Computational Surface Models for Chromaticity Differences

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In this study the purpose was to develop a computational surface model, which gives an equal computed chromaticity difference for equal perceived chromaticity difference in every part of the color space. The computational surface model assess chromaticity differences based on the surfaces defined by an ellipse data set and the two chromaticity points whose chromaticity difference is to be calculated. The distances along the surfaces are calculated by a method based on the Weighted Distance Transform On Curved Space (WDTOCS). The ellipse data sets are the MacAdam ellipses in the CIE 1931 (*x,y*)-chromaticity diagram and the ellipses which are fitted from the visual color difference measurements used in deriving the CIE DE2000 color difference formula in the CIELAB color space. In general the ellipse data set can be any set of planar chromaticity ellipses. The chromaticity differences calculated along the surfaces correct the planar chromaticity values, thus these differences match better with perceived chromaticity differences. The experiments are made in the vicinity of the chromaticity difference ellipses and the results are contrasted with CIE DE2000.

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Introduction

One of the oldest chromaticity difference ellipse data sets consists of the MacAdam chromaticity difference ellipses. The area inside each ellipse appears as an equal chromaticity so that the chromaticity differences cannot be perceived,[1] see Fig. 1.

The ellipses have various sizes in different parts of the CIE 1931 *x,y* chromaticity diagram, thus the equisized chromaticity differences in each part of the diagram are perceived unequally.[3] For example, in the bottom left corner in the blue area a small planar Euclidean distance yields a large perceived chromaticity difference and in upper part of diagram in the green area the same perceived chromaticity difference results in a much larger planar Euclidean distance. MacAdam has made also a model for chromaticity difference calculations.[4]

Recently color difference formulas have been mainly based on studies in which chromaticity differences are measured at various illumination levels. Color difference formulas are derived by combining the chromaticity difference with lightness difference measurements. In general the color difference formulas try to compensate for the lack of uniformity of the CIELAB color space, which occurs especially in the blue region.[5] The latest CIE recommended color difference formula, CIE DE2000, was developed with a set of variables for the parametric correction of the error from the CIELAB ΔE^*_{ab} formula.[6] The CMC model for textile industry divides the (a^*b^*)-plane into microfacets which are weighted differently in the different parts of the color space. The result is an error weighting surface, which compensates the planar color difference errors.[7]

The purpose was to develop a computational surface model, which gives an equal computed chromaticity difference for equal perceived chromaticity difference in every part of the color space. The computational surface model for chromaticity differences can be further developed to include also the color-difference measurements by adding the lightness difference to the model. In this study we have focused only on the chromaticity differences measurements.

Surfaces are derived from the chromaticity ellipses and along the surface the chromaticity difference between the two chromaticity points can be calculated. The surface formed responds to the variation of the ellipse sizes and rotations in the color space. Since the surface model is based on the ellipses, this leads to a rather simple computational model and therefore does not include demanding statistical calculations. The chromaticity ellipses are considered locally so the local variations and patterns of the ellipses have an influence on the surface. The computational surface model is sensitive to sectional variance in the color space, thus the reported shortcomings of the CIELAB color space in the blue area are taken into account.

Three computational models have been created. In the latest model the chromaticity differences can be calculated from the different illuminant levels. These levels include variance of the chromaticity ellipses when the illumination level changes.

Computational Surface Models

For calculating the chromaticity differences between two chromaticity points, two different approaches has been developed, named as the mixing model and the line model. In both approaches the chromaticity differences are calculated from the surfaces which are

