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Evaluating the Quality of an Image Acquisition Device Aimed at the Reconstruction of Spectral Reflectances Using Recovery Models

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Abstract. *Accurate recovery of spectral reflectances is important for color reproduction under a variety of illuminations, and several models have been proposed to recover them. To evaluate the quality (Q_r) of an image acquisition system aimed at recovery of spectral reflectances, we proposed an evaluation model based on Wiener estimation and showed that mean square errors between the recovered and measured spectral reflectances as a function of Q_r agree quite well with the prediction from the model, and that estimation of the noise variance of the image acquisition system is essential to the evaluation model. In this paper, the evaluation model was applied to two different reflectance recovery methods, and it is confirmed that the proposed model can be applied to different methods.* © 2008 Society for Imaging Science and Technology.
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INTRODUCTION

Colors are one of the most important characteristics of human visual response, and they have been heavily studied in order to acquire accurate information from color images. The acquisition of the colorimetric information is considered as the acquisition of accurate colorimetric values of objects through the use of sensor responses.[1,2] The accuracy depends on the spectral sensitivities of a set of sensors, the noise present in the acquisition device, the spectral reflectances of the objects, etc.[1,2] Therefore the evaluation of the set of sensors is important for the evaluation of the colorimetric performance or optimization of the spectral sensitivities of the sensors. Several models have been proposed to evaluate a colorimetric performance of a set of color sensors,[3–7] and the optimization of a set of sensors has been performed based on these evaluation models.[8,9] However, application of the evaluation models to real color image acquisition devices such as digital cameras and color scanners has not appeared because of the difficulty in estimating noise levels. Recently, one of the present authors proposed a new model to estimate the noise variance of an image acquisition system,[10] applied it to the proposed colorimetric evaluation model and a spectral evaluation model, and confirmed that the evaluation model agrees quite well with the experimental results from multispectral cameras.[11]

On the other hand, there is an alternative approach for color image acquisition; namely, acquisition of the spectral information of the objects being imaged. The purpose of this approach is the acquisition of the spectral reflectances of the imaged objects through the use of sensor responses.[3,12–29] The acquisition of accurate spectral reflectances of objects is very important in reproducing a color image under a variety of viewing illuminants.[30] The accuracy of the recovered spectral reflectances depends on the number of sensors, their spectral sensitivities, the objects being imaged, the recording illuminants, the noise present in a device, and the model used for the recovery. Therefore the evaluation of a camera intended for recovery of spectral reflectances is important for the optimization of an image acquisition system, and to get an intuitive understanding about the acquisition of the spectral information. One of the present authors already derived an evaluation model based on Wiener estimation.[31] The proposed model is formulated by $\mathrm{MSE}(\sigma^2)=E_{\max}(1-Q_r(\sigma^2))$, where $\mathrm{MSE}(\sigma^2)$ is the mean square error between the recovered and measured spectral reflectances with the estimated noise variance σ^2, $E_{\max}$ represents a constant that is determined only by spectral reflectances of objects, and $Q_r(\sigma^2)$ is the quality of the image acquisition system aimed at recovery of spectral reflectances with the estimated noise variance σ^2. It was shown that $Q_r(\sigma^2)$ is determined by the spectral sensitivities of the sensors, the spectral power distribution of the recording illuminant, the noise variance of the image acquisition device, and the spectral reflectances of the imaged objects. The model was applied to multispectral cameras and it was confirmed that it agrees quite well with experimental results; i.e., the $\mathrm{MSE}(\sigma^2)$ of the recovered spectral reflectances by the Wiener estimation is a linear function of the quality $Q_r(\sigma^2)$ of a set of sensors, when noise is taken into account. As $Q_r(\sigma^2)$ is derived by Wiener estimation, it is very important to confirm whether this model can be applied to other recovery methods, since the quality $Q_r(\sigma^2)$ is useful not only for the evaluation of the image acquisition device but also for the optimization of a set of sensors aimed at recovery of spectral reflectances.

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In this paper, it is shown experimentally that $Q_r(\sigma^2)$ is in linear relation to the MSE(σ^2) of the reflectances recovered by multiple regression analysis[18] and the Imai-Berns model.[26] Mathematical proofs of the equivalence of the Wiener model, the multiple regression model and the Imai-Berns model are given. It is shown that $Q_r(\sigma^2)$ is also appropriately formulated for these models. Once this linear relationship is confirmed, we can estimate $Q_r(\sigma^2)$ by the multiple regression model or the Imai-Berns model, without knowing the spectral sensitivities of the sensors or the spectral power distribution of the recording illuminant, or estimating the noise variance.[32]

This article is organized as follows. The outline of the evaluation model and the method to estimate the noise variance and the models tested are briefly reviewed. In the following sections, the experimental procedures and the results to demonstrate the trustworthiness of the proposal are described. The final section presents conclusions; mathematical proofs are presented in an Appendix.

