

Digital Watermarking of Spectral Images Using PCA-SVD

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Abstract. Kaarna et al. [*J. Imaging Sci. Technol.* **48**, 183–193 (2004)] presented a technique based on principal component analysis (PCA) to embed a digital watermark containing copyright information into a spectral image. In this paper, a hybrid watermarking method based on the pure PCA approach of Kaarna et al. and singular value decomposition (SVD) is proposed. The performance of the proposed technique is compared with a pure PCA based technique against attacks including lossy image compression, median, and mean filtering. The experiments show that the proposed method outperforms a pure PCA based technique.
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INTRODUCTION

Copyright protection is becoming more important in open networks since digital copying maintains the original data and copying can be easily made at high speed. Digital watermarks offer a possibility to protect copyrighted data in the information society. A watermarking procedure consists of two parts: watermark embedding and extraction algorithms. The watermarking procedure should maintain the following properties: the watermark should be undetectable and hidden from an unauthorized user, the watermark should be invisible or inaudible in the information carrying signal, and, finally, the watermark should be robust towards possible attacks.^{1,2}

Normal RGB images have three color bands and the information for those bands is integrated from the wavelengths of visible light. The spectral images have a large number of bands and they may contain information from a wider spectrum, also outside the visible range. Spectral imaging has various applications in remote sensing and can now be used in industrial applications including quality control, exact color measurement, and color reproduction. This evolution has been possible due to the development in the spectral imaging systems.^{3–5}

Several watermarking techniques have been developed for spectral images. The grayscale watermark can be embed-

ded in the transform domains such as by discrete wavelet transform (DWT) of the spectral image.^{6,7} Furthermore, the grayscale watermark can also be embedded in the principal component analysis (PCA) transform domain of the spectral image.^{5,8}

In this paper, a new watermarking method is proposed for spectral imaging. The paper is organized as follows: principal component analysis (PCA) and singular value decomposition (SVD) are briefly discussed in the next section. The proposed watermarking procedure is described in the third section. The performance measures for the watermarking techniques are introduced in the fourth section. Comparisons with the pure PCA approach of Kaarna et al.^{5,8} are given in the fifth section and the conclusions are drawn in the final section.

PCA AND SVD

In order to describe the new proposed method, the principal component analysis (PCA) and singular value decomposition (SVD) are briefly discussed here.

Principal Component Analysis

PCA has been widely applied to spectral image analysis and spectral image coding.^{5,9,10} Let $\Omega = \{x\}$ be a given data set containing N column vectors. The main features of the data set Ω can be extracted using the PCA algorithm, which has the following three steps:

- (1) Compute the mean of the data set:

$$\mu = \frac{1}{N} \sum_{x \in \Omega} x. \quad (1)$$

- (2) Compute the covariance matrix C defined by

$$C = \frac{1}{N} \sum_{x \in \Omega} (x - \mu)(x - \mu)^T. \quad (2)$$

- (3) Compute eigenvalues λ_i and eigenvectors s_i of the symmetric and semi-positive definite matrix C with $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n \geq 0$. Here n is the number of components

of the vector x ($x \in \Omega$), and normally $n < N$. Superscript T is the transpose of a vector or matrix.

The vectors s_i ($i=1, 2, \dots, n$) form a basis for space R^n (the set of column vectors with n components). Thus, for any $x \in \Omega$,

$$x = \sum_{i=1}^n (x^T s_i) s_i. \quad (3)$$

In general, a smaller integer $p (< n)$ can be chosen so that the first p eigenvectors' combination is a good approximation, i.e.,

$$x \approx \sum_{i=1}^p (x^T s_i) s_i. \quad (4)$$

Singular Value Decomposition

Every $m \times m$ real matrix A has the following decomposition:

$$A = U \Sigma V^T, \quad (5)$$

where U and V are $m \times m$ orthogonal matrices, respectively, and Σ is a diagonal matrix having the following form:

$$\Sigma = \begin{pmatrix} \sigma_1 & & 0 \\ & \ddots & \\ 0 & & \sigma_m \end{pmatrix} = \text{diag}(\sigma_1, \dots, \sigma_m). \quad (6)$$

Here σ_i are the singular values, and they satisfy

$$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_m \geq 0. \quad (7)$$

The above decomposition is called the singular value decomposition (SVD) of the matrix A . If we let u_i and v_i be the column vectors of the orthogonal matrices U and V , then

$$A = \sum_{i=1}^m \sigma_i u_i v_i^T. \quad (8)$$

Note that the SVD separates A into two parts: U , V similar to the eigenvectors or components in the PCA, and Σ similar to the eigenvalues in PCA. Hence, the main components of A are $u_i v_i^T$, and σ_i decides the proportion of those main components $u_i v_i^T$.

