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Multispectral Imaging: How Many Sensors Do We Need?

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Abstract. *The surface reflectance functions of natural and man-made surfaces are invariably smooth. It is desirable to exploit this smoothness in a multispectral imaging system by using as few sensors as possible to capture and reconstruct the data. In this paper we investigate the minimum number of sensors to use, while also minimizing reconstruction error. We do this by deriving different numbers of optimized sensors, constructed by transforming the characteristic vectors of the data, and simulating reflectance recovery with these sensors in the presence of noise. We find an upper limit to the number of optimized sensors one should use, above which the noise prevents decreases in error. For a set of Munsell reflectances, captured under educated levels of noise, we find that this limit occurs at approximately nine sensors. We also demonstrate that this level is both noise and dataset dependent, by providing results for different magnitudes of noise and different reflectance datasets.* **©** *2006 Society for Imaging Science and Technology.* [DOI: 10.2352/J.ImagingSci.Technol.(2006)50:1(45)]

INTRODUCTION

The information contained in a black and white image is insufficient to reproduce the scene's spectral information. For example, it is not possible to know if a shirt which appears gray in the image is red, green, blue, or yellow. This means that surfaces with different reflectance properties are likely to integrate to the same gray shade. This phenomenon, whereby spectrally different surfaces integrate to the same camera response, is known as metamerism.[1,2]

It is possible to reduce metamerism by increasing the number of channels in the device. For example, most commercially available cameras employ three channels, which are commonly chosen to be red, green, and blue. Three channel, or trichromatic, cameras significantly reduce the degree of metamerism encountered in black and white cameras. Unfortunately, trichromatic cameras are not able to fully eradicate metamerism.[2] Thus, like in the example of a gray shirt in a black and white image, many surfaces might integrate to the same trichromatic response, making surface separation an impossible task.

To further decrease the degree of metamerism it is necessary to make cameras with more than three color channels, such cameras are known as multispectral cameras.[3–6] Unfortunately, the drive to increase the number of sensors is restricted by the increased cost and memory requirements as well as manufacturing limitations. In light of these constraints and the need for an increased number of channels we are faced with the question that we address in this paper, namely, what is the minimum number of sensors needed in a multispectral imaging device such that the error in recovering the reflectances is minimal.

As a first approximation we might assume that the number of channels needed in a multispectral camera is limited, and relates to the underlying dimensionality of the captured data. Such an assumption is supported by a large body of research in spectral data dimensionality where it is agreed that a small number of basis functions is adequate to fully represent large data sets. From analyzing 150 out of 433 Munsell chips Jozef Cohen[7] concluded that their reflectance depends on three components. Among later studies of the Munsell colors, Eem et al.[8] proposed four, Maloney[9] proposed five to seven, Burns[3] proposes five to six, Lenz et al.[10] use six, Parkkinen et al.[11] and Wang et al.[12] argue that eight components are necessary, and in a recent study Hardeberg[13] demonstrated that as many as 18 basis functions may be needed. The reason behind the discrepancies between these studies is that different authors use different thresholds for the required similarity between the original and reconstructed data.

There are two main drawbacks with basing our estimate for the number of sensors needed in multispectral imaging devices on the aforementioned studies. Firstly, the basis functions derived in those studies do not correspond to physically feasible sensors.[14] Secondly, the data is assumed to be noise-free; an assumption which is not justified in an actual imaging system where many types of noise are known to corrupt the response data.[15] Hence, in this paper we present two methods to derive physically feasible sensors such that they are optimised to record and reproduce the spectral data. Using the spectral curves of these sensors we are able to synthesize their responses to a database of Munsell reflectance spectra.[11] Doing so allows us to add educated levels of quantization and shot noise,[15] which makes it possible to study the efficacy of increasing the number of sensors in the imaging device without having to assume perfect noise-free conditions.

Finally, the sensor design methods presented in this paper are derived such that, in the absence of noise, an increasing number of sensors is guaranteed to improve the reflectance estimates. Choosing sensors with this property allows us to concentrate on the question of the minimum number needed rather than the spectral properties of the sensors. In

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other studies that include variable numbers of sensors, among other factors, the sensors are often chosen to have arbitrary characteristics.[4,16] As a result the effect of sensor number is confounded by the particular sensor characteristics chosen.

In the first section of this paper we review the principles of reflectance recovery. The argument in this section is based upon existing theory that appears in previous treatments (e.g., Wandell[17]). We outline the argument here with specific emphasis on choosing the number of sensors in an imaging system. In the next section we introduce methods for deriving physically feasible sensors that are optimized for spectral recovery. We use these sensors in computational experiments described in method to assess the effect of sensor number on reflectance recovery in the presence of noise. In Simulation Results and Discussion we present results that suggest that sensor noise provides a natural limit to decide the best number of sensors.