MODELS FOR THE RECONSTRUCTION OF SPECTRAL REFLECTANCES

In this section, the derivation of the quality $Q_r(\sigma^2)$ to evaluate the color image acquisition system and the models used for the experiments are briefly reviewed.

Wiener Estimation Using Estimated Noise Variance

A vector space notation for color reproduction is useful in the problem. In this approach, the visible wavelengths from 400 to 700 nm are sampled at 10 nm intervals and the number of the samples is denoted as N. A sensor response vector from a set of color sensors for an object with an $N\times 1$ spectral reflectance vector $\mathbf{r}$ can be expressed by

$$\mathbf{p} = \mathrm{S}\mathrm{L}\mathbf{r} + \mathbf{e}, \tag{1}$$

where $\mathbf{p}$ is an $M\times 1$ sensor response vector from the M channel sensors, S is a $M\times N$ matrix of the spectral sensitivities of sensors in which a row vector represents a spectral sensitivity, L is an $N\times N$ diagonal matrix with samples of the spectral power distribution of an illuminant along the diagonal, and $\mathbf{e}$ is a $M\times 1$ additive noise vector. The noise $\mathbf{e}$ is defined to include all the sensor response errors such as the measurement errors in the spectral characteristics of sensitivities, an illumination and reflectances, and quantization errors in this work and it is termed as the system noise[10] below. The system noise is assumed to be signal independent, zero mean and uncorrelated to itself. For abbreviation, let $\mathrm{S_L}=\mathrm{SL}$. The mean square error (MSE) of the recovered spectral reflectances $\hat{\mathbf{r}}$ is given by

$$\mathrm{MSE} = E\{\|\mathbf{r} - \hat{\mathbf{r}}\|^2\}, \tag{2}$$

where $E\{\bullet\}$ represents the expectation. If $\hat{\mathbf{r}}$ is given by $\hat{\mathbf{r}}=\mathrm{W_0}\mathbf{p}$, the matrix $\mathrm{W_0}$ which minimizes the MSE is given by

$$\mathrm{W_0} = \mathrm{R_{SS}}\mathrm{S_L^T}(\mathrm{S_L}\mathrm{R_{SS}}\mathrm{S_L^T} + \sigma_e^2\mathrm{I})^{-1}, \tag{3}$$

where T represents the transpose of a matrix, $\mathrm{R_{SS}}$ is an autocorrelation matrix of the spectral reflectances of samples that will be captured by a device, and σ_e^2 is the noise variance used for the estimation. Substitution of Eq. (3) into Eq. (2) leads to[10]

$$\mathrm{MSE}(\sigma_e^2) = \sum_{i=1}^{N}\lambda_i - \sum_{i=1}^{N}\sum_{j=1}^{\beta}\lambda_i b_{ij}^2 + \sum_{i=1}^{N}\sum_{j=1}^{\beta}\frac{\sigma_e^4 + \kappa_j^{v^2}\sigma^2}{(\kappa_j^{v^2} + \sigma_e^2)^2}\lambda_i b_{ij}^2, \tag{4}$$

where λ_i are the eigenvalues of $\mathrm{R_{SS}}$, b_{ij}, κ_i^v, and β represent jth row of the ith right singular vector, singular value, and a rank of a matrix $\mathrm{S_L V\Lambda^{1/2}}$, respectively, σ^2 is the actual system noise variance, V is a basis matrix, and Λ is an $N\times N$ diagonal matrix with positive eigenvalues λ_i along the diagonal in decreasing order. It is easily seen that the MSE is minimized when $\sigma_e^2=\sigma^2$, and the MSE(σ^2) is given by

$$\mathrm{MSE}(\sigma^2) = \sum_{i=1}^{N}\lambda_i - \sum_{i=1}^{N}\sum_{j=1}^{\beta}\lambda_i b_{ij}^2 + \sum_{i=1}^{N}\sum_{j=1}^{\beta}\frac{\sigma^2}{\kappa_j^{v^2} + \sigma^2}\lambda_i b_{ij}^2. \tag{5}$$

Equation (5) can be rewritten as

$$\mathrm{MSE}(\sigma^2) = \sum_{i=1}^{N}\lambda_i\left(1 - \frac{\sum_{i=1}^{N}\sum_{j=1}^{\beta}\lambda_i b_{ij}^2 - \sum_{i=1}^{N}\sum_{j=1}^{\beta}\frac{\sigma^2}{\kappa_j^{v^2} + \sigma^2}\lambda_i b_{ij}^2}{\sum_{i=1}^{N}\lambda_i}\right). \tag{6}$$

Therefore, the quality of a set of color sensors in the presence of noise is formulated as

$$Q_r(\sigma^2) = \frac{\sum_{i=1}^{N}\sum_{j=1}^{\beta}\lambda_i b_{ij}^2 - \sum_{i=1}^{N}\sum_{j=1}^{\beta}\frac{\sigma^2}{\kappa_j^{v^2} + \sigma^2}\lambda_i b_{ij}^2}{\sum_{i=1}^{N}\lambda_i}. \tag{7}$$