Note also that in this paper it is assumed that when PCA or SVD is applied, the resulting eigenvalues λ_i or singular values σ_i are arranged in descending order of inequality (7). In addition, symbol $(A)_{ij}$ denotes the (i, j) position element or pixel of the matrix or image A .

THE PROPOSED WATERMARKING PROCEDURE

Consider an $m \times m$ spectral image having n bands. Thus each band image $S^{(k)}$ can be represented as a matrix of the following form:

$$S^{(k)} = \begin{pmatrix} s_{11}^{(k)} & \dots & s_{1m}^{(k)} \\ \vdots & \ddots & \vdots \\ s_{m1}^{(k)} & \dots & s_{mm}^{(k)} \end{pmatrix}, \quad k = 1, 2, \dots, n. \quad (9)$$

Let Ω be the set of the spectral vectors r_{ij} defined by

$$r_{ij}^T = (s_{ij}^{(1)}, s_{ij}^{(2)}, \dots, s_{ij}^{(n)}), \quad i, j = 1, 2, \dots, m. \quad (10)$$

Thus the set Ω has $N = m^2$ spectral vectors. In addition, we assume the watermark image W is a $m \times m$ gray scale image. Now we are ready to describe the proposed watermarking procedure:

Watermark Embedding Algorithm

(1) Apply SVD to the visual watermark image W , resulting in

$$W = U_w \Sigma_w V_w^T, \quad (11)$$

with

$$\Sigma_w = \text{diag}(\sigma_{w,1}, \dots, \sigma_{w,m}). \quad (12)$$

(2) Apply PCA to spectral domain of the spectral image, i.e., apply PCA to the set Ω , resulting in eigenvalues λ_i and eigenvectors s_i for $i=1, 2, \dots, n$. Thus, for each r_{ij} in Ω we have

$$r_{ij} = \sum_{k=1}^n (r_{ij}^T s_k) s_k. \quad (13)$$

Hence, for each k , ($k=1, 2, \dots, n$) the k th eigenimage $E^{(k)}$ can be defined by letting $(E^{(k)})_{ij}$ be $r_{ij}^T s_k$, i.e., $(E^{(k)})_{ij} = r_{ij}^T s_k$ with $i, j=1, 2, \dots, m$. Thus, the spatial size of each eigenimage $E^{(k)}$ is the same as that of the original spectral image.

(3) Choose an eigenimage $E^{(k')}$ with k' satisfying $1 \leq k' \leq n$ and apply SVD to it, resulting in

$$E^{(k')} = U_e \Sigma_e V_e^T \quad (14)$$

with

$$\Sigma_e = \text{diag}(\sigma_{e,1}, \dots, \sigma_{e,m}). \quad (15)$$

(4) Modify the singular values of the eigenvalue image $E^{(k')}$ by mixing the singular values of the watermark image W with those of $E^{(k')}$:

$$\bar{\sigma}_{e,i} = \alpha_e \sigma_{e,i} + \alpha_w \frac{\sigma_{w,1}}{\sigma_{w,i}}, \quad i = 1, 2, \dots, m, \quad (16)$$

where the coefficients α_e and α_w control the strength of the embedding.

(5) Obtain the modified eigenimage:

$$\bar{E}^{(k')} = U_e \bar{\Sigma}_e V_e^T, \quad \text{with } \bar{\Sigma}_e = \text{diag}(\bar{\sigma}_{e,1}, \dots, \bar{\sigma}_{e,m}). \quad (17)$$

(6) The watermarked spectral image is constructed by computing the modified spectral vectors \bar{r}_{ij}

($i, j=1, 2, \dots, m$) using the inverse PCA transform, the modified eigenimages, and the original spectral eigenvectors. Let

$$F^{(k)} = \begin{cases} E^{(k)}, & \text{if } k \neq k' \\ \tilde{E}^{(k')}, & \text{if } k = k'. \end{cases} \quad (18)$$

Then

$$\tilde{r}_{ij} = \sum_{k=1}^n (F^{(k)})_{ij} s_{k'}, \quad i, j = 1, 2, \dots, m, \quad (19)$$

where $s_{k'}$'s are the eigenvectors obtained in step (2).