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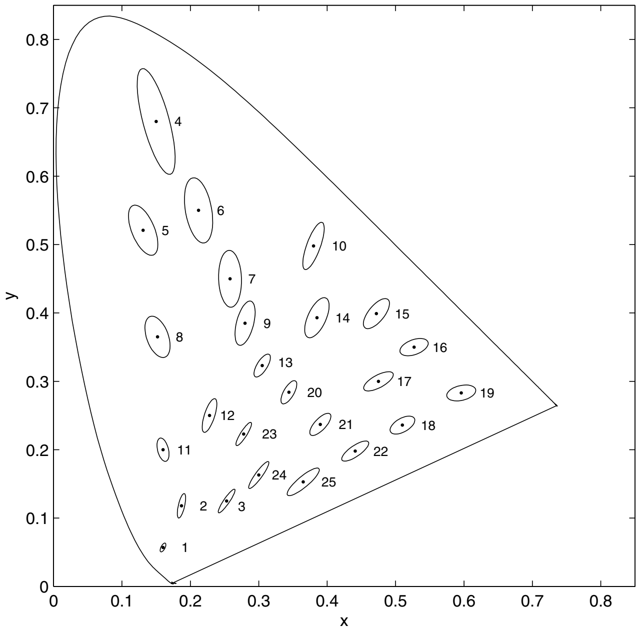

Figure 1. The MacAdam ellipses. The axes of plotted ellipses are 10 times their actual lengths.[2]

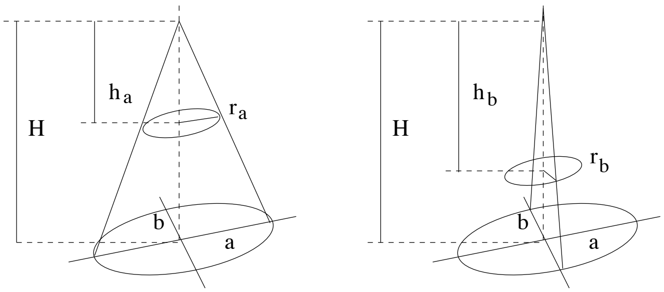

Figure 2. (a) Projection of the circle, major semiaxis; and (b) projection of the circle, minor semiaxis.

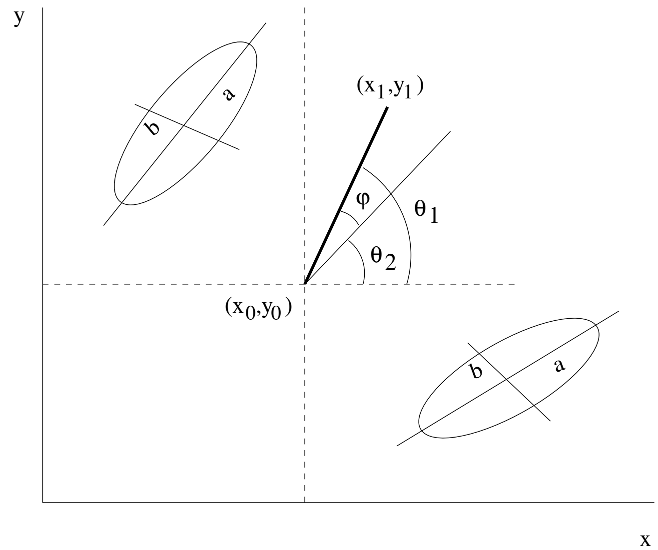

Figure 3. Angles θ_1, θ_2 and φ in the calculation of the parameter p.

defined by the chromaticity difference ellipse data set and by the two chromaticity points (x_0, y_0) and (x_1, y_1), whose chromaticity difference is under consideration. The main idea in both approaches is that the ellipses are projections of circles which lie on surface. In the mixing model (MM) the ellipses are perspective projections and in the line models parallel projections of circles on the surface.

The Mixing Model (MM)

The mixing model (MM) was originally named as Chromaticity Difference from Surfaces Defined from MacAdam Ellipses.[8] The chromaticity differences are calculated from the surface which is defined from the MacAdam ellipses; see Fig. 1. The surface is based on the parameters of all 25 ellipses.

