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Xerographic Printing System Performance Optimization by Toner Throughput Control

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Abstract. *For two component xerographic development systems the toner concentration and the average toner residence time in the developer sump are desired to be within optimal ranges for acceptable image quality. The toner concentration and the average toner residence time are coupled such that independent ranges are not typically achievable, and so a tradeoff must be managed. Also the required toner for any given job is highly variable and not known well in advance, yet significantly impacts the two aforementioned parameters. In this paper the authors propose and demonstrate a model predictive control design framework to optimally manage toner throughput to achieve performance objectives. © 2009 Society for Imaging Science and Technology.*
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BACKGROUND

In two component xerographic development systems the toner charge to mass ratio, i.e., the so-called toner tribo, is regulated in part by control of the ratio of toner mass to carrier mass in the developer sump [i.e., toner concentration (TC)]. Failure modes associated with high tribo (poor development and poor transfer) and low tribo (background and emissions) can be avoided by controlling the TC to within a predetermined latitude window. As the development system delivers toner to the latent image on the photoreceptor, the change in TC from a desired set point is measured by a sensor and used to trigger a dispense motor which delivers new toner to the developer sump. In practice, a proportional controller often forms the basis of the TC feedback control. In addition to feedback from the TC sensor, the anticipated toner usage based on customer image content is often used as a feedforward signal to improve the transient response.[1]

Toner aging is an important image quality degradation phenomena in two-component development systems.[2] Toners typically use surface additives for flowability and adhesion control. Toner functionality may degrade over time because additives get buried into the toner surface due to repeated collisions in the developer housing. Consequently, the average residence time of toner in the housing, frequently referred to simply as toner age, can often be used as a measure correlated with image quality.

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Simultaneously managing the TC and toner age parameters can be complex. The toner input and output rates necessary to regulate the TC are frequently not compatible with rates necessary to regulate the toner age within acceptable limits. Also the toner output rate is a strong function of the customer image content and so is highly variable. The system is nonlinear, multivariable, and has numerous physical as well as cost constraints.

To manage the system under such complex conditions, we propose a material state control architecture that utilizes toner dispense (a material input rate) and interdocument zone imaging (a material output rate) to manage both TC and toner age in an optimal sense. The design approach is based on the model predictive control (MPC) methodology and is easily applied to multivariable systems with constraints on inputs and outputs. MPC can also be economically advantageous since it permits operation close to constraint boundaries. Because of the aforementioned characteristics of MPC, it is one of the few advanced control methodologies that has made a significant impact in the process control industry since the mid 1970s.

MODEL PREDICTIVE CONTROL OVERVIEW

In general an MPC formulation is applied to a dynamic system or "plant" that is assumed linear and time invariant (the application to time varying linear systems is straightforward but does not concern us here, and extensions to some nonlinear systems have been made[3]). The inputs to the system may include unmeasured disturbances, measured disturbances, set points (or references), and manipulated variables (actuators). Outputs may be measured or unmeasured. In addition, a measurement noise model may be combined with the outputs to improve estimates of the variable to be controlled. MPC, as the name suggests, requires a model to predict the impact that inputs and measurable disturbances will have on the outputs. The model enables one to estimate the output trace due to a set of assumed inputs over a future specified time horizon p_1. An input or actuation sequence is computed to minimize a quadratic cost function subject to constraints. The actuation changes are constrained to vary over a "control" horizon p_2 that is necessarily less than p_1. The sequence of control inputs that minimizes the quadratic cost function is the solution to a convex optimization problem and is solved by quadratic programming. Only the first control input computed is in fact applied; after which the process is repeated at the next sample instant with updated measurements. The challenge is to tune the controller to meet the varied objectives. Tuning is achieved mainly by varying weights in the quadratic cost function.

In the remainder of this article we formulate the MPC problem for developer material state control. Through this example the basic features of MPC will be illustrated. For those interested in a more in depth discussion we refer to previous works.[4,5] In the next section we discuss a dynamic sump model. Next we describe the controller design, simulate the performance under a range of conditions, and lastly present conclusions.