Note that the watermark embedding algorithm given by Kaarna et al.⁵ is simpler than the one given above. Their algorithm does not need the singular value decompositions of the watermark image W [Eq. (11)] and k' 'th eigenimage (or multiplier image) $E^{(k')}$ [Eq. (14)]. Their watermark embedding can be simply expressed by

$$\tilde{E}^{(k')} = \alpha E^{(k')} + \beta W. \quad (20)$$

Thus, their method hides the whole watermark image into the original image. However, the watermark image W may consist of fine detail over a significant portion of a slowly varying background level. Hence the gray levels of W often change rapidly around the edges. Thus, the change from $E^{(k')}$ to $\tilde{E}^{(k')}$ [Eq. (20)] is inevitably not "smooth," which will result in greater visual difference between the original and the watermarked images, i.e., the watermark will be more visible. On the other hand, by using the singular values decomposition $W = U_w \Sigma_w V_w^T$, the watermark image is thus separated into two parts: main components U_w and V_w or $v_{w,i} v_{w,i}^T$ and Σ_w or $\sigma_{w,i}$. Our proposed method only hides Σ_w into the original image. Since W has m^2 values (pixels), while Σ_w has only m values, it is expected that our proposed method has a better embedding quality. In fact, from Eqs. (11) and (14) the modified eigenimage $\tilde{E}^{(k')}$ defined by Eq. (17) can be expressed by

$$\tilde{E}^{(k')} = U_e \tilde{\Sigma}_e V_e^T = \alpha_e U_e \Sigma_e V_e^T + \delta U_e \Sigma_w V_e^T. \quad (21)$$

Thus, the proposed change from $E^{(k')}$ to $\tilde{E}^{(k')}$ is expected to be "smooth" (avoiding sudden changes between pixels), which will result in less visual difference between the original and watermarked images.

Note also that naturally the watermarked spectral image differs from the original image. The difference between these images depends on the strength α_e and α_w in the watermark embedding according to Eq. (16). The strength balances between properties like robustness, invisibility, security, and false-positive detection of the watermark. The selection of a band or the integer k' in step (3) affects the visibility of the watermark in the watermarked image and the quality of the reconstruction of the watermark image against possible attacks. It is clear that the smaller the value of k' , the more visible the watermark will be. On the other hand, the larger the value of k' , the lower resistance against attacks. All these effects will be investigated below.

Watermark Image Extraction Algorithm

(1) Compute $e_{ij} = (\tilde{r}_{ij})^T s_{k'}$, with k' being defined in step (3) of the watermark embedding algorithm, and form matrix or image E by setting $(E)_{ij} = e_{ij}$ for $i, j = 1, 2, \dots, m$.

(2) Apply SVD to E , resulting in

$$E = U \Sigma V^T \quad \text{with } \Sigma = \text{diag}(\sigma_1, \dots, \sigma_m).$$

Note that the singular value σ_i is or is approximately equal to $\tilde{\sigma}_{e,i}$ in Eq. (16)

(3) Reconstruct or estimate the singular values of the watermark image by inverting Eq. (16):

$$\tilde{\sigma}_{w,i} = \frac{\sigma_i - \alpha_e \sigma_{e,i}}{\alpha_w \frac{\sigma_{e,1}}{\sigma_{w,1}}}, \quad i = 1, 2, \dots, m.$$

(4) Reconstruct the watermark image $W^{(r)}$ by computing

$$W_{PCA+SVD}^{(r)} = U_w \tilde{\Sigma}_w V_w^T \quad \text{with } \tilde{\Sigma}_w = \text{diag}(\tilde{\sigma}_{w,1}, \dots, \tilde{\sigma}_{w,m}).$$

Note that the above extraction algorithm needs some information from the embedding algorithm. They are the k' 'th eigenvector $s_{k'}$ generated from step (2), the watermark image's decomposition matrices U_w, Σ_w, V_w in Eq. (11), the singular values (Σ_e) of eigenimage $E^{(k')}$, obtained in step (3), and the strength α_e and α_w . Hence they should be kept by the owner. However, if storage space is critical, U_w, Σ_w, V_w need not be kept since the watermark image should be available to the owner, hence the values can be obtained by singular decomposition when needed.