BACKGROUND

By assuming that all surfaces are Lambertian, and that there is no fluorescence, the response of a digital camera at a single pixel can be modeled by

$$q_i = \int_{\lambda} Q_i(\lambda)E(\lambda)R(\lambda)d\lambda, \tag{1}$$

where q_i is the response of the ith sensor $(i=1,\ldots,P)$, $Q_i(\lambda)$ is the ith sensor response function, $E(\lambda)$ is the spectral power distribution of the illuminant and $R(\lambda)$ is the surface spectral reflectance function. Note that we are neglecting noise for the time being.

These continuous functions can be sampled at a number of discrete wavelength intervals n without a significant loss of accuracy, providing that the interval is sufficiently small.[18] In this work we sample functions on the range from 400 to 700 nm at 10 nm intervals, thus $n=31$. With this in mind Eq. (1) can be rewritten as:

$$q_i = \sum_{\lambda} Q_i(\lambda)E(\lambda)R(\lambda)\Delta\lambda. \tag{2}$$

This discrete sum is more conveniently expressed in terms of matrix-vector notation, thus we write:

$$\mathbf{q} = \boldsymbol{Q}^T\mathbf{r}, \tag{3}$$

where $\mathbf{q}$ is a $p\times 1$ vector of sensor responses and $\mathbf{r}$ is an $n\times 1$ reflectance vector. For compactness we represent the product of each sensor response function $Q_i(\lambda)$ and the illuminant $E(\lambda)$ as a single vector which forms the ith column of the $n\times p$ sensor matrix $\boldsymbol{Q}$.

The problem of recovering reflectance from camera responses can now be expressed as the problem of estimating the $n\times 1$ vector $\mathbf{r}$ given the $p\times 1$ vector of camera responses $\mathbf{q}$ and the matrix $\boldsymbol{Q}$. This is a system of p linear equations in n unknowns. For an exact solution it is sufficient to set the number of knowns equal to the number of unknowns, i.e., to use $p=31$ independent sensors in the imaging system. However, such a large number of sensors may not be necessary for reflectance recovery. Real reflectance spectra are known to be strongly constrained and may be represented accurately with fewer than 31 parameters.[13] A convenient way to express this is to write reflectance as the weighted linear sum of a small number of basis vectors,[9] i.e.,

$$\mathbf{r} = \sum_{i=1}^{m} \mathbf{b}_i\omega_i, \tag{4}$$

where $\mathbf{b}_i$ are the basis vectors, ω_i are the respective weights and $m \ll n$. This relation can be expressed in matrix vector notation thus:

$$\mathbf{r} = \boldsymbol{B\omega}, \tag{5}$$

where the columns of $\boldsymbol{B}$ are the basis vectors and $\boldsymbol{\omega}$ is a vector of weights. Replacing Eq. (5) into Eq. (3) we obtain

$$\mathbf{q} = \boldsymbol{Q}^T\boldsymbol{B\omega}. \tag{6}$$

This is a system of p equations in m unknowns. To solve uniquely for $\boldsymbol{\omega}$, and therefore $\mathbf{r}$, it is sufficient to set the number of independent sensors $p=m$. Providing that $\boldsymbol{Q}^T\boldsymbol{B}$ is invertible we can solve for the $\boldsymbol{\omega}$ as follows:[19,20]

$$\boldsymbol{\omega} = (\boldsymbol{Q}^T\boldsymbol{B})^{-1}\mathbf{q}. \tag{7}$$

The principal problem with this approach is that it is not straightforward to determine an objective value for m and, therefore, p. In order to understand this it is necessary to consider how to derive m from a statistical analysis using the singular value decomposition.

We can represent a set of k reflectance spectra as the columns of an $n\times k$ matrix $\boldsymbol{R}$. The singular value decomposition of $\boldsymbol{R}$ is given by

$$\boldsymbol{R} = \boldsymbol{U\Sigma V}^T, \tag{8}$$

where the matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ are both orthonormal, i.e., $\boldsymbol{U}^T\boldsymbol{U}=\boldsymbol{V}^T\boldsymbol{V}=\mathbf{I}$, and $\boldsymbol{\Sigma}$ is a matrix whose leading diagonal contains the singular values of $\boldsymbol{R}$ with zeros elsewhere. The columns of $\boldsymbol{U}$ are the eigenvectors of the matrix $\boldsymbol{RR}^T$ and are referred to as characteristic vectors. The characteristic vectors are a set of basis vectors for the columns of $\boldsymbol{R}$ that are ordered such that the first vector accounts for the most variability in the data, the second accounts for the most variability in the residual from the first vector, and so on. Thus by increasing the number of characteristic vectors in the linear model we are guaranteed to improve the reflectance estimate progressively. Furthermore, the first m characteristic vectors give the closest possible fit of any linear model for a given value of m.