Hence, the MSE(σ^2) is expressed as

$$\mathrm{MSE}(\sigma^2) = E_{\max}(1 - Q_r(\sigma^2)), \tag{8}$$

where $E_{\max}=\Sigma_{i=1}^{N}\lambda_i$. This equation shows that the MSE(σ^2) is linearly related to $Q_r(\sigma^2)$ and the slope of the line is $\Sigma_{i=1}^{N}\lambda_i$. The values of $\Sigma_{i=1}^{N}\lambda_i$ are dependent only on the surface spectral reflectance of the objects being captured. The MSE(σ^2) decreases as the $Q_r(\sigma^2)$ increases to unity.

If we let the noise variance $\sigma_e^2=0$ for the Wiener filter in Eq. (3), then the MSE(0) is derived by letting $\sigma_e^2=0$ in Eq. (4),

$$\mathrm{MSE}(0) = \sum_{i=1}^{N}\lambda_i - \sum_{i=1}^{N}\sum_{j=1}^{\beta}\lambda_i b_{ij}^2 + \sum_{i=1}^{N}\sum_{j=1}^{\beta}\frac{\sigma^2}{\kappa_j^{v^2}}\lambda_i b_{ij}^2. \tag{9}$$

The first and second terms on the right-hand side of Eq. (9) represent the MSE(0) for a noiseless case. We denote this

MSE as $\mathrm{MSE}_{\mathrm{free}}$; then the estimated system noise variance $\hat{\sigma}^2$ can be represented by

$$\hat{\sigma}^2 = \frac{\mathrm{MSE}(0) - \mathrm{MSE}_{\mathrm{free}}}{\sum_{i=1}^{N}\sum_{j=1}^{\beta}\frac{\lambda_i b_{ij}^2}{\kappa_j^{v^2}}}, \quad (10)$$

where $\mathrm{MSE}_{\mathrm{free}}$ is given by

$$\mathrm{MSE}_{\mathrm{free}} = \sum_{i=1}^{N}\lambda_i - \sum_{i=1}^{N}\sum_{j=1}^{\beta}\lambda_i b_{ij}^2. \quad (11)$$

Therefore, the system noise variance σ^2 can be estimated using Eq. (10), since the $\mathrm{MSE}_{\mathrm{free}}$ and the denominator of Eq. (10) can be computed if the surface reflectance spectra of objects, the spectral sensitivities of sensors, and the spectral power distribution of the illuminant are known. The MSE(0) can also be obtained experimentally using Eqs. (2) and (3), and applying the Wiener filter with $\sigma_e^2=0$ to sensor responses. Therefore, Eq. (10) provides a method to estimate actual noise variance σ^2.[10]

The quality $Q_r(\sigma^2)$ and $\mathrm{MSE}(\sigma^2)$ can be computed by substituting the estimated noise variance in Eqs. (7) and (3), respectively.

Multiple Regression Analysis

Let $\mathbf{p}_i$ be an $M\times 1$ sensor response vector that is obtained by image acquisition of a known spectral reflectance $\mathbf{r}_i$ of the ith object, where i represents a number. Let P be an $M\times k$ matrix that contains the sensor responses $\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_k$, and let R be an $N\times k$ matrix that contains the corresponding spectral reflectances $\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_k$, where k is the number of the learning samples. The pseudoinverse model is to find a matrix W that minimizes $\|\mathrm{R}-\mathrm{WP}\|$, where $\|\bullet\|$ represents the Frobenius Norm.[33] The matrix W is given by

$$\mathrm{W} = \mathrm{RP}^{+}, \quad (12)$$

where, P^{+} represents the pseudoinverse matrix of the matrix P. By applying a matrix W to a sensor response vector $\mathbf{p}$, i.e., $\hat{\mathbf{r}}=\mathrm{W}\mathbf{p}$, a spectral reflectance is estimated. Therefore this model does not use the spectral sensitivities of sensors or the spectral power distribution of the illumination; it uses only the spectral reflectances of the learning samples.

The Imai-Berns Model

The Imai-Berns model[26] is considered to be a modification of the linear model using multiple regression analysis between the weight column vectors, as basis vectors to represent the known spectral reflectances, and corresponding sensor response vectors.