Note also that any attack (filtering or compression) on the watermarked image will degrade the reconstructed watermark image. If the pure PCA approach is used for the watermark embedding, any attack will somehow directly affect the whole watermark image or U_w, Σ_w, V_w . Thus, the reconstructed watermark image is given by

$$W_{PCA}^{(r)} = \tilde{U}_w \hat{\Sigma}_w \tilde{V}_w^T.$$

While if our embedding method is used, an attack will only affect Σ_w or $\sigma_{w,i}$ since the main components U_w, V_w of the watermark image are not hidden in the watermarked image. Hence it is expected that the reconstructed watermark image is given by

$$W_{PCA+SVD}^{(r)} = U_w \tilde{\Sigma}_w V_w^T,$$

which is closer to $W = U_w \Sigma_w V_w^T$ than $W_{PCA}^{(r)} = \tilde{U}_w \hat{\Sigma}_w \tilde{V}_w^T$, if $\hat{\Sigma}_w$ and $\tilde{\Sigma}_w$ are not too different, which is confirmed by numerical simulations given in the fifth section.

QUALITY MEASUREMENTS OF THE EMBEDDING AND EXTRACTION

The watermark must be not only imperceptible but also robust so that it can survive some basic attacks and image distortions. Since spectral images are often very large in both

spectral and spatial dimensions, lossy image compression is usually applied to them. In general, however, lossy compression will lower the quality of the image and of the extracted watermark. 3D-SPIHT is the modern-day benchmark for three-dimensional image compression.¹¹

Other possible attacks are different kinds of filtering operations, such as median filtering and mean filtering.

The quality in embedding was measured by peak signal-to-noise ratio (PSNR), which is defined as

$$\text{PSNR} = 10 \log_{10} \frac{nm^2s^2}{E^d}, \quad (22)$$

where E^d is the energy of the difference between the original and watermarked images, n is the number of bands in the spectral image, m^2 is the number of pixels in the image, and s is the peak value of the original spectral image.

Note that the closer the original and watermarked spectral images, the smaller the value of E^d . Hence, the larger the value of PSNR, the closer will be the original and watermarked spectral images.

Another performance measure^{5,8} used is to test the average spectra for each band of the watermarked spectral image and to compare with the average spectra for the corresponding band of the original spectral image. The smaller difference indicates the better performance of the watermark embedding technique. The large changes may induce incorrect results as, for example, in classification applications.

Kaarna et al.⁵ used the correlation coefficient for measuring the similarity between the original and reconstructed watermark images. In this paper, the correlation coefficient (cc) is also used as a measure of the quality of the extracted watermark image and is defined as

$$\text{cc} = \frac{\sum_{i=1}^m \sum_{j=1}^m [(W)_{ij} - \bar{\mu}][(W^{(r)})_{ij} - \bar{\mu}^{(r)}]}{\sqrt{\left\{ \sum_{i=1}^m \sum_{j=1}^m [(W)_{ij} - \bar{\mu}]^2 \right\} \left\{ \sum_{i=1}^m \sum_{j=1}^m [(W^{(r)})_{ij} - \bar{\mu}^{(r)}]^2 \right\}}}, \quad (23)$$

where W and $W^{(r)}$ are the original and reconstructed watermark images, respectively, and where $\bar{\mu}$ and $\bar{\mu}^{(r)}$ are the mean values of the gray level values of the original and reconstructed watermark images, respectively. Note that the measure cc is equal to unity if the two images W and $W^{(r)}$ are the same. Hence, the closer to 1 the measure cc is, the closer to the original watermark image W the extracted watermark image $W^{(r)}$ will be.