Although, by increasing the number of bases m in the linear model, the approximation can always be improved, the inherent smoothness of reflectance spectra determines that there is a point when increasing m results in very small improvements in accuracy. Generally m is estimated as the point when the small improvement drops below some arbitrary threshold. However, if we intend to use m to determine the number of sensors in a multispectral imaging system,

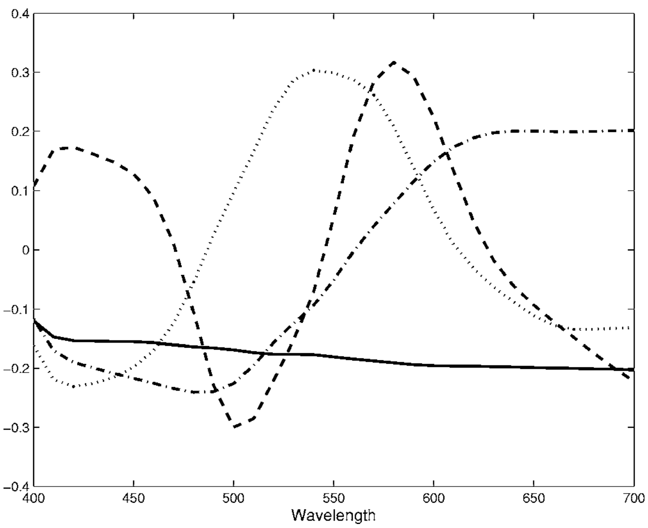

Figure 1. The first four characteristic vectors of the Munsell reflectance data. The data are plotted at 10 nm intervals and interpolated linearly.

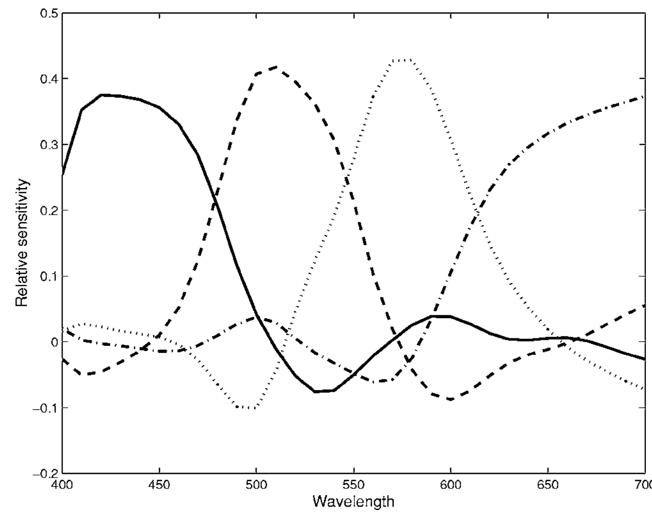

Figure 2. The first four characteristic vectors rotated by the varimax algorithm to be maximally positive. The data are plotted at 10 nm intervals and interpolated linearly.

then we must consider the point when the improvement in the accuracy of the linear model is cancelled by the detrimental effect of imaging noise. In order to assess the role of noise we need to make a real set of sensors to capture the data in typical noise conditions. This requires that we choose the sensor functions carefully, according to objective criteria.[21] Different sensor characteristics capture different information, and hence result in different reflectance estimates for the same sensor number. By not choosing carefully, the effect of increasing the number of sensors, and hence parameters in the linear model, will be confounded with the sensor characteristics. We would, therefore, like to choose sensors that are optimised for spectral recovery and thus guarantee that, in the absence of noise, increasing the number of sensors results in decreasing error. In order to do this we should choose sensors $\boldsymbol{Q}$ whose columns span the same vector space as the first m characteristic vectors.[22]

NON-NEGATIVE SENSORS

To guarantee that we choose the sensors to be within a linear transform of the basis vectors we must choose $\boldsymbol{Q}$ such that it satisfies the following relation:

$$\boldsymbol{Q} = \boldsymbol{B}\boldsymbol{A}, \tag{9}$$

where $\boldsymbol{A}$ is a linear transformation and $\boldsymbol{B}$ contains the first m characteristic vectors of $\boldsymbol{R}$ as columns. Initially, one might consider using the characteristic vectors themselves as sensors, i.e., let $\boldsymbol{A}$ be the identity matrix. However, as can be seen from Fig. 1, the characteristic vectors contain many negative values, yet real sensors must be non-negative everywhere. Further, the modulation of the characteristic vectors is proportional to their order, i.e., additional vectors have an increasing number of peaks and troughs. It is therefore desirable to transform these vectors into a non-negative vector space, such that their individual sensitivities are concentrated in distinct regions of the visible spectrum. For example, in a trichromatic camera system the sensors are commonly chosen to be red, green, and blue. Finally, the transformed vectors should ideally span the same space as the original.

