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Beyond Visual Aesthetics: The Role of Fractal-scaling Characteristics across the Senses†

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Abstract. *The investigation of aesthetics has primarily been conducted within the visual domain. This is not a surprise, as aesthetics has largely been associated with the perception and appreciation of visual media, such as traditional artworks, photography, and architecture. However, one doesn't need to look far to realize that aesthetics extends beyond the visual domain. Media such as film and music introduce a unique and equally rich temporally changing visual and auditory experience. Product design, ranging from furniture to clothing, strongly depends on pleasant tactile evaluations. Studies involving the perception of 1/f statistics in vision have been particularly consistent in demonstrating a preference for a $1/f$ structure resembling that of natural scenes, as well as systematic individual differences across a variety of visual objects. Interestingly, comparable findings have also been reached in the auditory and tactile domains. In this review, we discuss some of the current literature on the perception of $1/f$ statistics across the contexts of different sensory modalities.* © *2022 Society for Imaging Science and Technology.*
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1. NATURAL SCENE STATISTICS, FRACTAL-SCALING, AND AESTHETICS IN VISION

From the beginning, the study of aesthetics has often focused on the identification and definition of universal attributes. For example, attributes like symmetry, homogeneous texture, and certain spatial proportions are considered markers of facial attractiveness [41, 55]. Other features such as balance, complexity, and contrast have been put forward as determinants of beauty and preference in art [6, 16, 18, 23]. However, it is important to note that empirical aesthetics is not restricted to appreciation of artworks and is used more generally to refer to the attributes associated with sensory appeal and preference across a wide range of natural and synthetic objects.

Furthermore, our perception, representation, and interaction with the world is inherently multimodal and aesthetic experience is no exception. Yet, even in modern research, aesthetic perception in different modalities is studied in isolation from one another and still persists with a predominant focus on vision. Previously, we have argued for the close coupling between sensory tuning to the multi-scale and scale-invariant, properties of natural images, and visual preference [60–63]; here, we argue for an extension of this approach to preference in other sensory modalities. Specifically, we focus on the scale-dependent regularities in the statistical properties of visual images, three-dimensional surfaces, and tonal sequences to explore scaling-dependent preferences in visual, tactile, and auditory domains.

1.1 *Measuring Scale-Invariance: $1/f$ Fourier Spectrum and Fractal Dimension*

Natural scenes, even when displaying widely different environments and superficial different visual subjects, share a common statistical regularity — a distance-dependent degree of variations in their spatial structure. Namely, across natural scenes, the nearby regions are considerably more similar in their spatial properties such as luminance, chromaticity, orientation, and texture, compared to more distant regions. These regularities have been discussed in terms of scale-invariance and/or self-similarity and can be represented by the two different, but related, scaling measures: the Fourier amplitude spectrum and fractal dimension [9, 26].

Perhaps the most commonly used method of representing the distance-dependent variations in the intensity of individual points in natural scenes is through the properties of their spatial frequency amplitude spectra [13, 20, 57, 69], as illustrated in Figure 1. When an image of a natural scene is broken down into their component spatial frequencies (f), analysis of amplitude (i.e., average differences in pixel intensity) as a function of spatial frequency reveals a highly consistent inverse relationship between amplitude and spatial frequency, known as the $1/f^{\alpha}$ structure. The falloff of the amplitude spectra provides a simple snapshot of the relative differences in amplitudes across frequencies and is denoted by a value known as the amplitude spectrum slope (α). Early studies by Field [20] and Burton and Moorhead [13] found that the amplitude spectrum slope across a small set of a variety of natural scenes, averaged toward a value of 1. However, subsequent studies featuring larger samples of images elucidated a wider range of possible α values, with averages typically falling between 0.9 and 1.2 [21, 58, 69, 74]. Despite this variability, it is important to note that given the diversity of natural scene images, the majority share a highly regular statistical structure

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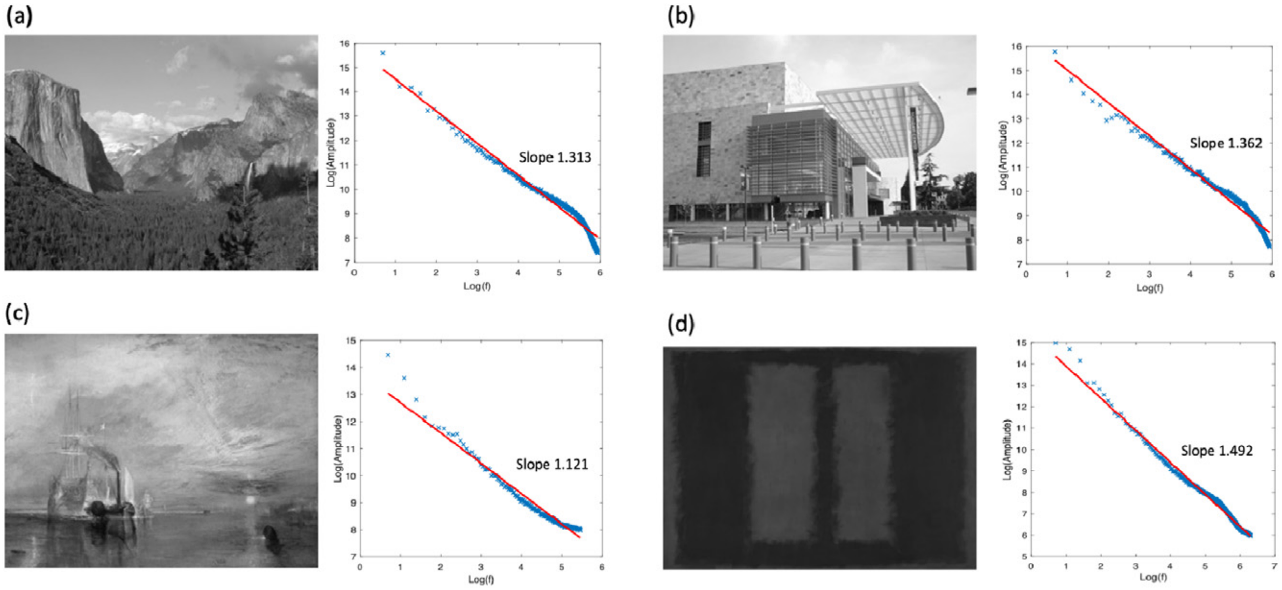

Figure 1. The Fourier amplitude spectra for (a) a natural scene image, (b) an urban scene image, (c) representational art (Willian Turner, The Fighting Temeraire, 1839), and (d) abstract art (Mark Rothko, Black on Maroon, 1959). The gradient of the line of best fit (in red) is the image's amplitude spectrum slope (α).

that comprises a relatively small proportion of all possible amplitude spectrum slope values.