Let Σ be a $d\times k$ matrix that contains the column vectors of the weights $\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\ldots,\boldsymbol{\sigma}_{\mathrm{k}}$ to represent the k known spectral reflectances $\mathbf{r}_1,\mathbf{r}_2,\cdots,\mathbf{r}_k$, and let P be a $M\times k$ matrix that contains corresponding sensor response vectors of those reflectances $\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_k$, where d is the number of the weights required to represent the spectral reflectances. The multiple regression analysis between these matrixes is expressed as $\|\Sigma-\mathrm{BP}\|$. A matrix B, which minimizes the Frobenius Norm, is given by

$$\mathrm{B} = \Sigma\mathrm{P}^{+}. \quad (13)$$

Since a weight column vector $\boldsymbol{\sigma}$ for a sensor response vector $\mathbf{p}$ is estimated as $\hat{\boldsymbol{\sigma}}=\mathrm{B}\mathbf{p}$, the estimated spectral reflectance vector is derived from $\hat{\mathbf{r}}=\mathbf{V}\hat{\boldsymbol{\sigma}}$, where a matrix V is the basis matrix that contains first d orthonormal basis vectors of the spectral reflectances. This model does not use the spectral characteristics of the sensors or the illumination.

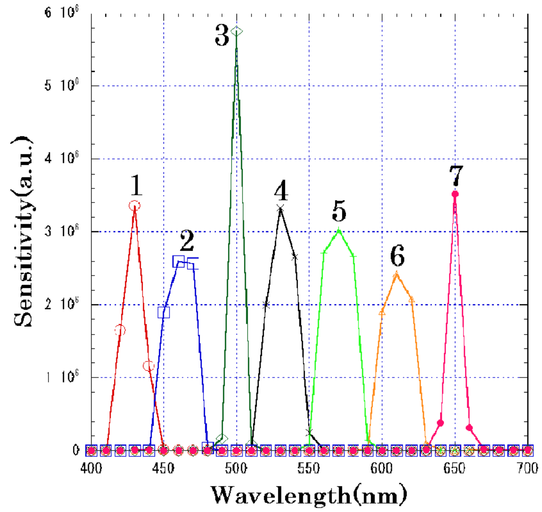

Figure 1. Spectral sensitivities of the sensors of the camera.

EXPERIMENTAL PROCEDURES

A multispectral color image acquisition system was assembled by using seven interference filters (Asahi Spectral Corporation) in conjunction with a monochrome video camera (Kodak KAI-4021M). Image data from the video camera were converted to 16-bit-depth digital data by an AD converter. The spectral sensitivity of the video camera was measured over wavelength from 400 to 700 nm at 10 nm intervals. The measured spectral sensitivities of the camera with each filter are shown in Figure 1. The illuminant used for image capture was the illuminant that simulates daylight (Seric Solax XC-100AF). The spectral power distribution of the illuminant measured by the spectroradiometer (Minolta CS-1000) is presented in Figure 2.

We denote the GretagMacbeth ColorChecker (24 colors) and the Kodak Q60R1 (228 Colors) as CC and KK, respectively, for abbreviation; these arrays were illuminated from the direction of about 45° to the surface, and the images were captured by the camera from the normal direction. The image data were corrected for nonuniformities in illumination and sensitivities of the pixels of the CCD. The computed responses from the camera to a given color, estimated using the measured spectral sensitivities of the sensors, the illuminant, and the surface reflectance of the color, were not equal to the actual sensor responses since the absolute spec-

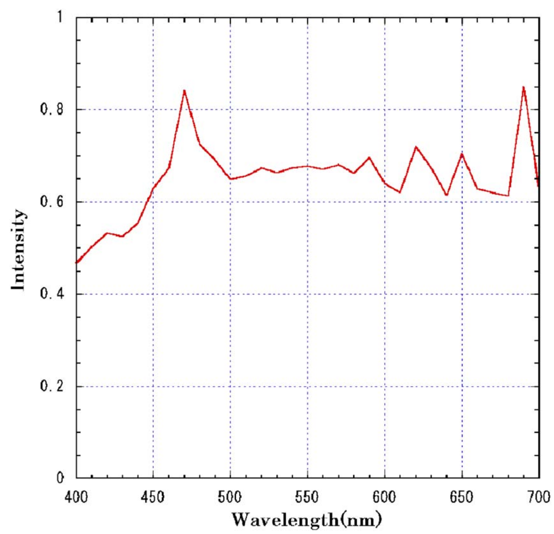

Figure 2. Spectral power distribution of the illumination.

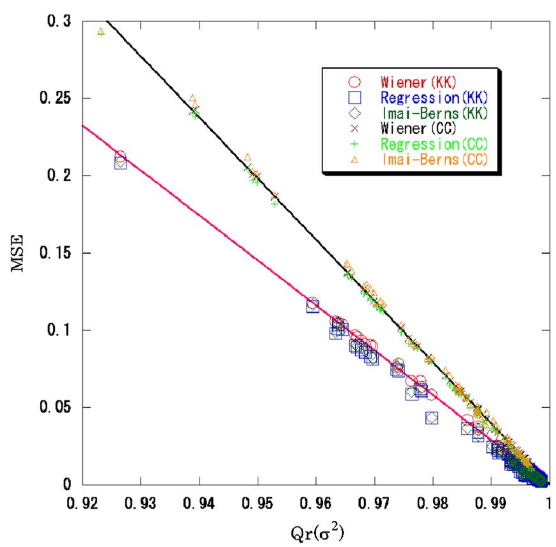

Figure 3. The MSEs of the recovered spectral reflectances by the Wiener, regression, and Imai-Berns method for the GretagMacbeth ColorChecker (CC) and the Kodak Q60R1 (KK) are plotted as a function of $Q_r(\sigma^2)$.