In the mixing model every circle is projected from the center of projection above each circle. The centers of projection are called illumination points and they all are at the same height H from the (x,y)-plane. The optimal height H was defined experimentally. The height h of the surface is obtained for each ellipse depending on the size of the ellipse. For a large ellipse the projected circle is closer to the illumination point and thus the height h gets a small value. For a small ellipse the height h gets a larger value. Two different surfaces are defined, the first one is based on the major semiaxes a and the second one on the minor semiaxes b of the ellipses; see Figs. 2(a) and 2(b). In the previous case, the projected circle has radius $r = r_a$ and in the latter case the projected circle has radius $r = r_b$. The heights of the surfaces are denoted as h_a and h_b, respectively.

The surface S_b defined from the minor semiaxis b lies higher than the surface S_a defined from major semiaxis a, because the height h of the surface is measured from the illumination point. The surface S used in the calculation of chromaticity differences is a mixture of these two surfaces defined as

$$S = pS_b + (1 - p)\, S_a \tag{1}$$

where the parameter $p = p(\theta_1, \theta_2)$, $0 \le p \le 1$. The angle θ_1 depends on the orientation of the two chromaticities and the angle θ_2 is the average value of the angles of the two ellipses closest to the two chromaticities; see Fig. 3.

The angles θ_1 and θ_2 depend on the two chromaticities whose difference will be calculated. The coordinates of these two chromaticities are (x_0, y_0) and (x_1, y_1); see Fig. 3. The difference between θ_1 and θ_2, angle φ, is defined as

$$\varphi = |\,\theta_1 - \theta_2\,|\,. \tag{2}$$

The coefficient p is calculated from the angle φ as

$$p = \frac{\varphi}{\pi/2} \;\text{ if }\; \varphi \le \pi/2 \tag{3}$$

or otherwise as

$$p = \frac{|\varphi - \pi|}{\pi/2}. \tag{4}$$

If $\theta_1 = \theta_2$ then $p = 0$ and the orientation of the chromaticities is parallel to the orientation of the ellipse's major semiaxis in that area. Now the surface S consists only of the surface S_a. If the line between the two chromaticities is perpendicular to the major semiaxis then $p = 1$ and the surface S consists only of the surface S_b. Normally, the surface S is a mixture of the both surfaces S_a and S_b; see Fig. 4.

In Fig. 5, the surfaces S_a and S_b are shown. The surface S_b is elevated by 0.03 units for better visualization of the two surfaces. These surfaces are not dependent on the chromaticities selected for the difference calculation.

In Fig. 6 is illustrated the surface S calculated from two chromaticities $(x_0, y_0) = (0{,}304; 0{,}433)$ and $(x_1, y_1) = (0{,}314; 0{,}453)$. The surface S depends on the two chromaticities selected, and it is valid only in the vicinity of these chromaticities.

The MacAdam ellipses cover the center of the *x,y* chromaticity diagram, but the edge of the diagram has to be defined in another way. The edge of the diagram was extrapolated on the basis of the contour diagrams of the covered area in the CIE 1931 *x,y* chromaticity diagram. The CIE-diagram was examined to decide where surface rises near the edge and where it falls. It was assumed that there were not any irregularities near the edge, but the slopes were in harmony with the covered areas. Another method is to extrapolate the edges through the Just-Noticeable-Differences (JNDs). JNDs are defined in the spectral locus and JNDs are three times larger than the corresponding standard deviation from the MacAdam ellipses.[1,2]

All the ellipse parameters are used in creation of the surface S. The major and minor semiaxes define the two surfaces and the surface used in the chromaticity difference calculation is a mixture of these two surfaces depending on the orientation of the selected pair of chromaticities.

The chromaticity difference is calculated as

$$\Delta E = f(S) \qquad (5)$$

where $f(S)$ is the distance along the surface S. The Weighted Distance Transform On Curved Space[9] was applied, where ΔE was replaced by piecewise Euclidean distances.

The Line Model

In the line model the chromaticity difference ellipses are parallel projections of circles. Two different line models were created. In the first one, named as the line model with two pairs of planes (LMPP), the surface is defined as determining two pair of planes based on the chromaticity points and the ellipse parameters. In the second one, named as the line model with chromaticity difference grid (LMCD), the surface consists of chromaticity difference grid, in which a chromaticity difference is calculated from one point to all other points in the grid.