Given these criteria, we would like to find the best transform $\boldsymbol{A}$ to solve for the sensors $\boldsymbol{Q}$. In this paper we propose to solve this problem using the varimax rotation algorithm described in Refs. 23 and 24. Starting from the $n \times m$ bases matrix $\boldsymbol{B}$, with elements b_{jk}, the varimax criterion is given by

$$V(\boldsymbol{B}) = \sum_k \left[\frac{1}{n} \sum_j b_{jk}^4 - \left(\frac{1}{n} \sum_j b_{jk}^2 \right)^2 \right]. \tag{10}$$

Verbally, Eq. (10) is the columnwise variances of the squared elements of $\boldsymbol{B}$. Given the varimax criterion in Eq. (10), the optimal transform $\boldsymbol{A}$ in Eq. (9) is any orthogonal rotation of $\boldsymbol{B}$ that maximizes the varimax criterion among all other orthogonal rotations. Constraining $\mathbf{A}$ to be an orthogonal transform means that the resultant sensors $\boldsymbol{Q}$ are themselves orthogonal. Maintaining the orthogonality of the sensors is important, as it makes the recovery of reflectance from their output maximally robust to sensor noise.[25] The sensors generated by this procedure are shown in Fig. 2.

The sensitivity of each sensor is clearly focused in a different region of the visible spectrum. However, the rotated vectors still contain some negative lobes, which means that they cannot be used as sensors. We, therefore, choose everywhere non-negative sensors that are as close as possible to the rotated sensors, denoted $\hat{\boldsymbol{Q}}$, but still within a linear transform of $\boldsymbol{B}$. Denoting $\hat{\boldsymbol{Q}}_i$ as the ith column of $\hat{\boldsymbol{Q}}$, we can do this sequentially for the ith sensor by solving the following optimization problem:

$$\min_{\mathbf{g}_i} \|\hat{\boldsymbol{Q}}_i - \hat{\boldsymbol{Q}}_i \mathbf{g}_i\|^2 \text{ subject to } \hat{\boldsymbol{Q}}_i \mathbf{g}_i \geqslant \mathbf{0}, \tag{11}$$

where $\mathbf{g}_i$ is a $p \times 1$ vector and $\mathbf{0}$ is a vector of zeros. This results in the sensors shown in Fig. 3, termed here the vari-

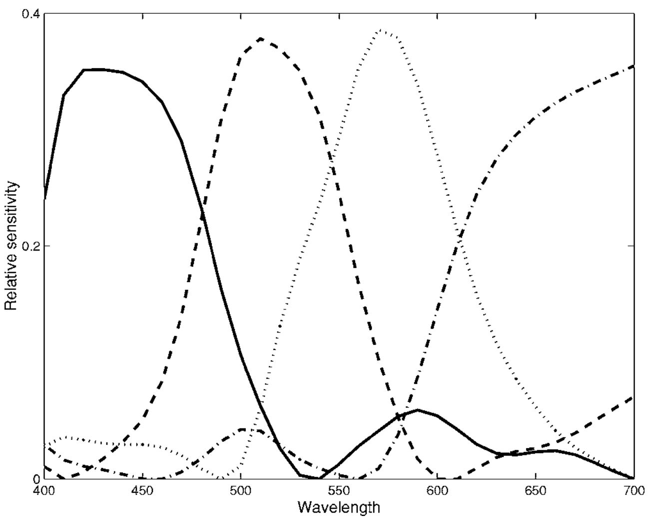

Figure 3. Non-negative sensors formed by varimax rotation with added positivity constraint. The data are plotted at 10 nm intervals and interpolated linearly.

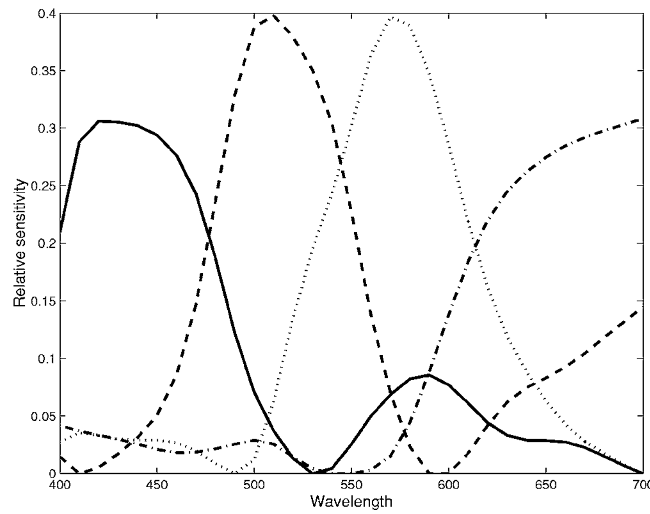

Figure 4. Non-negative sensors formed by Piché's procedure. The data are plotted at 10 nm intervals and interpolated linearly.

max sensors.