Besides, the root-mean-square (RMS) error defined by Eq. (24) below is also used as a similarity measure between the original and reconstructed watermark images:

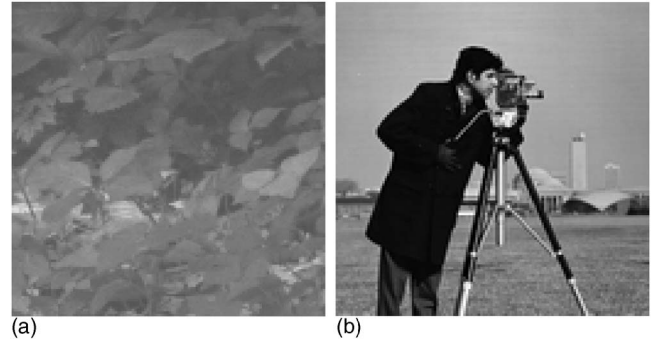


Figure 1. (a) The watermark image: (b) The band 30 image from the BRISTOL image.

$$\text{RMS} = \sqrt{\frac{1}{m^2} \sum_{i=1}^m \sum_{j=1}^m [(W)_{ij} - (W^{(r)})_{ij}]^2}. \quad (24)$$

Thus, if $\text{RMS}=0$, W and $W^{(r)}$ have no difference. Hence, the smaller the value of RMS, the better the embedding method.

SIMULATIONS AND COMPARISONS

In this section, the proposed method is compared with the pure PCA technique.^{5,8} The spectral image used in this comparison is the BRISTOL¹² image. The spectral range of the original BRISTOL image was in the human visual region. The image had 128 rows and 128 columns ($m=128$) with 8 bit resolution and had 32 ($n=32$) bands. The watermark was an 8 bit gray scale image having 128×128 in spatial dimension. The watermark used in this experiment is shown in Fig. 1(a) and the band 30 image of the BRISTOL spectral image is shown in Fig. 1(b).

The parameters α_e and α_w in Eq. (16) control the strength of the watermark embedding. $\alpha_e=1$ and $\alpha_w=0$ mean that there was no watermark information embedded. When $\alpha_e=0$ and $\alpha_w=\sigma_{w,1}/\sigma_{e,1}$, purely watermark information is embedded. In this study, we set $\alpha_e=0$, hence, the parameter α_w controls the strength of the embedding. Figure 2 shows the difference measured in terms of PSNR between the original and watermarked spectral images versus the band k' in which the watermark information was embedded. The value in the horizontal axis is the band k' , which varies from 1 to 32. The values in the vertical axis are the corresponding differences in terms of PSNR computed using Eq. (22). The value $\alpha_w=7$ was used. From this diagram it can be seen that PSNR value increases with the increase of the value k' , which means that when the watermark information was embedded in the smaller band k' , the energy E^d of the difference between the original and watermarked spectral images is larger, hence the watermark is more perceptible. While when the watermark information was embedded in the larger band k' , the energy E^d of the difference between the original and watermarked spectral images is smaller, hence the watermark is less perceptible. This reflects the main characteristics of the PCA transform. The first few

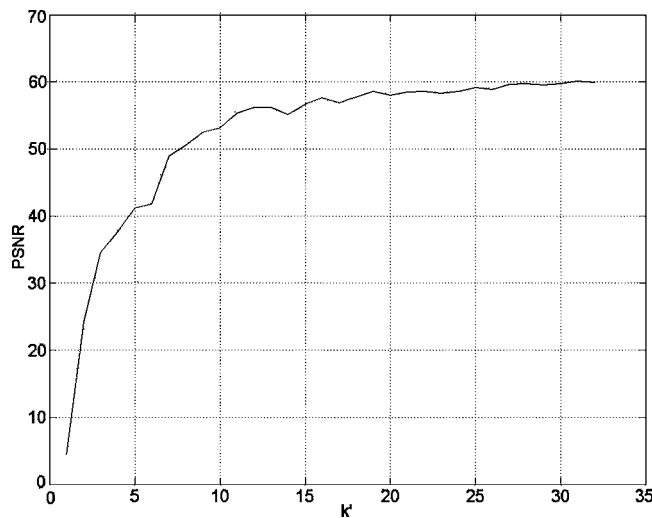


Figure 2. The difference in terms of PSNR (vertical axis) between the original and watermarked spectral images versus the band k' (horizontal axis) in which the watermark information was embedded.