It is interesting that these statistical regularities extend to a wide range of images (Fig. 1), including those of urban scenes and art images [24, 25, 43, 45, 53, 54]. Graham and Field [24] analyzed a sample of 124 paintings that featured both representational and non-representational works across a variety of cultures, subjects, periods, and artistic movements, and found an average amplitude spectrum slope of 1.23. This is not to say that all artworks, regardless of content, will approximate this slope; for example, abstract art was found to have a slight, but significantly shallower mean slope of 1.13 compared to landscapes and portraits [25]. However, like natural scenes, the range in average slope values across artworks shares very similar boundaries and overall exhibits a high degree of statistical regularity.

The scale invariance of natural scenes can also be captured by a geometric scaling parameter known as the fractal dimension (D), which focuses on the boundary edge between the paint-filled regions and empty regions in an image. A well-known method to quantify fractal dimension is the box-counting technique (Figure 2), which performs the scaling examination by covering an image with a mesh of identical squares ("boxes") of varying side lengths (L). The box-counting technique counts the number of squares, N, that contain part of the boundary edge with the count repeated for increasingly small squares within the mesh. Reducing the box size (i.e., smaller values of L) is equivalent to examining the image at finer spatial frequencies and N assesses the amount of space containing the pattern boundaries at these spatial scales. Even if the image content is represented as a surface in a three-dimensional volume (with the intensity axis as a third dimension), or in the case of three-dimensional objects, the same procedure can be performed by dividing the three-dimensional volume into progressively smaller three-dimensional cubes [40, 44, 65]. Like the Fourier amplitude spectrum slope analysis, the scale-invariance with the box counting method appears through the power law relationship $N \sim (1/L)^D$, by plotting $\log N$ as a function of $\log(1/L)$. The fractal dimension is the exponent D and is inversely related to the $1/f$ amplitude spectrum slope: higher alpha value is equivalent to low D value and vice versa [9, 24, 61].

The images in Figure 3 illustrate the relationship between variations in amplitude spectra of synthetic filtered noise images and fractal dimension more explicitly. The top row shows examples of synthetic filtered noise images with parametric variations in their amplitude spectrum slope α ranging from 0.75 (top left image) to 2.25 (top right image). The amplitude spectrum slopes for all the images in the top row are indicted at the bottom (α) alongside with the corresponding fractal dimension values (D) for the same images. It is important to emphasize that fractal dimension calculations are always performed by considering the degree of spatial variations along the edges of binarized, black and white regions in an image. Thus, to apply a box-counting procedure on the synthetic images varying in their amplitude spectra characteristics, as illustrated in the top row in Fig. 3, these images are first thresholded with respect to their mean luminance. During the thresholding procedure, all pixels with a value higher than the mean luminance are assigned as white and all pixels with values below the mean are assigned as black, resulting in images shown in the second row in Fig. 3. Edge-only image variations are generated by extracting the edges from the thresholded black and white images as illustrated in the third row in Fig. 3. While the

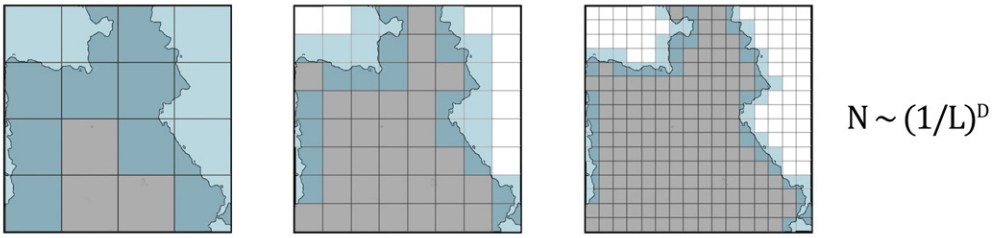

Figure 2. Demonstration of the box counting technique at three different values of L. As L decreases from left to right, the number of boxes (N) needed to measure the length of a boundary edge increases following a power law relationship defined by D.

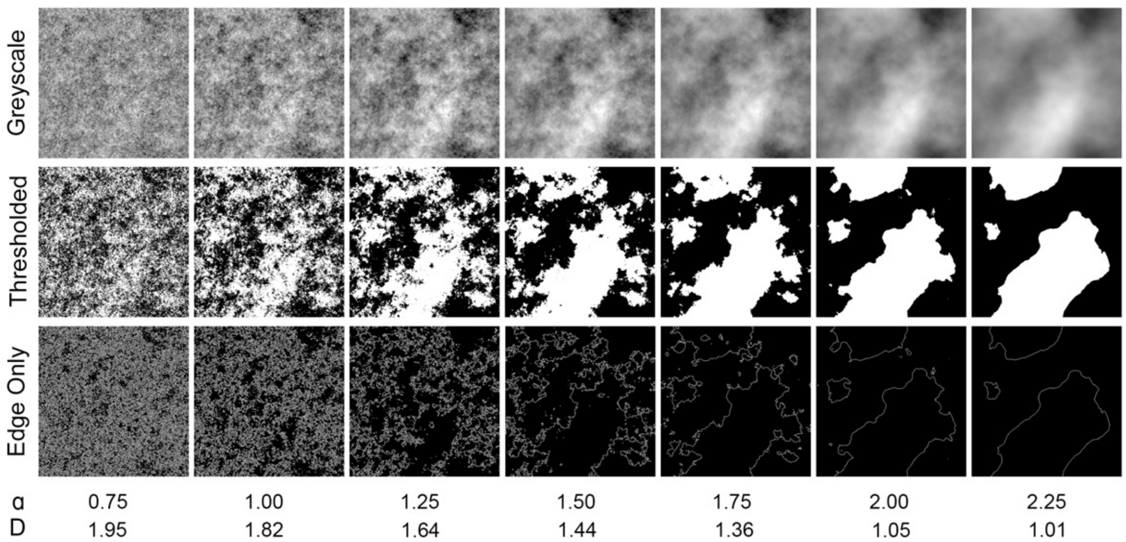

Figure 3. Examples of synthetic, computer-generated $1/f$ fractal noise images. The first row shows the the original $1/f$ grayscale images, created to have falloffs (amplitude spectrum slopes) ranging from 0.75 to 2.25 in increments of 0.25. The middle row shows the binarized (or ''thresholded'') versions of the same $1/f$ images. The third row depicts the ''edge-only'' variations. The input amplitude spectrum slope used to create the original grayscale images are shown in the row labeled ''α,'' and the approximate fractal dimensions of each column are shown in the row labeled ''D.''

thresholding and edge extraction transformation procedures alter the measured photometric and amplitude spectrum slope values of the corresponding derived images, their geometrical, fractal-scaling properties remain essentially identical.