SYSTEM MODEL

The dynamics of the sump are described by mass balance expressions in the form of discrete difference equations that represent the evolution of TC and toner age over time. To arrive at the expressions we first define the following variables: $k=1,2,3,\ldots$ represents the sampling time instances which for a constant speed printer and given sheet size can be reparametrized by print; $m_t(k)$ is the toner mass level in the sump; m_c is the carrier mass level in the sump which is assumed to be constant; $Age(k)$ is the average toner residence time in the sump; $\mathrm{TC}(k)$ is the percent toner concentration; and $\mathrm{AC}(k)$ is the area coverage of the customers job and will act as a disturbance to both the $Age(k)$ and $\mathrm{TC}(k)$ variables. In digital printing applications the future values for $\mathrm{AC}(k)$ are known over some future time horizon. Knowledge of disturbances well in advance can be exploited by the MPC approach since output predictions over a future time horizon are required.

The system also includes two manipulated variables; $\mathrm{Disp}(k)$ is the toner dispense rate and $\mathrm{AC\_act}(k)$ is the additional area coverage that may be printed in the inter document zone. The actuator $\mathrm{AC\_act}(k)$ may be necessary to remove toner from the system when the customer job's area coverage is too low to do so. Though use of $\mathrm{AC\_act}(k)$ is occasionally necessary, it is a costly actuator and its use will be discouraged through an appropriate weighting term in the cost function. Costs are incurred because the toner is simply cleaned from the photoreceptor and sent directly to the waste collection bottle.

Using the terminology from above, we now describe the relevant plant dynamics. A toner mass balance expression yields

$$m_t(k+1) = m_t(k) + \alpha[\mathrm{Disp}(k) - \mathrm{AC\_act}(k) - \mathrm{AC}(k)], \tag{1}$$

which is a function of the printer developed mass per unit area (assumed constant) and print speed, and where α is a scaling parameter. The normalization is such that $\mathrm{Disp}(k)$, $\mathrm{AC\_act}(k)$, and $\mathrm{AC}(k)$ all range from 0 to 1, where 1 corresponds to a 100% area coverage. So, for example if $\mathrm{AC}(k)=0.4$, then 0.4α is the amount of mass per unit time removed from the sump at a 40% area coverage. Also $\mathrm{Disp}(k)$ is defined to have equivalent units to area coverage (though it is an input, not an output). The equivalence can be achieved by the inclusion of a gain lumped with the actuator in software. Although the normalization is such that the actuators all range from 0 to 1, in practice the maximum of $\mathrm{Disp}(k)$ and $\mathrm{AC\_act}(k)$ are less than 1 because of admix and interdocument zone size constraints. From Eq. (1) the toner mass level at the next sampling instant, $k+1$, is equal to the toner mass level at the previous sampling instant plus the toner mass dispensed at time k, minus the toner mass lost from the sump at time k due to inter document zone development and the rendering of the customer's job. Although toner enters and leaves the system continuously between sample instants k and $k+1$, we consider the toner that has entered or left the sump as having occurred at the instant k. The output to be controlled is TC which is related to toner mass by

$$\mathrm{TC}(k) = 100\frac{m_t(k)}{m_c(k)}. \tag{2}$$

To express the evolution of $Age(k)$ we have

$$Age(k+1) = \frac{(m_t(k) - \alpha(\mathrm{AC}(k) + \mathrm{AC\_act}(k)))(Age(k)+1) + \alpha\mathrm{Disp}(k)^{*}1}{m_t(k) + \alpha(\mathrm{Disp}(k) - \mathrm{AC}(k) - \mathrm{AC\_act}(k))}. \tag{3}$$

Equation (3) models the age progression as a weighted sum of the mass not lost during the interval k to $k+1$, which advances by one time interval, and the newly dispensed mass that has advanced by one time interval. It is assumed that none of the dispensed toner at time k is developed between times k and $k+1$. This assumption is reasonable since typically the toner dispense port is located far from the development zone to allow time for admix. Rearranging the age expression we have

$$Age(k+1) = \frac{1}{1 + (\mathrm{Disp}(k)/((m_t(k)/\alpha) - \mathrm{AC}(k) - \mathrm{AC\_act}(k)))} \times Age(k) + 1. \tag{4}$$