tral sensitivities of a camera depend on the camera gain. Therefore, the sensitivities were calibrated using an achromatic color in the charts. In this work, the constraint is imposed on the signal power given by $\rho = \mathrm{Tr}(S_L R_{SS} S_L^T)$, where $\rho = 1$ was used so that the estimated system noise variance could be compared for different sensor sets.

By using various combinations of sensors from the three to seven in Fig. 1, the system noise variance was estimated by the methods described above for each combination of sensors. The estimated noise variance was then used to recover the spectral reflectances by the Wiener estimation, and the MSE(σ^2) of the recovered spectral reflectances was computed. The spectral reflectances were also recovered by both the multiple regression model and the Imai-Berns model. By using the estimated noise variance, the quality $Q_r(\sigma^2)$ for each combination of sensors was computed using Eq. (7).

RESULTS AND DISCUSSION

The values of the MSE(σ^2) of the CC and KK as a function of $Q_r(\sigma^2)$ for the 80 sets of sensors are shown in Figure 3. The lines in the figure indicate the theoretical relationship between MSE(σ^2) and $Q_r(\sigma^2)$ as given by Eq. (8) for the two color charts, where $E_{\max} = \Sigma_{i=1}^{N} \lambda_i$ was used for the determination of the slopes of the line for each color chart. The experimental results for MSE as a function of $Q_r(\sigma^2)$ by the multiple regression analysis and the Imai-Berns method agree well with the theoretical lines.

To show the importance of considering the noise variance, let $Q_r(0)$ be the value of $Q_r(\sigma^2)$ when the noise variance $\sigma_2 = 0$; i.e.,

$$Q_r(0) = \frac{\sum_{i=1}^{N} \sum_{j=1}^{\beta} \lambda_i b_{ij}^2}{\sum_{i=1}^{N} \lambda_i}. \tag{14}$$

Note that Eq. (9) was used for the MSEs of the recovered reflectances by use of the Wiener filter with zero noise variance. The relationship between MSE and $Q_r(0)$ is shown in Figure 4.

In Fig. 4, plots disagree not only with the theoretical lines but also scatter more in comparison to Fig. 3. Especially the plots for KK data recovered by the Wiener model scatter far above the theoretical line. These scattered plots indicate the importance of the accurate estimation of the noise variance in the images. We also confirmed that plots of CC data scatter more in Fig. 4 when the 6-bit AD converter is used, instead of the 16-bit converter, to digitize the sensor responses.

The results in Fig. 3 agree well with the theoretical predictions, which means that $Q_r(\sigma^2)$ can be used for multiple regression analysis and for the Imai-Berns model. As a matter of fact, the multiple regression model and the Imai-Berns model are mathematically equivalent to the Wiener estimation, i.e., the matrixes W in Eq. (12) and B in Eq. (13) are equivalent to the matrix of the Wiener filter W_0 in Eq. (3), which can be proved by mathematical analysis. For these proofs of the equivalences, see the Appendix.

Typical examples of the recovered reflectances and the reproduced color images for three cases of the $Q_r(\sigma^2)$ are shown in Figures 5–7. It is very clear that the error of the recovered spectral reflectances increases with a decrease in the $Q_r(\sigma^2)$, and faithfulness of the reproduced colors decreases with a decrease in the $Q_r(\sigma^2)$. Typical examples of the maximum and minimum values of the $Q_r(\sigma^2)$ and MSE(σ^2) by the three models for each number (three to seven) of sensor sets are shown in Table I. It is very interesting that a set of four sensors (sensor number "2457") has a larger $Q_r(\sigma^2)$ than a set of six sensors (sensor number "123456"). It is not always true that the $Q_r(\sigma^2)$ increases when the number of the sensors increases.