In summary, both the slope of the amplitude spectrum and the fractal dimension are scaling measures that quantify the relationship between the coarse and fine spatial detail in an image, depending either on the contrast amplitude or density of fine spatial detail in an image. While both can be used to refer to the scale-invariant properties of natural images (either regarding the distribution of contrast or structural complexity across different spatial scales), by themselves, these measures are not diagnostic of whether an image is natural or fractal per se. Indeed, the amplitude spectrum slope can be used to quantify the spatial scale-dependent distribution of contrast in urban, artistic, or any types of images. Similarly, while the fractals have been defined as patterns that self-repeat at different levels of magnification [46], it is important to emphasize that they also come in a variety of forms. Typically, the most salient fractals we encounter are those that appear identical at different levels of magnification, for example, Koch's snowflake or the Mandelbrot set, and are known as the exact or mathematical fractals. However, the term "fractal" is not restricted to these mathematically constructed patterns, but can be also extended to incorporate forms that appear similar in their statistical properties at different levels of magnification.

1.2 *Fractal-Scaling, Complexity, and Aesthetics*

The realization that both natural scenes and a wide range of art images share a common statistical characteristic has received much attention and has led researchers to consider several possible explanations. For example, Redies and his colleagues [53, 54] propose that artists seem to have implicit knowledge of natural statistics, a view consistent with the idea that the human visual system evolved to recognize and recreate natural image statistics [24]. On the other hand, Graham and Field [25] have proposed the so-called "perceptibility" hypothesis according to which artists' adherence to specific natural scene statistics are done in order to make the image "visible" to the human eye. In this view, the statistical regularities found in art are a corollary of artistic production and attributed to the constraints of the visual and motor systems involved in creating and inspecting works of art. Simply, the artworks with a natural amplitude spectrum are both more easily created and more easily processed by the visual system, compared to the artworks with non-natural statistical characteristics, also thought to be more challenging to produce. It is interesting that the visibility hypothesis remains agnostic regarding the aesthetic appeal of images which do or do not possess particular spatial statistics.

In contrast, others have explicitly linked the natural scene statistics and fractal structure to their aesthetic appeal [3, 31, 42, 60–63]. Initially referred to by Graham and Field as the "affect" hypothesis [25], this type of approach proposes that a human's intrinsic ability to efficiently process natural scenes is directly responsible for higher preference for aesthetic objects, like artworks, that share similar statistical regularities. Indeed, a number of studies have shown that images with spatial and chromatic properties departing from the $1/f$ statistics found in natural scenes were less preferred [60–63, 66, 70] and rated as more uncomfortable [19, 42]. Collectively, these studies show an enhanced aesthetic preference for intermediate amplitude spectrum slope and fractal dimension values across a wide variety of images with fractal-like statistics. They are also broadly consistent with the well-known concept of perceptual/processing fluency [52, 67], which argues that if human perception is most efficiently tuned to the statistical characteristics of natural scenes, the same efficiency in perceiving images of similar structures is perceived as pleasant, non-effortful, and as a result, translate to higher scores on experimental measures of aesthetic evaluation and preference. Indeed, there is both psychophysical and neurological evidence to suggest we are more visually sensitive to images that have natural amplitude spectra compared to images with slopes that deviate from this structure [38, 62]. Rogowitz and Voss [56] have also shown that patterns with low fractal dimension evoke the perception of nameable objects, thus highlighting the role of fractal complexity in shape perception.

As discussed above, the relationship between natural scene statistics and perceived qualities is indeed multifaceted and the mechanisms mediating preference for characteristics such as fractal-scaling remain unclear. We have argued that the overall preference for intermediate fractal-like scaling exponents is reminiscent of and consistent with the previous findings that patterns with moderate degrees of complexity are preferred [16, 23, 63, 71] and that the perceived complexity might act as a mediator between the physical complexity and preference. This relationship has also been suggested by Boon, Casti, and Taylor [12] who have argued for the importance of characterizing both objective and subjective complexity of complex spatial and temporal systems including visual art and music.

Prior to the wide adoption of image statistics and analysis of frequency spectra, visual complexity was manipulated by systematically adjusting a variety of features or combinations of features such as quantity, heterogeneity, and variety of image elements. The stimuli used in experiments were typically abstract patterns that varied along some spectrum along these properties [2, 6, 7, 72]. These seminal studies formed the foundation for the role of perceived complexity in aesthetic preference, whereby stimuli of "intermediate" complexity were generally regarded as more pleasing over less complex or more complex stimuli. However, the introduction of fractal-like scale invariance and consistent statistical structures found in natural scenes became an important tool for expanding the understanding of the relationship between aesthetic perception and image complexity. Indeed, because both amplitude spectrum slope (α) and fractal dimension (D) values chart the ratio of coarse-to-fine geometrical structure in a pattern, they can be considered a powerful and generic measure of physical complexity generated by repeating patterns that is also directly related to their perceived complexity [14, 51, 63]. For example, the panel in Figure 4(a) plots the average perceived complexity for the synthetic grayscale, thresholded, and edges-only images as a function of their fractal-scaling characteristics (physical complexity). The preference ratings for the same grayscale, thresholded, and edges-only images plotted as a function of their perceived complexity is illustrated in Fig. 4(b), confirming the highest average preference for all three types of images for the intermediate level of perceived complexity [63].