The age expression above can be seen to be a first order difference equation with a constant input of unit value. The pole given by

$$\frac{1}{1 + (\mathrm{Disp}(k)/((m_t(k)/\alpha) - \mathrm{AC}(k) - \mathrm{AC\_act}(k)))} \tag{5}$$

is generally time varying, depending at any given time on the values of the inputs and outputs. When $\mathrm{Disp}(k)=0$, the pole assumes unit value, and the expression reduces to that of an integrator, i.e., the age of the toner that remains in the housing simply increases by one time unit at each sampling interval. For any nonzero value for dispense, $\mathrm{Disp}(k)>0$, the pole is necessarily <1. For constant inputs and outputs the pole assumes a fixed value and applying the final value theorem,[6,7] the age asymptotically converges to $(\mathrm{Disp}+((m_t/\alpha)-\mathrm{AC}-\mathrm{AC\_act}))/\mathrm{Disp}$. For time varying inputs and outputs the age expression is nonlinear (actuators and states are multiplicative). Since the general implementation of MPC requires linear system models,[3,4] a linear approximation is considered.

Recall that $\mathrm{Disp}(k)$, $\mathrm{AC\_act}(k)$, and $\mathrm{AC}(k)$ are constrained in the interval from 0 to 1. For typical production printers $m_t(k)$ is on the order of 75 to 150 g, α ranges from 0.5 to 2, and so the ratio $\mathrm{Disp}(k)/m_t(k)/\alpha-\mathrm{AC}(k)-\mathrm{AC\_act}(k)\ll 1$. Since $1/1+\beta\cong 1-\beta$ for $\beta\ll 1$, it follows that

$$Age(k+1) \approx \left(1-\frac{\mathrm{Disp}(k)}{(m_t(k)/\alpha)-\mathrm{AC}(k)-\mathrm{AC\_act}(k)}\right) \times Age(k)+1,$$

which can be further simplified to

$$Age(k+1)\approx Age(k)-\mathrm{Disp}(k)\alpha\frac{\bar{A}ge(k)}{\bar{m}_t(k)}+1. \qquad (6)$$

Because the MPC controller is required to project the toner age evolution over a time horizon from k to $k+p_1$, and the MPC computation requires a linear time invariant expression for $Age(k)$, the values for age and toner mass level are assumed fixed from k to $k+p_1$, and are set at their respective values at time k. The fixed values over the horizon are indicated as $[\bar{A}ge(k)]/[\bar{m}_t(k)]$. At the next sampling instant in which the MPC controller repeats the computation of the optimal actuator commands, the values for $\bar{A}ge(k)$ and $\bar{m}_t$ are updated. $\bar{A}ge(k)$ is updated via the nonlinear expression [Eq. (3)] that can run in the background, and $\bar{m}_t$ can be updated by expression, Eq. (2), in which the TC sensor output and the assumed carrier mass m_c are substituted. For systems with carrier mass trickle the variation in m_c may need to be taken into account. In this way a linear approximation to the age expression can be used for the MPC computation, and at each new sampling instant the coefficients are updated. The result is a toner age expression approximated as a first order integrator with input $1-\alpha[\bar{A}ge(k)]/[\bar{m}_t(k)]\mathrm{Disp}(k)$, i.e., the input to the integrator is 1 less an amount proportional to $\mathrm{Disp}(k)$. Figure 1 is an example of the toner age evolution computed with Eq. (3), and the results produced by the corresponding linear approximation for typical values of $\mathrm{Disp}(k)$, $Age(k)$, α, and $m_t(k)$. The age projection degrades with time, but at a $\sim$100 minute level the error remains under $\sim$10 s throughout a 3 min duration.

CONTROL DESIGN

The MPC controller is such that we can specify either soft or hard constraints. Soft constraints can be violated (though violation is discouraged) to ensure a feasible solution during optimization. Hard constraints cannot be violated; rather it is preferable for a machine to shut down and call for a service intervention.[8]

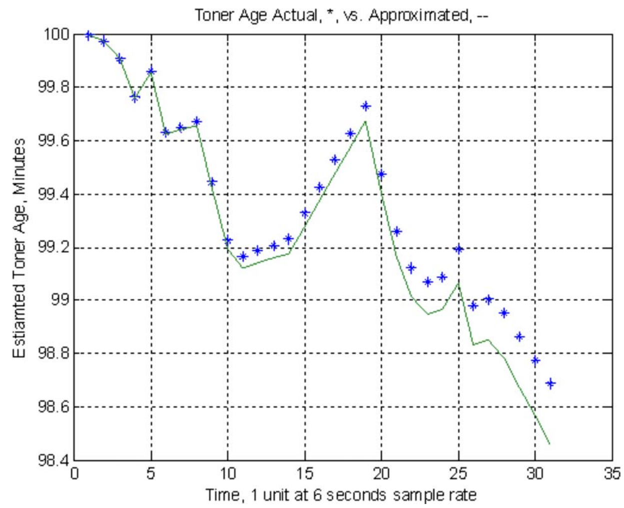

Figure 1. Toner age, simulated actual vs linear approximation.