The Line Model with Two Pairs of Planes (LMPP)

The line model with two pairs of planes (LMPP) was previously named as the Enhanced Model for Chromaticity Differences.[10] In the LMPP the chromaticity differences are calculated from the surfaces which are based on the parameters of the 25 MacAdam ellipses; see Fig. 1.

A two-dimensional model, a curve, was first defined; see Fig. 7. The idea is that the longest semiaxis of all the ellipses, the radius r, is horizontal, and the rest of the semiaxes are projections of the radius r. These projections are achieved through vertical rotations of

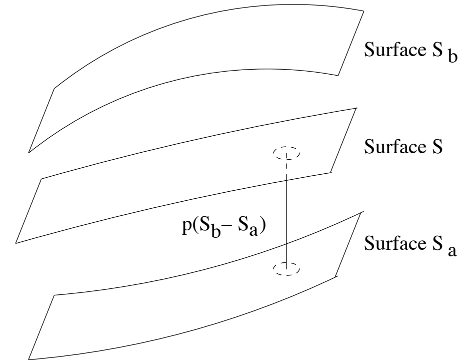

Figure 4. Defining surface S from the surfaces S_a and S_b. The distance between S_a and S is $p(S_b - S_a)$.

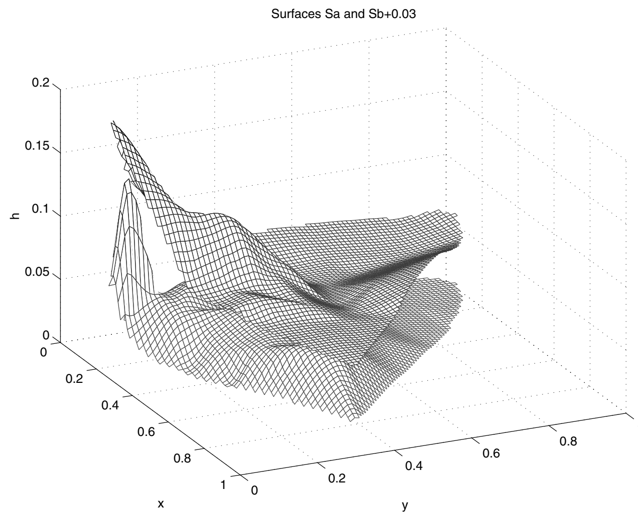

Figure 5. Surfaces S_a and S_b, which was elevated for visualizing; see text.

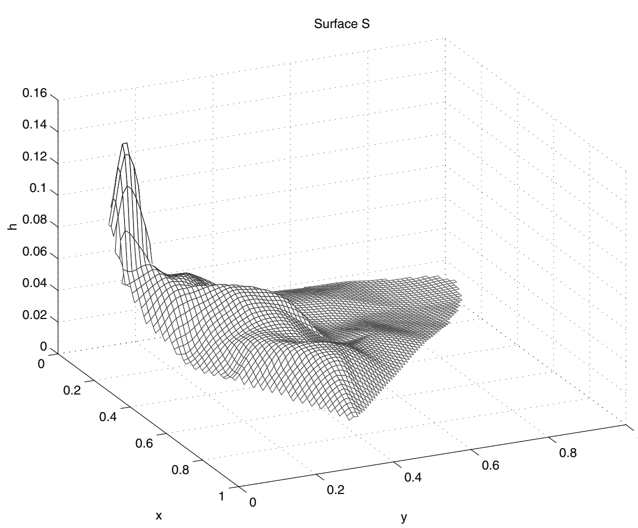

Figure 6. Surface S according to the two chromaticities (x_0, y_0) and (x_1, y_1); see text.