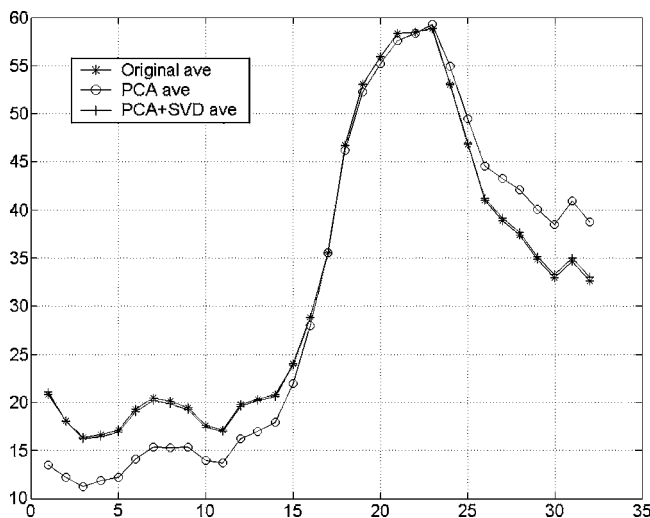


Figure 3. Average spectra (vertical axis) of the original (marked*), watermarked images using the pure PCA technique (marked with O) and using the proposed method (marked with +) versus band k (horizontal axis).

eigenvectors carry the main feature of the spectral set Ω , while the additional eigenvectors share little main features of the spectral set.

Embedding Quality Comparison

We now compare the embedding quality using our proposed technique and a pure PCA-based watermarking technique. Note that the performances of both techniques depend on the choice of the band k' and embedding strength parameters. In this experiment k' was fixed to a value of 3 for both methods. For a fair comparison, the strength parameter for each method was adjusted so that the difference between the original and watermarked spectral images measured in terms of PSNR computed using Eq. (22) is approximately equal to a given value (PSNR=34.50 dB in this experiment).

Table I. Correlation coefficients between the original and extracted watermark images using median and trimmed mean filters for the proposed and pure PCA methods.

	PCA+SVD	Pure PCA
Median 3×3	0.999	0.946
Median 5×5	0.998	0.916
Trimmed mean 3×3	0.995	0.948
Trimmed mean 5×5	0.984	0.923

Table II. RMS error between the original and extracted watermark images using median and trimmed mean filters for the proposed and pure PCA methods.

	PCA+SVD	Pure PCA
Median 3×3	10.730 010 1	20.839 572 5
Median 5×5	15.438 597 1	25.182 329 5
Trimmed mean 3×3	13.611 175 5	20.057 059 7
Trimmed mean 5×5	20.874 862 6	24.133 671 1

Since each of the embedding methods has the same value of PSNR, the comparison of the embedding quality is made based on the measure of average spectra.^{5,8} Figure 3 shows the average spectra from the original image (curve with “*”), the watermarked images using the PCA (curve with “O”) and the proposed methods (curve with “+”) versus band k . The vertical values are the average spectral values, and the value in the horizontal axis is the band k varying from 1 to 32. From this diagram, it can be seen that the average spectra curve for the watermarked image embedded using the proposed method is nearly overlapping with that of the original image, while the average spectra curve of the watermarked image using the purely PCA method is markedly different from that of the original image at two end bands (bands 1–15, and bands 25–32). This diagram clearly shows the proposed method is better than the pure PCA approach.

Reconstruction Quality Against Attacks

It is often that image processing techniques can be applied to the watermarked spectral image. The processed or attacked image will affect the quality of the reconstructed watermark image. The difference between the original watermark image W and the extracted watermark image $W^{(r)}$ measures the performance of the watermark embedding method. The smaller the difference, the better the method performs, and the method is more robust against attack. Also, the small difference indicates the closeness between the two images. Here the correlation coefficient cc defined by Eq. (23) and RMS defined by Eq. (24) are used as measures of the closeness between W and $W^{(r)}$, or measures as the robustness of the watermark embedding method. Median filtering and