It has now been confirmed that the MSE(σ^2) of the spectral reflectances recovered from the multiple regression model and from the Imai-Berns model have a linear relation

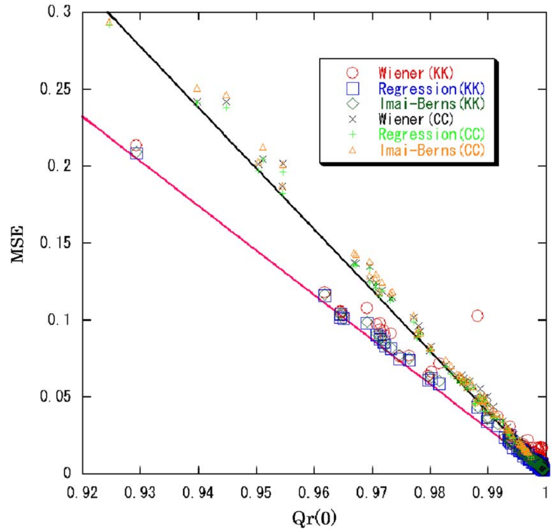

Figure 4. The MSEs of the recovered spectral reflectances by the Wiener, regression, and Imai-Berns method for the GretagMacbeth ColorChecker (CC) and the Kodak Q60R1 (KK) are plotted as a function of $Q_r(0)$, which is the value of $Q_r(\sigma^2)$ when the estimated noise variance is zero. This is the case without consideration for the noise.

to the quality $Q_r(\sigma^2)$ of the image acquisition system. Once this linear relation is confirmed, we can estimate $Q_r(\sigma^2)$ by the multiple regression or by the Imai-Berns model without the spectral sensitivities of sensors, the spectral power distribution of the recording illuminant or the noise present in the image acquisition system since it ($Q_r(\sigma^2)$) can be easily estimated by marking the value of the MSE by the models on the theoretical line; i.e., the corresponding $Q_r(\sigma^2)$ of the point gives the estimate. Now it is possible to estimate the quality $Q_r(\sigma^2)$ by the multiple regression model or by the Imai-Berns model with only the spectral reflectances and the captured images of the object.

CONCLUSION

The evaluation of an image acquisition system aimed at recovery of spectral reflectances, which is derived based on the Wiener estimation, was applied to the multiple regression analysis and the Imai-Berns method. The experimental results by multispectral cameras agree quite well with the proposed model. From this result, it is concluded that the proposed evaluation model is appropriately formulated and that the estimation of the noise variance of an image acquisition system is essential to evaluate the quality $Q_r(\sigma^2)$. This result also gives us an easy way to estimate the quality $Q_r(\sigma^2)$ and provides us an easier way to evaluate an image acquisition system aimed at reconstruction of spectral reflectances without the spectral sensitivities of sensors, the spectral power distribution of the recording illuminant, or the noise present in the image acquisition system.

ACKNOWLEDGMENTS

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(a)

(b)

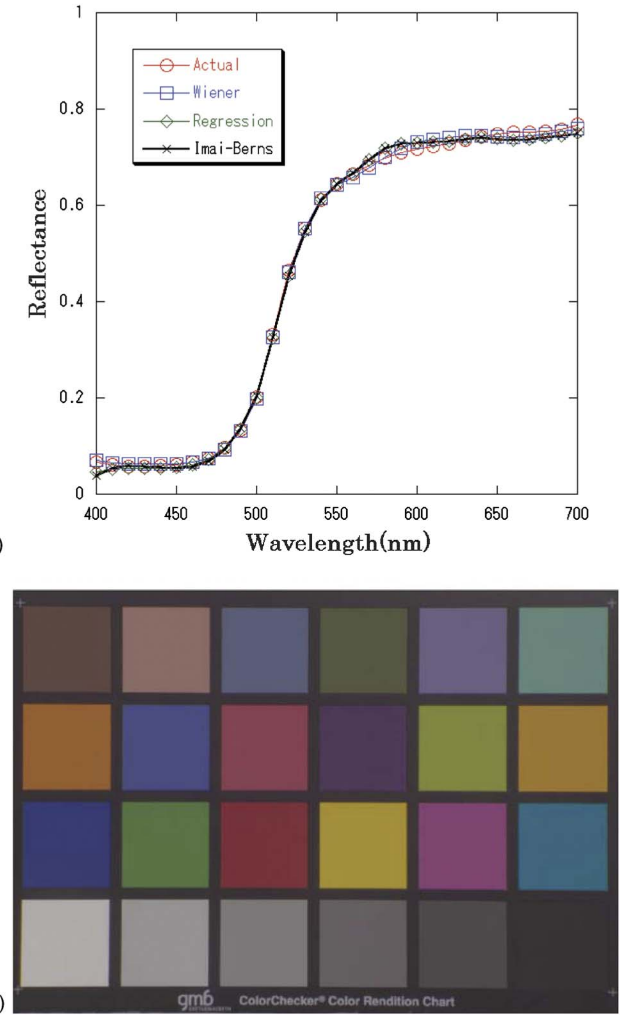

Figure 5. (a) Typical example of the recovered spectral reflectance of the color red at a large $Q_r(\sigma^2)$($Q_r(\sigma^2)=0.996\,894$). (b) The color reproduction of the GretagMacbeth ColorChecker by the recovered spectral reflectance at $Q_r(\sigma^2)=0.996\,894$.