There are a number of constraints relevant to a typical production printer. A nonzero lower limit on dispense ensures at least a continuous, although small, introduction of fresh toner entering the sump. This is beneficial in managing toner properties that degrade with age. At the very least a lower constraint of zero must be imposed since dispense cannot physically assume a negative value. In addition, although dispense may in principle keep up with 100% area coverage rates, a smaller upper limit may be set to ensure proper material admix. Under high throughput conditions a lower TC limit may be reached necessitating a dead cycle and retone process, again adversely impacting productivity.

For $\mathrm{AC\_act}(k)$ there is an upper limit due to limited photoreceptor space between customer images. An image skip mode can be used to increase the upper limit if necessary but that is not considered here because it adversely impacts productivity. Hard constraints are set to $0\le \mathrm{AC\_act}(k)\le 0.15$.

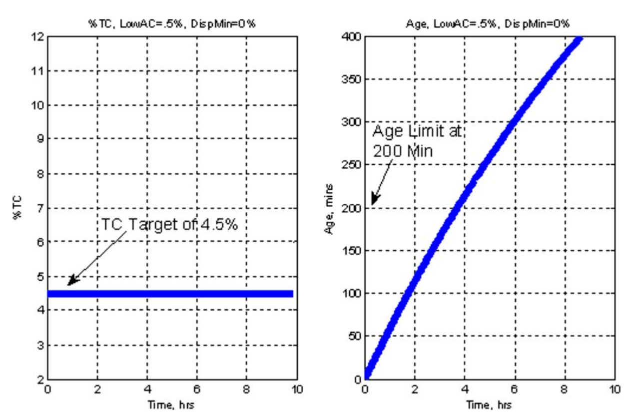

Figure 2. Plant outputs TC and age vs time. Controlling to fixed %TC target of 4.5%.

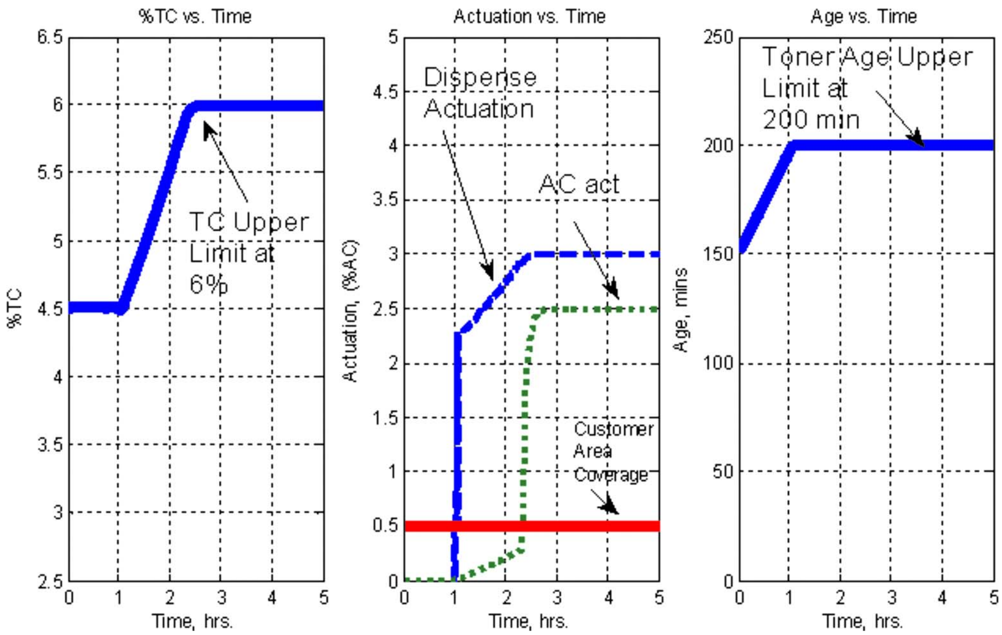

Figure 3. Plant outputs TC and age vs time. Full MPC controller implemented and constraints fully satisfied.