In summary, the images that possess natural amplitude spectrum statistics or an intermediate level of fractal structural complexity may afford the "optimal" amount of the perceived "subjective" visual complexity – not too simple to be boring, and not too complex as to be overstimulating [6]. It is also argued that the fractal structures most commonly encountered in nature elicit unique physiological responses related to relaxation [31, 66] and this familiarity makes them the most preferred. Certainly, studies that have utilized images with fractal-scaling have consistently shown an average preference for synthetic images and patterns with natural $1/f$ statistics and/or intermediate levels of complexity over those with higher and lower complexity [3, 60, 62, 70]. Of course, on its own, fractal-like scaling characteristics do not provide a full account of the entire human aesthetic experience. They do, however, offer an intuitive quantitative measure of complexity and fractal structure across many forms of visual images. Additionally, it is also linked with low-level processes in human visual perception and efficient perceptual processing, providing a foundation upon which the relationship between visual processing and the aesthetic experience can be further explored.

1.3 *Patterns of Individual Differences in Preference for Fractal-Scaling Characteristics*

Though the inverted-U complexity-preference function is still quite a domineering presence in aesthetics literature, its purported universality has been diminished to some degree, with a redirected focus on more nuanced, individual patterns of aesthetic preferences [27, 64]. While the preference for

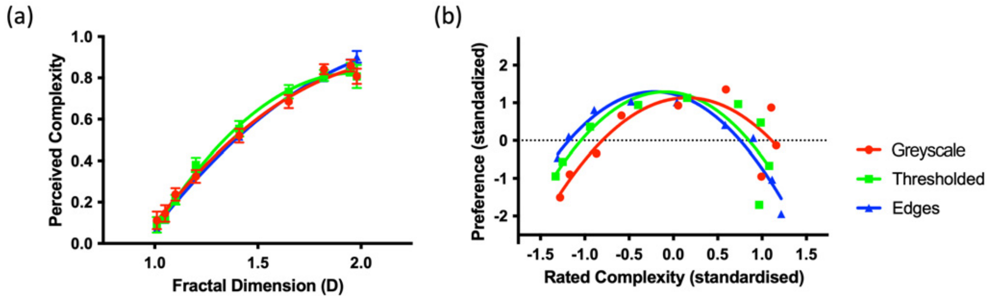

Figure 4. (a) Average perceived complexity in grayscale, thresholded, and edges-only images as a function of their fractal dimension. (b) Average preference for these patterns plotted as a function of perceived complexity. Adapted from Spehar et al. [63].

intermediate complexity and natural scene statistics is very robust on an average population basis, it should come as no surprise that the emphasis on the "average" often obfuscates the importance of individual differences. Even though the investigation of individual differences may seem at odds with the paradigm of universal aesthetics, the degree to which aesthetic preferences are shared versus individual is becoming a question of growing interest. Should individual differences be disregarded as just inexplainable noise, or taken into account as something more systematic and to an extent, predictable?

Recent lines of research regarding the preference for visual complexity has suggested individual differences are indeed worth investigating [27, 63, 64]. For example, Street et al. [64] investigated whether age, gender, or culture predicted preferences for synthetic fractal patterns of varying complexity. They measured complexity in fractal dimension (*D*), in which a higher *D* value corresponds with greater visual complexity. They found significantly different preference patterns between men and women, with women's preference peaking for images with 1.6*D* and men's preference peaking at 1.2*D*. Preference patterns also varied across geographical locations, with North American participants' preference peaking for higher D images compared to Central Asian or African participants. Güçlütürk et al. [27] found that complexity preferences not only differed across distinct populations, but also between participants within a single population. While they found a curvilinear relationship between average preference and complexity, a cluster analysis showed that this curve was rather composed of the average of two distinct, linear preference patterns; specifically, a group of participants that preferred simple patterns and another group that preferred complex ones.

Similarly, Spehar et al. [63] presented participants with a range of $1/f^{\alpha}$ synthetic images that varied systematically in amplitude spectra and fractal dimension. In addition to the standard $1/f^{\alpha}$ noise images (Fig. 3), they also presented thresholded, edge, cross-section, and terrain varieties generated from the same base $1/f^{\alpha}$ image. They found average preference peaked at a slope of around 1.25 across all image varieties and possessed the typical inverted-U shape, whereby preference was on average lowest for the most "non-natural" slopes further from the intermediate value. However, the examination of individual preferences using clustering analysis revealed divergent patterns of preferences that were systematic across all variations of stimuli (Figure 5). These preference patterns were also relatively evenly distributed across the population of participants and can be defined into three distinct preference patterns: low complexity (high slope, low *D*), intermediate complexity ("natural" slope, mid *D*), and high complexity (low slope, high *D*). In addition to emerging across five visually distinct types of $1/f^{\alpha}$ stimuli, we also found that there was a remarkable degree of internal consistency within individuals.

This finding was replicated and extended in a subsequent study that investigated whether individual preference patterns extended beyond not just synthetic $1/f^{\alpha}$ stimuli but also toward art images that possessed the same statistical structures [70]. Art images from a range of periods and subject matter were selected and separated into low, intermediate, and high fractal dimension groups. Synthetic images of matching $1/f^{\alpha}$ statistics were generated, as well as the thresholded and edge variations. Participants were presented with these images in a three alternative rank choice task, as well as rated individually for pleasantness, complexity, and interest. The results supported previous findings, showing a peak in average preference for intermediate fractal dimension images in both synthetic and art categories. Cluster analysis revealed the same three types of preference patterns across both categories: "smooth," "intermediate," and "sharp." Furthermore, correlations between preference patterns within participants on average were moderate and positive, again demonstrating a high degree of internal consistency in preference for specific $1/f^{\alpha}$ structures.

Sherman, Grabowecky & Suzuki [59] have also shown that aesthetic preference for complexity correlated with the visual working memory of individual observers: individuals with higher visual object working memory capacity preferred artworks of higher complexity compared to individuals with lower visual object working memory. Thus, we believe that the focus on individual differences in preference in general and for complexity is warranted, given the relative stability and internal consistency in preference for specific $1/f^{\alpha}$ statistics across distinct stimuli in the visual domain and their link with the visual working memory processes. We propose that investigating systematic individual differences offers a