APPENDIX

Proof of the Equivalence of the Multiple Regression Model to the Wiener Model

The multiple regression model minimizes

$$\|\mathrm{R}-\mathrm{WP}\|, \tag{A1}$$

where P is a $M\times k$ matrix that contains the sensor response vectors $\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_k$ and let R be a $N\times k$ matrix that contains the corresponding spectral reflectances vectors $\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_k$, where k is the number of the learning samples. The $N\times M$ matrix W, which minimizes Eq. (A1), is given by

$$\mathrm{W}=\mathrm{RP}^{+}, \tag{A2}$$

where P^{+} represents the pseudoinverse matrix of the matrix P;

$$\mathrm{P}^{+}=\mathrm{P}^{T}(\mathrm{PP}^{T})^{-1}, \tag{A3}$$

because $M<k$ holds in the image acquisition devices and $\mathrm{Rank(P)}=M$. Let

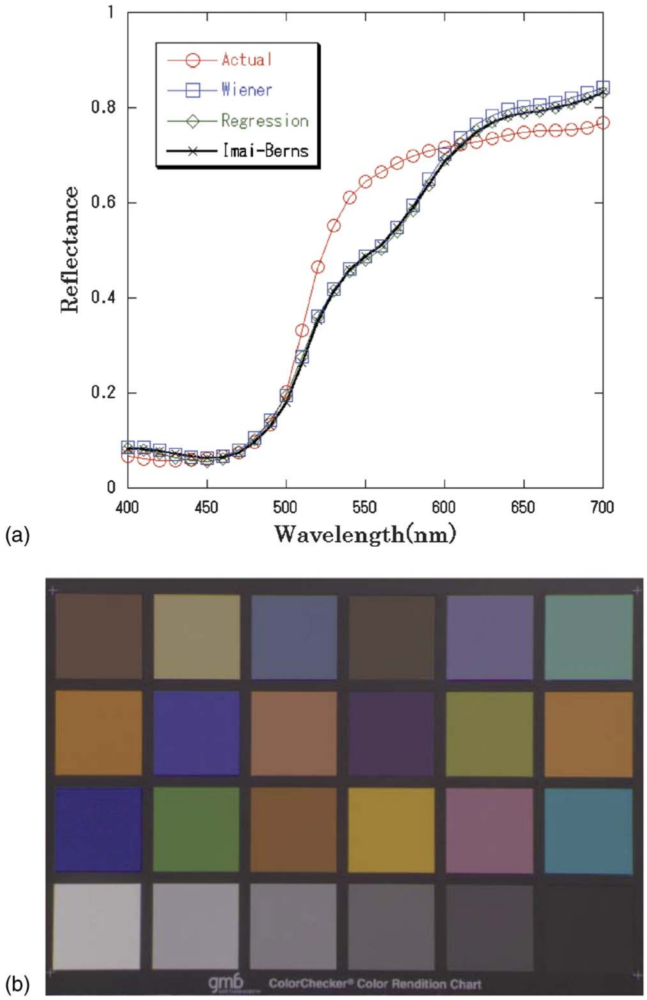

Figure 6. (a) Typical example of the recovered spectral reflectance of the color red at a middle $Q_r(\sigma^2)(Q_r(\sigma^2)=0.965\,348)$. (b) The color reproduction of the GretagMacbeth ColorChecker by the recovered spectral reflectance at $Q_r(\sigma^2)=0.965\,348$.