The TC latitude window is dependent on the development system characteristics. Exceeding an upper limit may result in toner emissions. Exceeding a lower limit will result in a supply limit. Because of inaccuracies in the TC sensor, a TC target is usually set between the upper and lower limits with deviations from target allowed but discouraged. For the simulation we choose hard constraints of $3\% \le \mathrm{TC}(k) \le 6\%$ and a TC target of 4.5%. Through experimentation an acceptable upper threshold of toner age can be determined by correlating age with toner related image quality degradation. We impose a hard constraint of $Age(k) \le 200$ min.

Subject to the constraints outlined above the MPC controller will compute an actuator sequence that is permitted to change over the future time horizon from k to $k+p_2$ that minimizes a quadratic cost function over the larger time horizon k to $k+p_1$. The actuator values computed are implemented at time $k+1$ after which the computation is repeated with updated measurements. The two actuators are expressed in vector notation as, $u(k+i|k)=[\mathrm{Disp}(k+i|k),\mathrm{AC\_act}(k+i|k)]$ where the integer i ranges from 1 to p_2. The notation $k+i|k$ signifies the determination of a value at time $k+i$ based on information available at time k. To minimize the $TC(k)$ deviation from target and penalize the use of the actuator $\mathrm{AC\_act}(k)$, the cost function proposed is,

$$\min_{u(k+1|k)\ldots u(k+p_2|k),\varepsilon}\left\{\sum_{i=0}^{p_{1-1}}(\omega^{\mathrm{TC}}[\mathrm{TC}(k+i+1|k)-\mathrm{TC}_{Setpoint}]^2\right.$$
$$\left.+\,\omega^{\mathrm{AC\_act}}[\mathrm{AC\_act}(k+i+1|k)]^2+\rho_\varepsilon\varepsilon^2)\right\}.$$

Weights ω^{TC} and $\omega^{\mathrm{AC\_act}}$ can be time varying, but in the simulation are constant and of equal value. The slack variable ε permits soft constraints to be violated in case we are faced with an infeasible problem (e.g., a large disturbance such that it is impossible to stay within the constraints). The soft constraints are modified by ε which is always ≥ 0. So, for example, the constraint that the TC remain between 3% and 6% can be modified as $3\% - \varepsilon\mathrm{TC}_{\mathrm{min\_slack}} < \mathrm{TC}(k) < 6\% + \varepsilon\mathrm{TC}_{\mathrm{max\_slack}}$ for some $\mathrm{TC}_{\mathrm{min\_slack}}$ and $\mathrm{TC}_{\mathrm{max\_slack}}$. The latter slack variables define a hard constraint if set to 0 or a soft constraint if nonzero, with the relative magnitudes determining the degree of concern if the constraint is violated.

SIMULATION RESULTS

Three scenarios are simulated using the MATLAB® model predictive control toolbox MPCTOOL®. Many initial conditions in area coverage, TC, and age are possible. First we consider TC regulation only as illustrated in Figure 2. In this case we regulate the %TC to a fixed target of 4.5% under a low area coverage condition of 0.5% and with toner age simply as an outcome. The %TC is well regulated but due to the low area coverage the age which is initially zero exceeds 200 min within about 4 h running time. Once the 200 min age is exceeded some form of intervention policy would be required to bring the image quality performance back within specification.

In the next simulation we consider an MPC based approach in which toner age is included as an output that is constrained to be less than 200 min. %TC is permitted to range over 3%–6% although %TC deviations from the 4.5% target are penalized. Also the additional actuator, $\mathrm{AC\_act}(k)$ is included with a penalty for any deviations in value from zero. Referring to Figure 3, we see that at the start of the low area coverage run of 0.5% there is a gradual rise in %TC and a significant increase in toner age. When toner age reaches 200 min the dispense actuator quickly increases so as not to