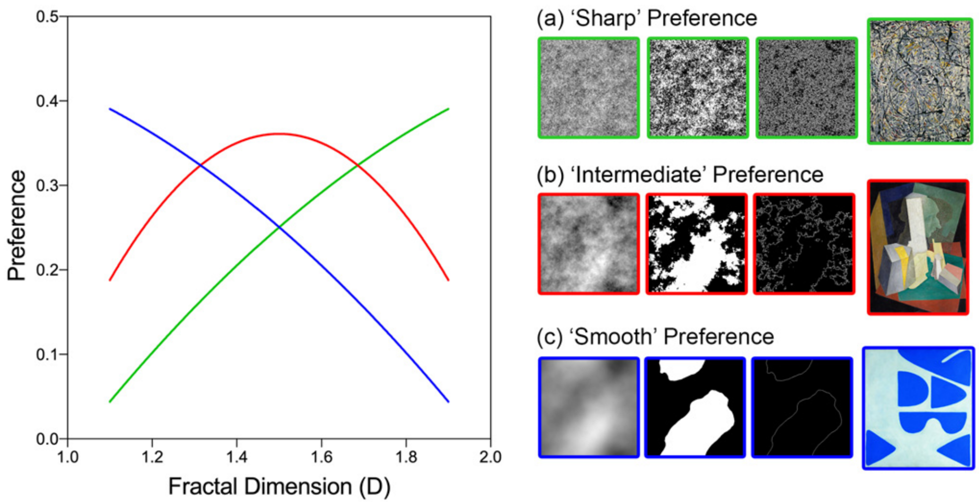

Figure 5. The graph on the left shows the typical clusters of individual preferences that emerge when investigating a sample for aesthetic preferences for a range of $1/f$ stimuli. The green line represents participants who preferred more complex stimuli, such as those in (a). The red line represents participants who preferred stimuli with "natural" $1/f$ structure, which also corresponds to intermediately complex stimuli as seen in (b). The blue line represents those who preferred more simple stimuli like those seen in (c). These preference clusters are based on findings from Spehar, Walker [63], and Viengkham and Spehar [70]. Artworks displayed – top: Jackson Pollock, Watery Paths, 1947; middle: Diego Rivera, Composition with Bust, 1916; bottom: William Scott, Berline Blues 6,1966.

unique framework from which one can explore aesthetic preferences across multiple sensory domains.

2. FRACTAL-SCALING CHARACTERISTICS AND AESTHETICS BEYOND STATIC VISUAL PATTERNS

The question that is raised through this line of research is how far $1/f^{\alpha}$ statistics predicts aesthetic preference across a larger variety of aesthetic stimuli. Our perception, representation, and interaction with the world are inherently multimodal and aesthetic experience is no exception. Though it is not restricted to any single modality, we typically investigate different modalities in isolation from one another and maintain a predominant focus in the visual domain. One advantage of $1/f^{\alpha}$ statistics and similar fractal-scaling manipulations are their flexibility to be applied across a wide variety of stimulus types. As such, it offers a common control of complexity across stimuli from different modalities and can allow us to investigate whether we perceive $1/f^{\alpha}$ structures the same way across different sensory modalities.

2.1 *Dynamic Fractal Patterns: 1/f Structure in Spatiotemporal Stimuli*

The natural world surrounding us is not a purely static one. There is constant movement introduced both through the motion of objects in the world itself as well as in the biological movement of the observer (i.e., eyes, head, body). Early studies have analyzed the spatiotemporal amplitude spectra of dynamic natural scenes and have consistently found a $1/f^{\alpha}$ amplitude spectra across time, where α was approximately 1 [11, 15, 30].

In order to more precisely investigate the interplay between temporal and spatial frequencies, synthetic spatiotemporal fractals were created, in which the $1/f^{\alpha}$ structure could be manipulated in both time and space [11]. Here, altering the temporal slope changes how much energy is in certain temporal frequencies in a dynamic pattern. For example, as temporal slope approaches 0, equal amounts of energy are distributed equally across all temporal frequencies, resulting in a large rate of change at all frequencies from low to high. As temporal slope increase, more energy is distributed to low frequencies, resulting in a large rate of change at low frequencies and relatively smaller change at high. This is most clearly perceptible in the speed and predictability a pattern changes over time, with high sloped stimuli changing slowly over time and lower slopes changing at a rapid, more random pace – similar to a flicker.

Billock et al. [11] investigated the tuning of the visual system toward the $1/f^{\alpha}$ amplitude spectrum in both space and time and found that just noticeable difference thresholds were lowest for the most natural slopes (α between 0.8 and 1.0), and became greater as a temporal slope diverged from this point (becoming particularly bad at detecting differences at the lowest slopes). These findings demonstrated visual sensitivity to natural temporal slopes, as well as further reconsolidating a sensitivity for natural spatial slopes.

However, even until recently, there has been a general dearth of studies exploring the aesthetic appeal of spatiotemporal textures of varying slopes. A study by Toet et al. [68] used a variety of real-world natural textures, like moving water, and asked individuals to label them with a number of affective terms related to emotional descriptions including pleasure, relaxation, and arousal. While the study did not measure the $1/f$ temporal structure of the movies, they were rated for several spatial and temporal characteristics such as temporal regularity, speed, amplitude, and regularity. They found that qualities like high speed and amplitude were both negatively correlated with pleasure, and the former with relaxation. However, it is important to note that all the movies used to feature natural scenes and therefore, natural dynamics. It is yet to be understood whether preferences for the same dynamics are preserved when non-natural temporal structure is introduced.

Recently, Isherwood, Clifford, Schira, Roberts & Spehar [39] measured discrimination sensitivity and preference for dynamic filtered noise movies that varied in their spatiotemporal amplitude spectra. Interestingly, they found that sensitivity and visual preference did not closely overlap. While the sensitivity of the visual system was highest for our stimulus with an intermediate modulation rate (temporal amplitude spectrum slope of 1.25), which is presumably most abundant in nature, the stimulus with the slowest modulation rate in their study (temporal amplitude spectrum slope of 2.25) was most preferred. They argued that preference for the temporal variations might be related to what these properties signal in the natural world with slower stimuli signaling the safer and thus more preferred environmental characteristics. However, their dynamic stimuli consisted of only 128 frames which were presented in a loop. Even though the drop-off in temporal frequency remains the same, with shorter durations the power at higher temporal frequencies is boosted relative to the low temporal frequencies. In other words, what was the perceived speed/rate of change at different temporal slopes is unknown, which might have affected the shape of the preference function for different temporal amplitude spectrum slopes.