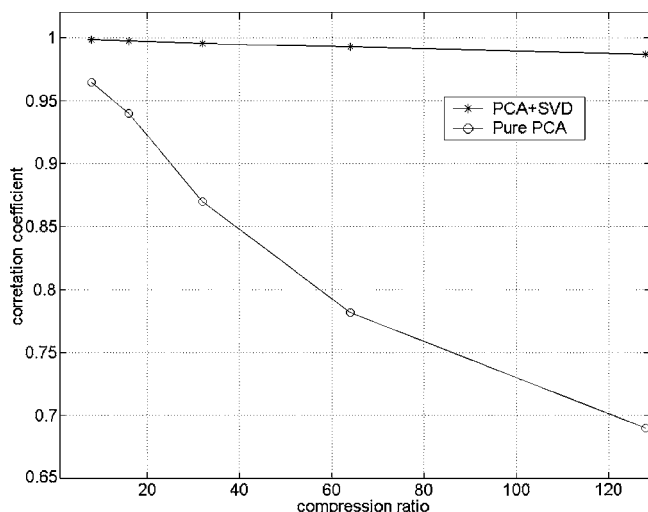


Figure 4. Correlation coefficients between the original and extracted watermark images for the proposed and pure PCA methods versus the compression ratio using the 3-D SPIHT lossy compression method.

trimmed mean filtering were first applied to the watermarked image with the filter size being 3×3 and 5×5 . The correlation coefficient (cc) and root-mean-square (RMS) results are listed in Tables I and II respectively. In each table, the values in the second column indicate the performance or robustness of the proposed method (PCA+SVD), and the values in the last column reflect the performance of the pure PCA approach. For example, from the second row of Table I, the correlation coefficients were computed after the watermarked image was processed using a 3×3 median filter. The correlation coefficient (second row and column) for the proposed method is 0.999, while the correlation coefficient (second row and third column) for the pure PCA approach is 0.946. In fact, when comparing the results in each row of Table I, it can be seen that values in the second column are always closer to a value of unity than the corresponding values in the last column, indicating the proposed method outperforms the pure PCA approach. Similarly, from Table II, the values in the second column are always smaller than the corresponding values in the last column, indicating once again the proposed method outperforms the pure PCA approach.

Lossy compression was the second attack that corrupted the watermarked image. A 3D-SPIHT¹¹ compression method was used. The correlation coefficient and RMS error (vertical axis) between W and $W^{(r)}$ versus the compression ratio (horizontal axis) are shown in Figs. 4 and 5, respectively. The curve marked with “*” corresponds to the proposed method (PCA+SVD) and the curve marked with “○” corresponds to the pure PCA approach. It can be seen from Fig. 4, that the curve marked with “*” is much higher than the curve marked with “○,” telling us that for each of the chosen compression ratios, the correlation coefficient for the proposed method is closer to unity than that for the pure PCA approach. Figure 5 shows that the RMS error curve of the proposed method is always located below that of the

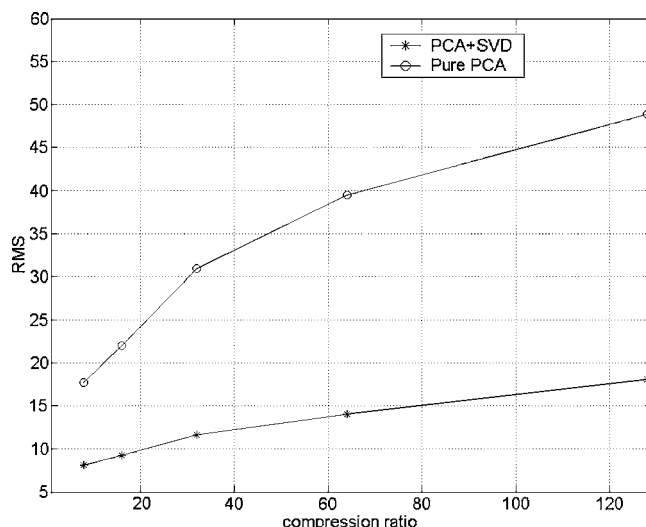


Figure 5. RMS errors between the original and extracted watermark images for the proposed and pure PCA methods versus the compression ratio using the 3-D SPIHT lossy compression method.

pure PCA approach. Both measures show the proposed method is more robust than the pure PCA method.

CONCLUSIONS

In this paper, a digital watermarking technique has been proposed based on principal component analysis (PCA) and singular value decomposition. This work was motivated by the pure PCA approach of Kaarna et al.^{5,8} The robustness of the proposed method was tested using attacks such as lossy compression, median, and trimmed mean filtering. Simulation results have shown the proposed method is more robust than the pure PCA approach. However, comparing with the PCA approach our method involves further singular value decompositions of the watermark image and selected band of eigenimage $E^{(k')}$, which will give no problems with modern computing power considering the nature of the applications.

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