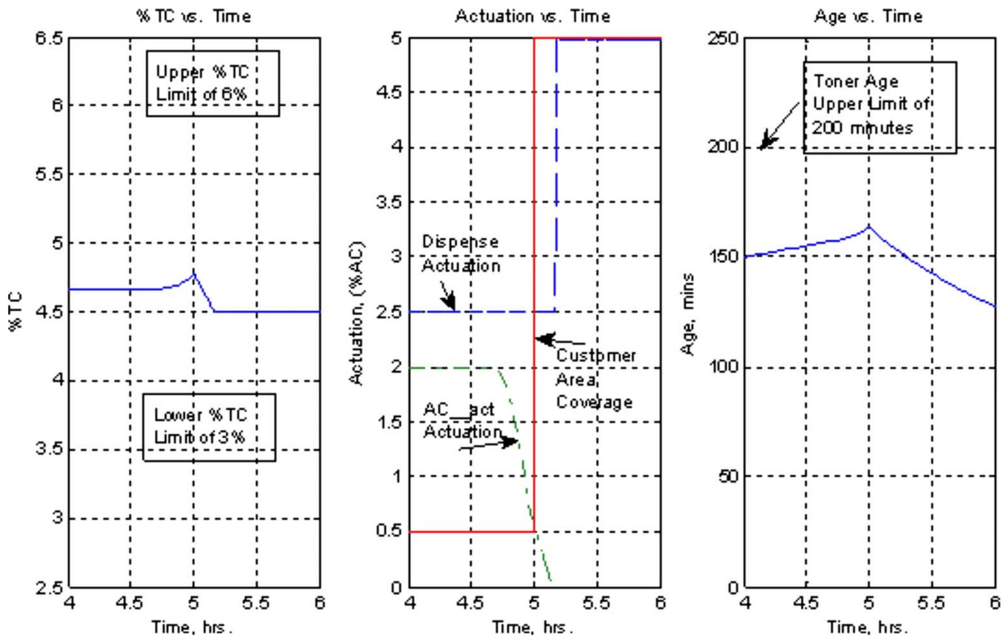

Figure 4. Plant inputs dispense, AC_*IDZ*, and area coverage AC.

allow the toner age to exceed 200 min. The %TC level however continues to increase from target because the penalty weight on actuator AC_act(k) exceeds that of the penalty for %TC deviations from target. Nonetheless AC_act(k) does reluctantly increase as the MPC controller sees the %TC approach the upper constraint of 6%. The increase is gradual until the %TC level reaches the upper constraint after which AC_act(k) quickly increases to prevent exceeding the upper threshold. In this way constraints are fully satisfied.

The final simulation, shown in Figure 4, is similar to the MPC application in Fig. 3 but with a minimum dispense level set to 2.5% and with a step change in customer area coverage at 5 h from 0.5% to 5%. The MPC controller, because of the look-ahead horizon, sees the step change in customer area coverage to 5% in advance and so begins to decrease actuator AC_act(k) about 15 min prior to the actual increase in customer area coverage. In this way the amount of toner sent to the waste bottle is reduced.

CONCLUSION

In conclusion an exploration of the suitability of the model predictive control framework to the design of a xerographic toner dispense system in which toner age and TC are controlled outputs has been presented. Models for the TC and toner age were derived and constraints relevant to a typical production printer were considered. The simulations demonstrated complex yet desirable behavior. A comparison to alternative methods should involve simulation over a representative distribution of jobs.

REFERENCES

1. M. Scheuer, "Toner concentration control for an imaging system", US Patent 6,175,698 (1999).
2. P. Ramesh, "Quantification of toner aging in two component development systems", *Proc. IS&T's NIP21* (IS&T, Springfield, VA, 2005) pp. 544–547.
3. B. Kouvaritakis and M. Cannon, *Nonlinear Predictive Control, Theory and Practice* (The Institutuion of Electrical Engineers, London, UK, 2001).
4. J. M. Maciejowski, *Predictive Control with Constraints* (Pearson Education Limited, Essex, UK, 2002).
5. E. F. Camacho and C. Bordons, *Model Predictive Control* (Springer, London, UK, 2004).
6. B. Kuo, *Automatic Control Systems*, 5th ed. (Prentice-Hall, Englewood Cliffs, NJ, 1987).
7. K. Ogata, *Discrete Control Systems* (Prentice-Hall, Englewood Cliffs, NJ, 1987).
8. E. Gross and E. Hamby, "A method and system for job scheduling in a productive system to improve where purge routines are implemented", US Patent Application 2003172-US-NP XERZ 2 00892 (2005).