2.2 *Fractal-Scaling Characteristics in Tactile Domain*

The $1/f^{\alpha}$ amplitude spectra variations can also be rendered in the domain of 3D surfaces. Here, the standard $1/f^{\alpha}$ noise image of varying slope can be mapped in such a way that individual luminance values of the pixels in the image corresponds to a specific height [51]. Computer-generated 3D models of the $1/f^{\alpha}$ textures have previously been used in studies of aesthetics, both to gauge perceived roughness [50] as well as aesthetic value [63]. In the case of aesthetic value, preference was found to be greatest for intermediate slope values, similar to the average preference patterns for static $1/f^{\alpha}$ images. However, in both experiments, the 3D surfaces were presented on screen and visually inspected as opposed to touched.

The quality of roughness is a particularly important descriptor in tactile aesthetics. The smooth-rough dimension is consistently defined in multivariate multidimensional models of haptic perception [4, 32, 33, 49]. Furthermore, ratings of everyday materials have reliably found a negative relationship between roughness and perceptions of pleasantness; that is, the more rough a surface is, the less pleasant it is perceived to be [17].

We recently conducted the same preference study using 3D printed physical versions of these same $1/f^{\alpha}$ surfaces [71]. The surface textures were generated from $1/f^{\alpha}$ noise images that range from slopes of 1.25 to 2.75 and were printed onto blocks made from a matte plastic type material (Figure 6). Different groups of participants were asked to inspect the blocks visually, tactilely, and through both sensory modalities. Similar to the preferences for dynamic $1/f^{\alpha}$ visual patterns, we found that preference was linear and peaked for surfaces generated from the highest $1/f^{\alpha}$ slopes. Surfaces made from these slopes were also perceived to be the most smooth and pleasant. These results are consistent with what has been found previous research of material perception from a non-$1/f^{\alpha}$ framework in which normal materials are typically used. Here, materials that are high in qualities of smoothness and softness are the most preferred compared to materials on the opposite end of those dimensions [17].

Average preferences for $1/f^{\alpha}$ tactile surfaces were not curvilinear like what was consistently found for the static $1/f^{\alpha}$ noise images and its variations. However, it is interesting that similar systematic differences in preference patterns also emerged for these stimuli. Specifically, a small proportion of the population showed a greater preference for stimuli with low slope values – rougher, sharper surfaces – as well as the intermediate slope values. Moreover, correlations between tactile preferences and static visual $1/f^{\alpha}$ images remained unanimously positive across all image variations [71].

2.3 *Auditory Perception of 1/f Melodies*

Finally, we'll explore what has been established so far on the role of $1/f^{\alpha}$ statistical structures in the auditory domain. Music consists of numerous components that influence its complexity and aesthetic value. This has resulted in a rather wide and vague definition of the "fractal nature" of music, whereby different types of analyses, musical features, and parameters have all been used to identify, quantify, or dispute the relevance of fractal statistics in music.

Early research on the spectra of complex noise signals found through the analysis of a variety of music, sound, and talk show programs on the radio possessed a $1/f^{\alpha}$ structure. Hour long segments from various radio programs were analyzed and found to have a slope averaging the natural intermediate range, same as those found in natural scenes [73]. Noise signals ranging from classical music to rock music to speech — all share a common "natural" $1/f^{\alpha}$ structure. It is especially in the analysis of the existing discography of classical and contemporary real-world music that the relationship between fractal-scaling statistics in music and its aesthetic value or perception have become somewhat unclear. For example, some studies have found that different genres of music differ in their fractal dimension [10, 34, 35], while others have found no discernible consistencies in the role of fractal-scaling statistics at all [37].

More research has been completed to determine the perceived complexity and melodicity of $1/f^{\alpha}$ synthetic melodies [5]. That is, when music is in its most basic form – a sequence of single notes of same duration spaced equally to form a melody – does a $1/f$ manipulation of this sequence alter their perceived aesthetic value? This structure was later applied directly to simple melodic sequences in which a

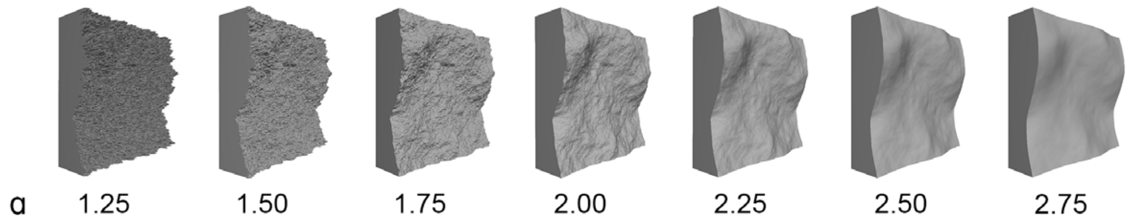

Figure 6. Examples of $1/f$ textured surfaces recently used in [71]. The texture blocks were created from $1/f$ grayscale images with the corresponding amplitude spectrum slopes.

Figure 7. Examples of synthetic $1/f$ melodies of the corresponding amplitude spectrum slope input values.

sequence of notes (essentially equivalent to a sequence of numbers) followed a $1/f^{\alpha}$ structure based on three slope values: 0, 1, and 2. Similar to their visual counterparts, melodies created from $1/f$ structures with a slope of 0 are random, where no one note predicts any subsequent note. As the slope increased toward 2, the sequence becomes more predictable and the change in consecutive notes is more gradual. An informal study carried out at the time revealed a greater preference for sequences that followed the intermediate $1/f^{\alpha}$ structure over those with slopes of 0 or 2 [73]. These findings have been more recently supported by Beauvois [5], who generated a greater series of synthetic $1/f^{\alpha}$ melodies with slopes ranging from approximately 0 to 2.0 in increments of 0.2. Here, preference showed a curvilinear function and peaked for slope values of approximately 1.38. Furthermore, slope was found to correspond closely to perceived complexity where complexity rating decreased monotonically with increasing slope.

However, when a fractal-like structure is applied to music in its simplest form – a sequence of single notes placed in equal spacing to form a melody – it is essentially isolating a single aspect of complexity and removing all other factors (Figure 7). Based on the existing research highlighted, we can see that $1/f^{\alpha}$ structure is a reliable quantifier of objective complexity of melodies and that similar to what is found in the visual domain, aesthetic preference appears to peak for values closest to the most natural slope range. While it is still unclear whether the same preference patterns found in the visual and tactile domain also emerge for the auditory domain, a recent study offers some possible insight. Güçlütürk and van Lier [28] measured participant liking ratings for 25 song excerpts from a variety of musical genres that had been previous rated for instrumental complexity. Analysis of preferences clustered the participants into two relatively equal sized groups, where one group showed a preference for more complex songs and the other group showed a preference for more simple songs. Therefore, it is expected that looking beyond the average curvilinear preference found for $1/f^{\alpha}$ melodies, more distinct and systematic preference patterns will also be found.

