the same scale-invariant property of the model used.

Real-world grayscale texture images

Finally, we applied the two methods to the grayscale images shown in 3, chosen to represent a wide range of textures from the Amsterdam Library of Textures (ALOT) [6].

The results of the standard stationarity test are reported in Table 1. It can be seen that Fig. 3a is the only image classified as non-stationary. These results can be clarified by the multi-scale analysis, whose output is reported in Fig. 4: for example, Fig. 3a is stationary at fine scales, but p is strongly influenced by the lack of stationarity at larger scales. The direction of analysis l can also give additional insight on the nature of the texture, such as for Fig. 3d. Results from more images can be found at `https://gist.github.com/micheleconni/abe79a8b90357559563d7bc8e4a4fb9c`.

Table 1: p-values resulting from the application of the bootstrap stationarity test to the ALOT images.

	155	181	182	185	193	204	212	241
p	0.01	1.00	0.79	1.00	0.78	0.96	0.52	0.10

Conclusions

The results obtained confirm that it is suitable to use the p value introduced in [4] to characterize the statistical attributes of a textured image. Furthermore, it is clear that the $\mathbf{p}$ vector can be an interesting tool to fathom these attributes at different scales.

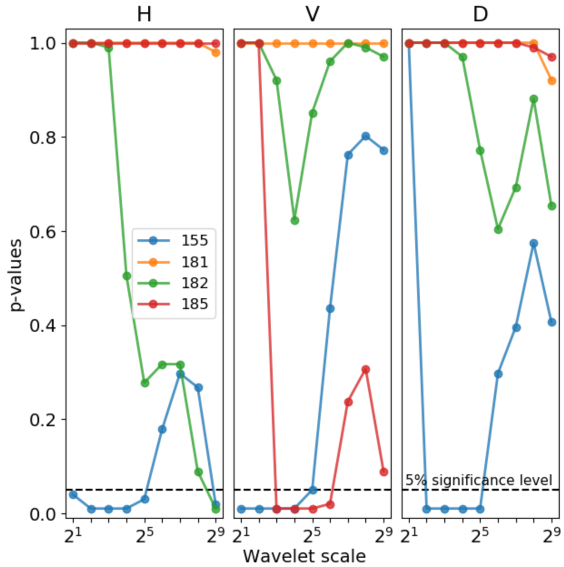


Figure 4: Results of the **p** vector analysis for Fig. 3. The vector was obtained for dyadic scales, which are shown in the abscissa, for the horizontal, the vertical and the diagonal direction, which in figure are indicated with the H, V and D label, respectively.

As discussed, for now this tool is limited to a dyadic scaling, but it would be interesting to develop a corresponding continuous function, so to have a more precise idea of how this value varies through different scales. It would be also interesting to examine the classification capabilities of this system, since that is a usual benchmark for novel texture features.

An extension of this approach can also be introduced and developed for the multivariate image domain. In this regard, we note that one has been developed based on the stationary wavelet transform [16], taking on an integrative multi-channel color texture analysis approach [19]. Therefore, it gets computationally very expensive when the number of channels of the image increase, making it unsuitable spectral imaging applications.

Another possible application is the development of a texture analysis routine based on **p**. Noting that various analysis techniques are better for non-stationary texture or vice versa [1], it would therefore be possible to employ different techniques at different scales, depending on the structure of **p**. Finally, since stationarity is fundamental mathematical property, it would be interesting to consider its relationship with human perception, by matching **p** with the results of proper psycho-physical tests.

Acknowledgments

This work is partially supported by FRIPRO FRINATEK Metrological texture analysis for hyperspectral images (projectnr. 274881) funded by the Research Council of Norway.

References

[1] M. Petrou and P. G. Sevilla, *Image processing: dealing with texture*. Wiley, 2006.

[2] N. Bhushan, A. R. Rao, and G. L. Lohse, "The texture lexicon: Understanding the categorization of visual texture terms and their relationship to texture images," *Cognitive Science*, vol. 21, no. 2, pp. 219–246, 1997.