In earlier work Piché[26] generates non-negative combinations of the characteristic vectors that explicitly maximize the mutual orthogonality of the sensors. He points out that the orthogonality of the sensors can be measured directly by the *condition number* of $\boldsymbol{Q}$, where the condition number is given by

$$cond(\boldsymbol{Q}) = \|\boldsymbol{Q}\|_2 \|\boldsymbol{Q}^{\dagger}\|_2. \tag{12}$$

Here $\|\cdot\|^2$ denotes the spectral norm of a matrix, which is given by its largest singular value, and † denotes the pseudoinverse operation. Given that the characteristic vectors are guaranteed to be orthogonal, the condition number of $\boldsymbol{Q}$ is determined solely by the condition number of $\boldsymbol{A}$. Piché, therefore, generates transformations of the characteristic vectors that explicitly attempt to minimize the condition number of $\boldsymbol{A}$. That is, he minimizes the following objective function

$$\min_{\mathbf{A}} \|\mathbf{A}\|_2 \|\boldsymbol{A}^{\dagger}\|_2 \text{ subject to } \boldsymbol{Q} = \boldsymbol{B}\boldsymbol{A} \geqslant \mathbf{0}, \tag{13}$$

where $\mathbf{0}$ is now a matrix of zeros. This optimization problem can be tackled directly using iterative nonlinear optimization methods. Sensors generated using this procedure, termed here Piché sensors, are shown in Fig. 4. Note the similarity between the sensors in Fig. 4 and those generated by the varimax procedure in Fig. 3. We also find that the varimax and Piché procedures generate sensors with equally low condition numbers, even though Piché minimizes condition number explicitly.

METHOD

To generate each set of p sensors we choose the first p characteristic vectors of a set of reflectance spectra and transform them into non-negative sensors using both the Piché and varimax procedures. We use the synthesized responses of these sensors to assess the effect of increasing sensor number on reflectance recovery performance in the presence of noise. Synthetic camera responses are generated according to the following camera model:

$$\mathbf{q} = \boldsymbol{Q}^T \mathbf{r} + \mathbf{n}_{\text{shot}} + \mathbf{n}_{\text{quant}}, \tag{14}$$

where the vectors $\mathbf{n}_x$ denote sources of noise. Shot noise $\mathbf{n}_{\text{shot}}$ arises from the inherent uncertainty in the generation, reflection, and capture of light. This is a Poisson process,[15] thus the variance of the shot noise component increases with increasing input intensity. This is modeled using multiplicative Gaussian noise, thus

$$\mathbf{n}_{\text{shot}} = [\zeta_1 \mathbf{q}_1, \zeta_2 \mathbf{q}_2, \ldots, \zeta_p \mathbf{q}_p]^T, \tag{15}$$

where each of the ζ_i is a pseudorandom variable taken from a Gaussian distribution with zero mean and variable standard deviation and $\mathbf{q}_i$ represents the ith sensor response. Quantization noise $\mathbf{n}_{\text{quant}}$ is incorporated by directly quantizing the simulated responses after the application of shot noise. Other sources of noise, such as dark noise, are assumed to be negligible or corrected for.

In all calculations the equal energy illuminant E is used and the columns of $\boldsymbol{Q}$ all sum to 1, thus ensuring a camera response of 1 to a perfect reflecting diffuser. Reflectance is estimated from camera responses using Eqs. (5) and (7). The difference between original and estimated spectra is measured in terms of absolute route-mean-squared error, given by

$$\text{rms} = \sqrt{\frac{(\mathbf{r} - \hat{\mathbf{r}})^T (\mathbf{r} - \hat{\mathbf{r}})}{n}}, \tag{16}$$

where $\hat{\mathbf{r}}$ is the reflectance estimate and $\mathbf{r}$ is the original.

Given that a pseudorandom variable is used to simulate noise, the results of these simulations will vary from trial to

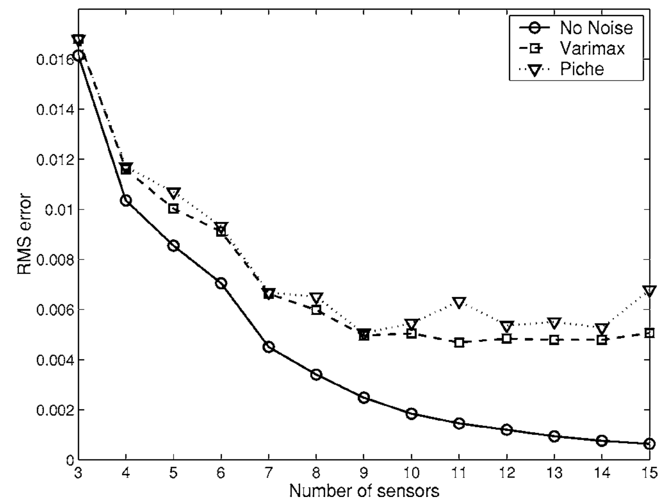

Figure 5. Effect of increasing sensor number with 12 bit quantization and 1% shot noise on Munsell reflectance data.