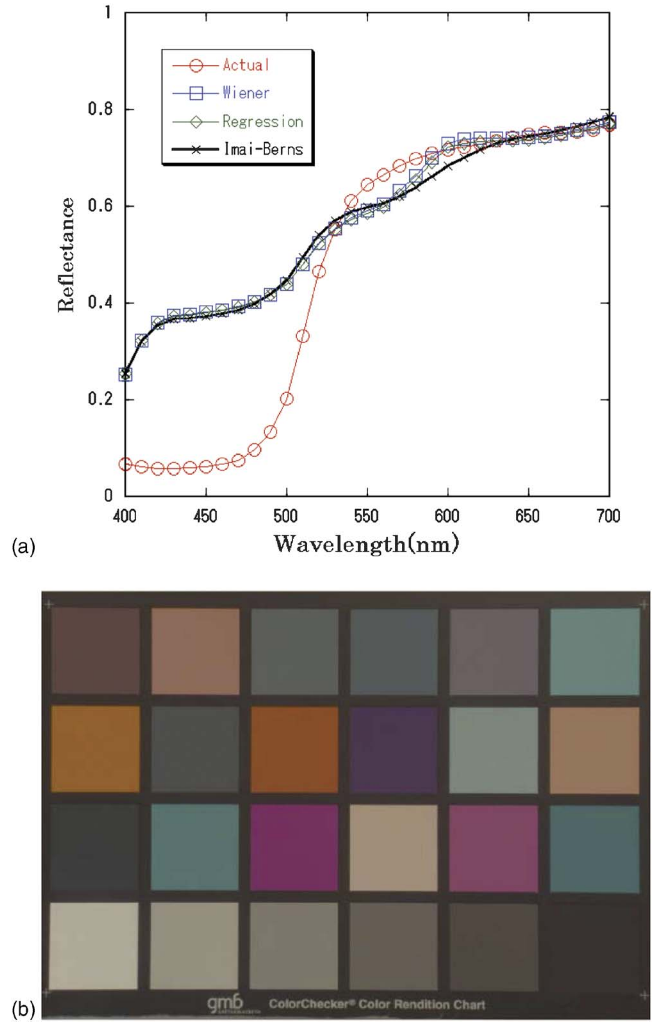

Figure 7. (a) Typical example of the recovered spectral reflectance of the color red at a small $Q_r(\sigma^2)(Q_r(\sigma^2)=0.938\,896)$. (b) The color reproduction of the GretagMacbeth ColorChecker by the recovered spectral reflectance at $Q_r(\sigma^2)=0.938\,896$.

$$\mathrm{P} = \mathrm{SLR} + \mathrm{E}, \tag{A4}$$

where S is a $M \times N$ matrix of the spectral sensitivities of sensors in which a row vector represents a spectral sensitivity, L is an $N \times N$ diagonal matrix with samples of the spectral power distribution of an illuminant along the diagonal, and E is a $M \times N$ matrix that contains the additive noise vectors. For abbreviation, let $\mathrm{S_L}=\mathrm{SL}$. Substitution of Eq. (A3) and Eq. (A4) into Eq. (A2) leads to

$$\mathrm{W} = \mathrm{R}(\mathrm{S_L R} + \mathrm{E})^{\mathrm{T}}((\mathrm{S_L R} + \mathrm{E})(\mathrm{S_L R} + \mathrm{E})^{\mathrm{T}})^{-1}. \tag{A5}$$

Hence, W is rewritten as

$$\mathrm{W} = \mathrm{R_{SS} S_L^T}(\mathrm{S_L R_{SS} S_L^T} + \sigma_e^2 \mathrm{I})^{-1}, \tag{A6}$$

because $\mathrm{RR^t}$ is an autocorrelation matrix of R, and $\mathrm{EE^t}$ gives the noise variance; $\mathrm{RE^T}=\mathrm{ER^T}=0$ as the spectral reflectances and the error have no correlation. Thus, the matrix W is equivalent to that of the Wiener filter.

Proof of the Equivalence of the Imai-Berns Model to the Wiener Model

Let Σ be a $d \times k$ matrix that contains the vectors of the weights to represent the k known spectral reflectances, where d is a number of the weights to represent the spectral reflectances and let P, R, $\mathrm{S_L}$, and E be as defined in the Appendix.

A $d \times M$ matrix B, which minimizes

$$\|\Sigma = \mathrm{BP}\| \tag{B1}$$

is given by

$$\mathrm{B} = \Sigma \mathrm{P}^{+}. \tag{B2}$$

From the Eqs. (A2) through (A6), it is easily understood that

$$\mathrm{VB} = \mathrm{V}\Sigma\mathrm{P}^{+} = \mathrm{V}\Sigma(\mathrm{R^T S_L^T} + \mathrm{E^T})(\mathrm{S_L RR^T S_L^T} + \mathrm{EE^T})^{-1}. \tag{B3}$$

From the definition of the method, $\mathrm{R}=\mathrm{V}\Sigma$, where V is an orthonormal basis matrix. Hence,

Table I. The maximum and minimum $Q_r(\sigma^2)$ for each number of sensors of the Macbeth ColorChecker.

$Q_r(\sigma^2)$	Number of sensors	Sensors	$Q_r(0)$	MSE(σ^2) (Wiener)	MSE(0)	MSE (regression)	MSE (Imai-Berns)
0.996 894	7ch	1234 567	0.997 397	0.012 164	0.012 423	0.010 682	0.011 500
0.996 324	6ch	134 567	0.996 891	0.014 354	0.014 631	0.013 326	0.014 258
0.995 627	5ch	13 467	0.996 312	0.017 177	0.017 348	0.016 063	0.016 816
0.993 334	4ch	2 457	0.993 690	0.026 408	0.026 374	0.025 583	0.026 071
0.989 134	6ch	123 456	0.990 872	0.043 106	0.043 771	0.037 802	0.038 943
0.987 814	3ch	257	0.988 318	0.048 248	0.048 213	0.047 758	0.048 249
0.923 114	5ch	12 345	0.924 614	0.304 426	0.304 508	0.292 043	0.293 503
0.856 650	4ch	1 234	0.860 630	0.569 539	0.569 701	0.558 052	0.563 130
0.787 850	3ch	123	0.791 501	0.838 561	0.839 893	0.817 497	0.826 465

$$VB = R_{SS}S_L^T(S_L R_{SS} S_L^T + \sigma_e^2 I)^{-1}. \quad (B4)$$

Thus, the Imai-Berns model is equivalent to the Wiener model.

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