3. CONCLUSION

Studies which have investigated cross-modal aesthetic perception have often been done from a more top-down perspective of genre categorification [1], or ensuring a congruent cross-sensory consumer-minded experience [36, 75]. Given the multitude of scale-invariant, fractal-like scaling manifestation across many physical and biological domains, we explore the potential of this approach when considering aesthetic experience across different sensory domains.

Table I. Summary of preference outcomes as a function of fractal-scaling manipulations in visual, auditory, and tactile stimuli. Shaded cells represent studies from tactile and auditory modalities.

Author (Year)	Stimulus	Measured statistic	Range	n	Analysis	Task	Shape of Preference Function	Individual Differences	Comments/Other
Aks & Sprott (1996) [3]	324 computer-generated chaotic pattern	Fractal dimension (correlation dimension, F)	F = 0.49 to 1.78	24	Correlation	Ranked 4AFC	Inverted-U Preference peaked at 1.26*D*	Measured Compared preferences based on gender, self-reported levels of creativity, and science-math abilities.	Also measured Lyapunov exponent (L)
Voss & Clarke (1978) [73]	Atonal melodies	Power spectrum slope	3 slope levels (0, −1, −2)	n/a	Not specified	Free response	Inverted-U Preference peaked for −1	Not measured	Informal data acquisition
Spehar, Clifford, Newell & Taylor (2003) [60]	11 natural fractals (photographs), 15 computer-generated fractals (coastlines), 40 human fractals (painting segments)	Fractal dimension	Natural images (1.1*D* to 1.9*D*), mathematical fractals (1.33*D*, 1.5*D*, 1.66*D*), human fractals (1.12*D*, 1.5*D*, 1.66*D*, 1.89*D*)	220	Not specified	2AFC	Inverted-U Preference peaked between 1.3*D* and 1.5*D*	Not measured	No measures regarding perceived complexity
Hagerhall, Purcell & Taylor (2004) [29]	80 landscape silhouettes 52 landscape silhouettes (removing images with water or hills)	Fractal dimension	∼1*D* to ∼1.5*D*	119	Correlation	Preference rating	No relationship Linear, preference increased with increasing *D*	Not measured	Second outcome obtained by performing analysis on a subset of the stimuli from the first.
Beauvois (2007) [5]	Computer-generated atonal melodies	Power spectrum slope	11 slope levels (−2 to 0)	18	Trend analysis	Melodicity and complexity rating	Inverted-U Preference peaked at slope of −1.38	Not measured	Perceived complexity increased as slope increased
Fernandez & Wilkins (2008) [19]	45 non-representational artworks (grayscale, square cropped) Computer-generated noise images	Amplitude spectrum slope	Range of artworks selected	47	Wald Chi-Squared Test, Pearson correlation	Preference, discomfort rating	Higher discomfort for images with non-intermediate amplitude spectra Negative relationship between preference and discomfort	Not measured	No measures regarding perceived complexity

Table I. Continued.

Author (Year)	Stimulus	Measured statistic	Range	n	Analysis	Task	Shape of Preference Function	Individual Differences	Comments/Other
Hagerhall, Purcell & Taylor (2008) [31]	Computer-generated fractal patterns	Fractal dimension	4 levels (1.14*D*, 1.32*D*, 1.51*D*, 1.70*D*)	31	Repeated-measures ANOVA, trend analysis	EEG (alpha, delta, and beta waves)	Inverted-U Alpha peak at 1.32	Not measured	No measures regarding perceived complexity
Graham, Friedenberg, McCandless & Rockmore (2010) [26]	20 landscape paintings, 20 portrait/still-life paintings, 18 abstract paintings	Amplitude spectrum slope	Range of artworks selected	33	Pearson correlation	2AFC, preference rating	Linear Positive relationship between slope and preference	Not measured	
Juricevic, Land, Wilkins & Webster (2010) [42]	Computer-generated noise images	Amplitude spectrum slope	5 levels (−2 to 0)	61	Repeated-measures ANOVA, post-hoc Holm-Sidak t-tests	Discomfort, artistic merit rating (1 − 7)	U-shaped Discomfort decreased as slope increased toward intermediate range Artistic merit peaked at −1 slope	Not measured	Also examined the effect of color contrast and chromatic axes
Forsythe, Nadal, Sheehy, Cela-Conde & Sawey (2011) [22]	240 images, varied (abstract and figurative artworks, design objects, environmental photographs)	Fractal dimension	Range of artworks selected	120	Correlation	Beauty rating	Linear Positive relationship between fractal dimension and beauty	Not measured	No measures regarding perceived complexity
Spehar & Taylor (2013) [61]	Computer-generated noise images	Amplitude spectrum slope	9 slope levels (−2.5 to −0.5)	54	Repeated-measures ANOVA, Bonferroni t-tests	2AFC	Inverted-U Preference peaked between −1.5 and −1.25	Not measured	No measures regarding perceived complexity
Spehar, Wong, van de Klundert, Lui, Clifford & Taylor (2015) [62]	Exp 1a: computer-generated noise images Exp 1b: computer-generated noise images	Amplitude spectrum slope	6 slope levels (Series 1: −1.6 to −0.1, Series 2: −1.7 to −0.2) 9 slope levels (−2.5 to −0.5)	22 47	Repeated-measures ANOVA, post-hoc Holm-Sidak t-tests	2AFC	Inverted-U Preference peaked between −0.8 and −1.4	Not measured	Found visual sensitivity correlated positively with preference

Table I. Continued.