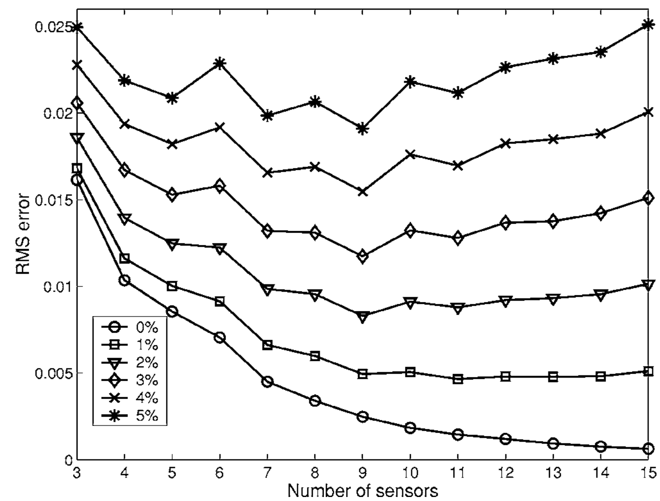

Figure 6. Results for the Munsell reflectance data with different levels of shot noise.

trial. In order to discount this effect the simulations are repeated 10 times and the average is recorded.

SIMULATION RESULTS

In Fig. 5 we present results for a set of 1269 Munsell reflectance spectra captured under 1% shot noise and 12 bit quantization along with noise-free estimation results. This shows the effect of increasing the number of sensors on mean rms reconstruction error. When there is noise in the sensor responses the recovery error does not decrease monotonically with increasing sensor number as it does when there is no noise. Minimum error is reached at 11 sensors and nine sensors for the varimax and Piché methods respectively, although the varimax sensors show little improvement beyond nine sensors. The value of nine corresponds to previous estimates of the dimensionality of this dataset made using different decision criteria by Parkkinen et al.[11] and Wang et al.[12]

Although we have made every effort to reduce the dependency of the best number of sensors on external factors, it is still both dataset and noise dependent. Figure 6 shows the effect of increasing noise levels on the best number of sensors. In the figure the number of sensors, generated for the Munsell reflectance data using varimax rotation, is plotted against mean rms error for increasing levels of shot noise. As expected, increasing the level of noise increases the overall reconstruction error. Furthermore, as the noise level increases so does the variability in error for different sensor numbers. Therefore, although the minimum error remains at nine sensors for noise levels of 2% and above, for high noise levels the advantage of increasing the number of sensors above four could be outweighed by the potential for increasing reconstruction error. Recent measurements of noise levels in a trichromatic camera[27] suggest that realistic levels of shot noise are between 1% and 2%, thus the best number of sensors for a real system is likely to be close to nine. In additional experimental data, not shown here, it was found that quantization noise has a negligible effect.

The sensors that are generated in each condition are derived directly from the characteristic vectors of the data, and are therefore, clearly data dependent. Figure 7 shows the four-sensor sets of varimax sensors derived from four different reflectance datasets; the Munsell data, the patches of the Macbeth ColorChecker DC,[28] the patches of the Esser calibration target[29] and a set of natural reflectance spectra, consisiting of leaves, bark and flowers, measured by Owens.[30] Only varimax rotation is used here, as Piché's method took too long to converge for some sets of sensors. The sensors from the ColorChecker DC and Munsell reflectance data are very similar to each other, since the characteristic vectors of these sets are closely related. The sensors derived from the Esser target differ slightly from those of the Munsell data, while the natural sensors appear markedly different.

In Figs. 8–10 we plot sensor number versus rms error for the three additional datasets, again deriving sensors using the varimax method. In each case there is 1% shot noise and 12 bit quantization. For the Macbeth ColorChecker DC (Fig. 8), the error has stopped decreasing monotonically at nine sensors, with a minimum reached at 13 sensors. For the Esser target (Fig. 9) the minimum is reached at 13 sensors, although the error decreases monotonically up to 13 sensors in this case. The natural data (Fig. 10) seems to show a tailing off at around eight sensors, although by plotting up to 25 sensors we can see that there is in fact a clear minimum at around 17 sensors.

DISCUSSION

In these experiments we have used carefully chosen sensors to find the minimum number of sensors such that they provide minimal reflectance reconstruction error. The number that we find is dependent both upon the data set for which the sensors are optimized, and on the noise level. This number is a theoretical limit but it is also based upon realistic noise estimates.