[3] I. A. Eckley, G. P. Nason, and R. L. Treloar, "Locally stationary wavelet fields with application to the modelling and analysis of image texture," *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, vol. 59, no. 4, pp. 595–616, 2010.

[4] S. L. Taylor, I. A. Eckley, and M. A. Nunes, "A test of stationarity for textured images," *Technometrics*, vol. 56, no. 3, pp. 291–301, 2014.

[5] D. J. Field, "Wavelets, vision and the statistics of natural scenes," *Philosophical Transactions of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, vol. 357, no. 1760, pp. 2527–2542, 1999.

[6] G. J. Burghouts and J.-M. Geusebroek, "Material-specific adaptation of color invariant features," *Pattern Recognition Letters*, vol. 30, no. 3, pp. 306–313, 2009.

[7] A. Ephraty, J. Tabrikian, and H. Messer, "A test for spatial stationarity and applications," in *Proceedings of 8th Workshop on Statistical Signal and Array Processing*, pp. 412–415, IEEE, 1996.

[8] A. Ephraty, J. Tabrikian, and H. Messer, "Underwater source detection using a spatial stationarity test," *The Journal of the Acoustical Society of America*, vol. 109, no. 3, pp. 1053–1063, 2001.

[9] R. Blanc, J.-P. Da Costa, Y. Stitou, P. Baylou, and C. Germain, "Assessment of texture stationarity using the asymptotic behavior of the empirical mean and variance," *IEEE Transactions on Image Processing*, vol. 17, no. 9, pp. 1481–1490, 2008.

[10] S. Bose and A. O. Steinhardt, "Invariant tests for spatial stationarity using covariance structure," *IEEE transactions on signal processing*, vol. 44, no. 6, pp. 1523–1533, 1996.

[11] M. Fuentes, "A formal test for nonstationarity of spatial stochastic processes," *Journal of Multivariate Analysis*, vol. 96, no. 1, pp. 30–54, 2005.

[12] G. Nason, *Wavelet methods in statistics with R*. Springer Science & Business Media, 2010.

[13] G. P. Nason, R. Von Sachs, and G. Kroisandt, "Wavelet processes and adaptive estimation of the evolutionary wavelet spectrum," *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, vol. 62, no. 2, pp. 271–292, 2000.

[14] D. Taubman and M. Marcellin, *JPEG2000 image compression fundamentals, standards and practice: image compression fundamentals, standards and practice*, vol. 642. Springer Science & Business Media, 2012.

[15] I. A. Eckley and G. P. Nason, "Efficient computation of the discrete autocorrelation wavelet inner product matrix," *Statistics and Computing*, vol. 15, no. 2, pp. 83–92, 2005.

[16] S. L. Taylor, I. A. Eckley, and M. A. Nunes, "Multivariate locally stationary 2D wavelet processes with application to colour texture analysis," *Statistics and Computing*, vol. 27, no. 4, pp. 1129–1143, 2017.

[17] G. R. Lee, R. Gommers, F. Wasilewski, K. Wohlfahrt, and A. O'Leary, "PyWavelets: A Python package for wavelet analysis," *Journal of Open Source Software*, vol. 4, no. 36, p. 1237, 2019.

[18] B. Sciolla, "Generator of 2D gaussian random fields," July 2017.

[19] C. Palm, "Color texture classification by integrative co-occurrence matrices," *Pattern recognition*, vol. 37, no. 5, pp. 965–976, 2004.

Author Biography

Michele Conni received his BS and MS in engineering physics from the Polytechnic University of Milan (2015), with specialization in Optics and Photonics. He is currently studying for a PhD in computer science at the Norwegian University of Science and Technology (NTNU), in collaboration with Barbieri Electronic, where he works in the research and development group.

Hilda Deborah received her BSc in computer science from University of Indonesia (2010), MSc from Erasmus Mundus Color in Informatics and Media Technology (2013), and PhD in computer science from NTNU and University of Poitiers (2016). She is currently Marie Curie Postdoctoral Fellow at NTNU, with mobility to University of Iceland. Her research focuses on the development of metrological framework for hyperspectral images.