Author (Year)	Stimulus	Measured statistic	Range	n	Analysis	Task	Shape of Preference Function	Individual Differences	Comments/Other
Bies, Blanc-Goldhammer, Boydston, Taylor & Sereno (2016) [8]	Exp 1: computer-generated exact fractals	Fractal dimension	9 levels ($1.1D-1.9D$)	42	Repeated-measures ANOVA, trend analysis	Preference rating	Linear Preference increased with increasing D	Measured (K-Means) Exp 1: Two preference patterns (increasing, decreasing)	No measures regarding perceived complexity
	Exp 2: Sierpinski carpet, symmetric dragon, golden dragon, and Koch's snowflake fractals			18				Exp 2: Two linearly increasing preference patterns	Effects of recursion, symmetry, and segment quantity also examined
Spehar, Walker & Taylor (2016) [63]	Computer-generated noise images	Amplitude spectrum slope, fractal dimension	9 slope levels (-2.5 to -0.5)	278 50	Mixed-measures ANOVA, repeated-measures ANOVA, post-hoc Holm-Sidak t-tests	2AFC	Inverted-U Preference peaked at slope of -1.25	Measured (Q-Mode, K-Means) Inverted-U (50%), high (20%), low (20%)	Perceived complexity increased linearly with increasing slope/D ($n=32$)
Street, Forsythe, Reilly, Taylor & Helmy (2016) [64]	Computer-generated noise images	Fractal dimension	9 levels ($1.1D-1.9D$)	443	Repeated measures ANOVA, linear mixed effects models	2AFC	Inverted-U Preference peaked at $1.2D$	Measured (linear mixed effects models) Compared complexity preferences between genders, continents, and ages	Complexity measured using GIF compression ratios
Isherwood, Schira & Spehar (2017) [38]	Computer-generated noise images	Fractal dimension, amplitude spectrum slope	5 slope levels (-2.25 to -0.25)	21	Repeated measures ANOVA	fMRI BOLD response, aesthetic rating	Inverted-U Preference and BOLD responses peaked between -1.25 and -1.75	Not measured	
Viengkham & Spehar (2018) [70]	Computer-generated noise images, artworks	Fractal dimension, amplitude spectrum slope	3 levels ($1.1D$, $1.4D$, $1.7D$)	171	Repeated measures ANOVA, correlations	Ranking Pleasantness, interesting complexity ratings	Inverted-U Preference peaked for $1.4D$ images	Measured (k-means) 3 preference patterns (intermediate, high, low)	Complexity ratings increased with greater D

Table I. Continued.

Author (Year)	Stimulus	Measured statistic	Range	n	Analysis	Task	Shape of Preference Function	Individual Differences	Comments/Other
Viengkham & Spehar (2019) [71]	Computer-generated noise images (three variations) 3*D* printed 1/*f* surfaces	Fractal dimension, amplitude spectrum slope	9 levels (1.01*D* to 1.82*D*) 7 levels (2.01*D* to 2.82*D*)	51	Repeated measures ANOVA	2AFC	Inverted-U Preference peaked for 1.2*D* images Linear Preference decreased with increasing *D*	Measured (Q-Mode) 2 preference patterns (high, low)	No measures regarding perceived complexity
Ogawa & Motoyoshi (2020) [48]	Computer-generated noise images	Amplitude spectrum slope	4 slope levels (−2 to −0.5)	12	One-way ANOVA	Unpleasantness rating	Linear Unpleasantness greatest for slope of −2	Not measured	Also manipulated orientation spectra
Wu & Chen (2020) [76]	Computer-generated noise images	Amplitude spectrum slope	5 slope levels (−2 to 0)	43	Repeated measures ANOVA	Beauty rating	Flat	Not measured	Also manipulated symmetry in addition to slope
Isherwood, Clifford, Schira, Roberts & Spehar (2021) [39]	Computer-generated noise images and videos	Amplitude spectrum slope	3 slope levels (−2.25 to −0.25)	33	Repeated-measures ANOVA	2AFC	Inverted-U Preference peaked at −1.25	Not measured	Also manipulated temporal slope in addition to spatial slope
Nguyen & Spehar (2021) [47]	Computer-generated noise images	Amplitude spectrum slope	5 slope levels (−2.25 to −0.25)	58	Repeated-measures ANOVA, post-hoc Holm-Sidak t-tests	2AFC	Inverted-U Preference peaked at −1.25	Measured (k-means) 3 preference patterns (intermediate, high, low)	Adaptation shifted preference in direction of adapted stimulus slope

While a fair amount of research has been completed on the aesthetic evaluation of scale-invariant $1/f^{\alpha}$ stimuli across vision, audition, and touch, these have mostly been done independently of each other (see Table I). Though it is obvious that the relationship between fractal-scaling and preference differs across studies both within and between different sensory domains, this is not very surprising. A closer inspection reveals that these studies differ enormously in the type of stimuli they use, the way in which the fractal-scaling characteristics are manipulated, and whether they measure preference, beauty, discomfort, complexity, or some combination of different measures. As previously noted, there is paucity of studies in tactile and auditory domains.

Nevertheless, in all sensory domains and across a wide range of different types of stimuli, there is evidence that fractal-scaling characteristics influence reported preference for these stimuli. Our review of $1/f^{\alpha}$ statistical manipulations across all these separate domains demonstrates its consistency as an objective measure of stimulus complexity. Importantly, it also seems to be directly and linearly related with the perceived or subjective stimulus complexity across sensory domains. Synthetic images with a spatial slope of 2.25 are generally considered to be less complex compared to images with slopes of 1.25, just as melodies, surfaces, or dynamic images with slopes of 2.25 are also considered less complex than their 1.25 variations.

Within a single domain, such as in vision, preferences for specific $1/f^{\alpha}$ slopes in a population and within an individual were found to be systematic and relatively consistent. That is, across a set of visually distinct visual stimuli, if an individual preferred an intermediate slope for one variation, this preference would remain internally consistent across the others. From this, we highlighted how individuals also fell into three preference types that emerged across all image variations: low, intermediate, and high complexity [8, 9, 27, 63, 70, 71]. A small number of studies within other domains such as tactile surfaces and dynamic patterns have also found stable preferences, as well as similar patterns of individual differences between visual and tactile domains. However, the structure and reasons for the patterns of individual differences in preference within and between different sensory domains remain underexplored and in a need of further empirical and theoretical focus.

In summary, despite some superficial differences, the underlying dimensional structure mediating the preference across sensory domains is encouragingly similar, suggesting systematic patterns of preference for fractals in vision, audition, and touch. Based on the evidence so far [27, 70, 71], we propose that there exists a strong link between fractal-scaling statistics and perception of dynamic, expressive, and affective aspects of sensory stimulations in different modalities. To be able to elucidate the contribution of shared and individual contributions to preference we believe that the perceived structural (roughness, complexity, regularity) and affective (pleasantness, liking, harmony, interest) representations of fractal-scaling properties should be studied concurrently and in a comparable manner across different sensory domains.

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