In order to verify these results in a practical setting, one might attempt to manufacture different sets of optimal sen-

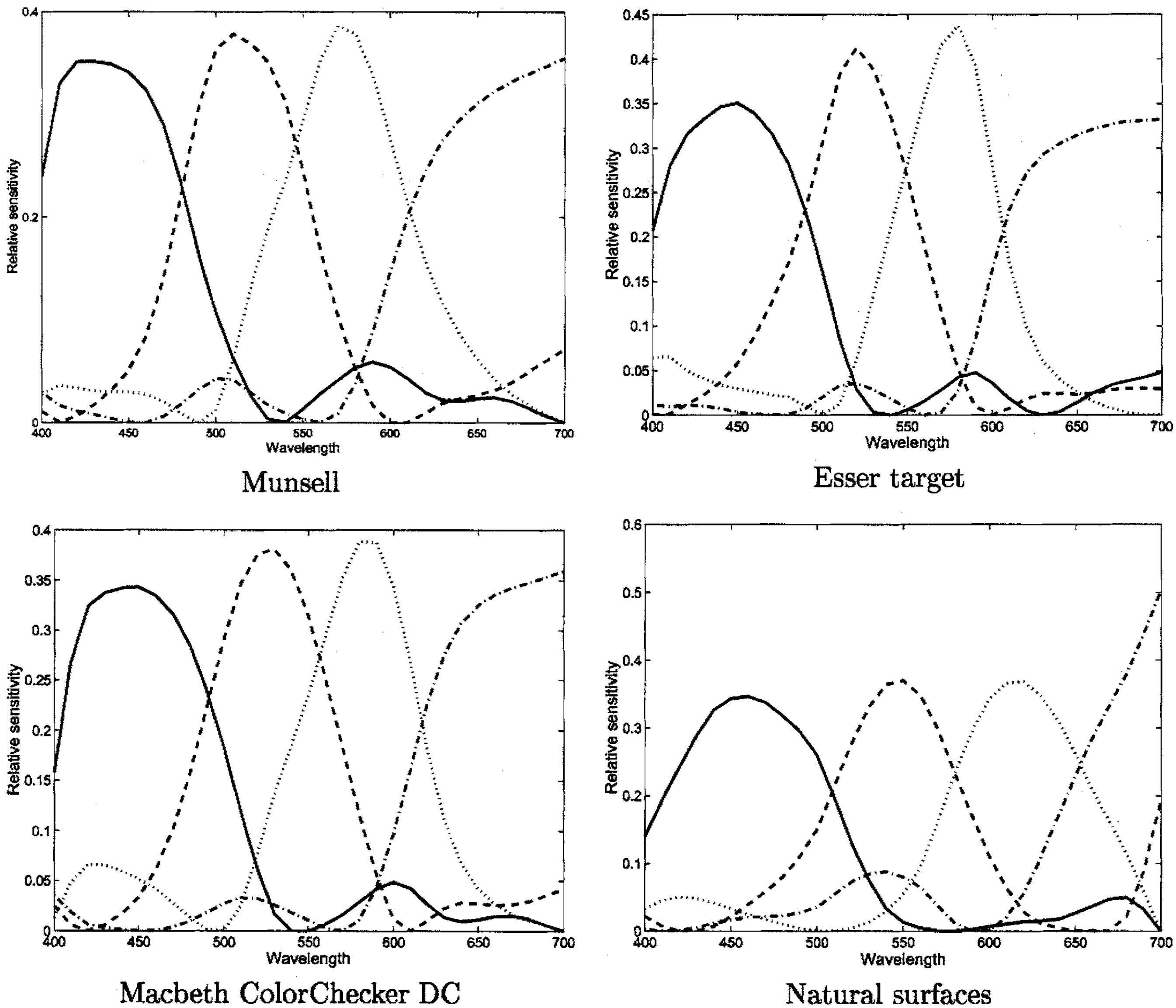

Figure 7. Sensors derived by varimax rotation for four different reflectance datasets. The data are plotted at 10 nm intervals and interpolated linearly.

sors. However, any real sensors are sure to deviate from optimal sensors due to variability in the manufacturing process. As such we expect that the best number of real sensors would be different from the theoretical values presented here. In most cases the best number is likely to increase relative to the theoretical case, since more sensors would be needed to capture the same amount of information as the theoretical sensors. However, the best number may also decrease, e.g., if errors in manufacturing the sensors were to increase sensitivity to noise as a function of sensor number.

As stated previously, the sensors we derive here are data dependent. Therefore, the sensors are no longer optimal when they are used to capture novel surfaces, i.e., surfaces not used to derive the sensors. The best number of sensors for the novel dataset will be determined by both the dimensionality and the spectral characteristics of the two datasets. For example, imagine that we have two surface sets, a derivation set for which the sensors are optimized and a novel set. If the novel set has a lower dimensionality than the derivation set, then it is likely that a smaller number would suffice for that data. Similarly, if the novel set has a higher dimension than the derivation set, or even a similar dimensionality but with very different reflectance characteristics, we expect the number of sensors required to increase.

In our experiments we make no distinction between the underlying sensitivity of the sensor and the filter placed in front of the sensor. However, practically these two processes are often separated, since a given camera generally has a single CCD chip, whose sensitivity is fixed, and the problem of deriving optimal sensors is one of deriving optimal filters. The way we approach the problem here can be seen directly as optimizing filters with the assumption that the camera has an equal sensivity at all wavelengths. However, even if this is not the case, it would be straightforward to adapt our

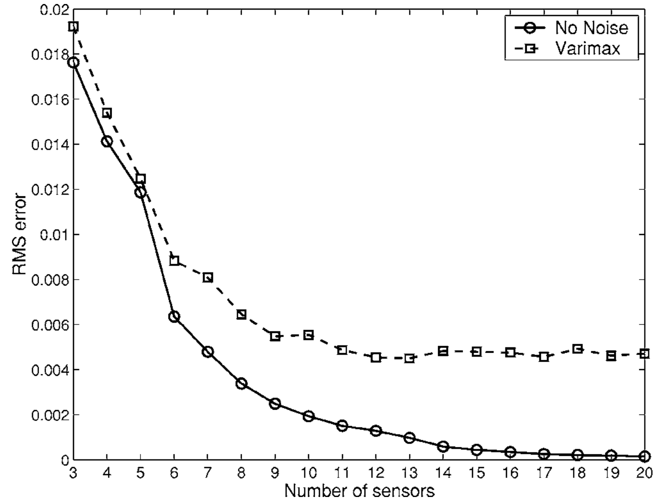

Figure 8. Effect of increasing sensor number with 12 bit quantization and 1% shot noise on Macbeth ColorChecker DC data.

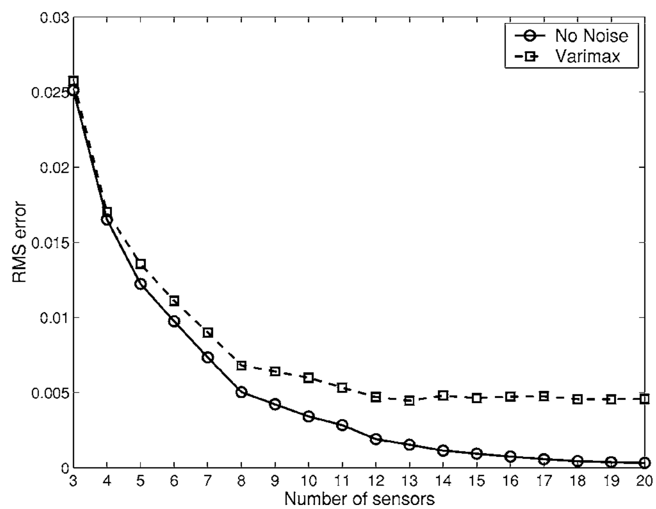

Figure 9. Effect of increasing sensor number with 12 bit quantization and 1% shot noise on the Esser target.

method, assuming that the underlying sensor sensitivity is known, by incorporating that information into the spectral data prior to performing the optimization, i.e., by simply multiplying the sensor sensitivity by the spectral data at each wavelength.

In addition to being data dependent, the results are also dependent upon the validity of the noise model. Implicit in the model that we employ is the assumption that image independent noise, such as that introduced by the sensor electronics or dark noise is minimal. Such an assumption is valid for cooled CCD cameras, which are commonly used in such applications (e.g., Ref. 31). However, for other imaging systems we might expect this not to hold, and the number of sensors therefore, to, be different.

CONCLUSIONS

In this work we have generated sensors for a simulated multispectral imaging device that are both optimized for recovering a particular set of reflectances and are maximally robust to noise. We have used these to find the minimum number of sensors such that they provide minimal reconstruction error. Using fewer sensors leaves potential room for improvement, whereas using more sensors does not decrease rms error due to the effect of noise. In a typical noise environment, with 1% shot noise and 12 bit quantization, we find that the limit occurs at approximately nine sensors for a set of Munsell reflectances, nine for the Macbeth ColorChecker DC, 13 for the Esser target and eight for a set of natural data. For the Munsell data, the value of nine corresponds to previous estimates of the dimensionality of this dataset made using different decision criteria by Parkkinen et al.[11] and Wang et al.[12]

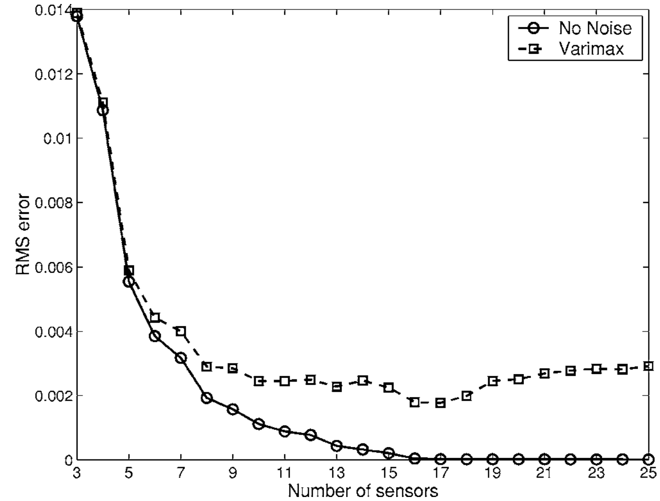

Figure 10. Effect of increasing sensor number with 12 bit quantization and 1% shot noise on natural reflectance data.

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