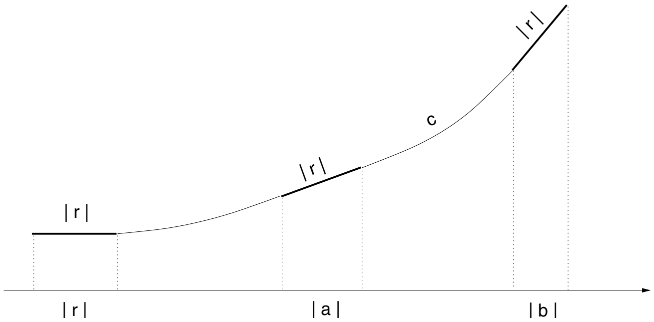

Figure 7. The definition of the curve in the two-dimensional model.

the radius r. The curve c is obtained by fitting the angles of ascent of the radius r for the curve.

In three-dimensional surface model the idea is that the radius r rotates a full circle on a plane and projects an ellipse to the (x,y)-plane. To simplify the model, only the directions of the a- and b-semiaxis are taken from the circle and they are projected to the (x,y)-plane. An ellipse is formed with these parameters on the (x,y)-plane. Ellipses with different shapes and sizes are obtained by rotating the plane.

MacAdam defined 25 ellipses, which fail to cover the whole area inside the CIE x,y chromaticity diagram. The attributes of the MacAdam ellipses, the lengths of the a- and b-semiaxes and the rotation angle θ between the a-semiaxis and the x-axis are interpolated for the whole area inside of the diagram in order to define the parameters of the ellipses in every point at the (x,y)-plane inside the diagram. For every chromaticity value a corresponding plane can be created, and it projects an ellipse to the (x,y)-plane. The ellipse corresponds to the interpolated parameters of the MacAdam ellipses on the corresponding point.

The rotating plane is defined from the three attributes of the ellipse: the lengths of the a- and b-semiaxes and the rotation angle, θ, between the a-semiaxis and the x-axis. The directions of the a- and b-semiaxes, *a–direction* and *b–direction* are obtained from the rotation angle θ. The plane is rotated vertically along a- and b-semiaxis. The vertical rotation angles to the *a–direction,* α_a, and *b–direction,* α_b, are defined as

$$\alpha_a = \arccos(a/r),\ \alpha_b = \arccos(b/r) \tag{6}$$

where a and b are the corresponding lengths of the a- and b-semiaxis and r is the longest semiaxis of the MacAdam ellipses.

For the measured pair of chromaticities two pairs of planes are defined. In the first pair of planes, S_{a1} and S_{a2}, the angle α_a defines the angle of ascent of the plane to *a-direction* and the angle α_b defines the angle of rotation of the plane to *b-direction*; see Figs. 8(a) and 8(b). In the second pair of planes, S_{b1} and S_{b2}, the angle α_b defines the angle of ascent of the plane to *b-direction* and the angle α_a defines the angle of rotation of the plane to *a-direction*.

The differences in heights, h_a and h_b, between the measured pair of planes in the same direction are defined as

$$h_a = d \cdot \arctan(\alpha_a),\ h_b = d \cdot \arctan(\alpha_b) \tag{7}$$

where α_a and α_b are the angles of ascent, and d is the distance between the chromaticities on the (x,y)-plane; see Fig. 8(b). The planes S_{a1} and S_{a2} are connected as one surface S_a through biharmonic spline interpolation.[11] The planes S_{b1} and S_{b2} are similarly connected as one surface S_b; see Fig. 9.

The distance d between the chromaticities (x_0, y_0) and (x_1, y_1) is measured along *a-direction* and *b-direction* on the (x,y)-plane; see Fig. 8(a). The distances d_a and d_b are measured from the point $(x_0, y_0, 0)$ to (x_a, y_a, h_a) and from $(x_0, y_0, 0)$ to (x_b, y_b, h_b) along the surfaces S_a and S_b, respectively. The heights, h_a and h_b are the corresponding heights calculated with Eq. (7).

The chromaticity difference is calculated as

$$\Delta E = \sqrt{\left(d_a \cdot \cos(\varphi)\right)^2 + \left(d_b \cdot \sin(\varphi)\right)^2} \tag{8}$$

where d_a and d_b are the distances measured along the corresponding surfaces S_a and S_b, and φ is the angle

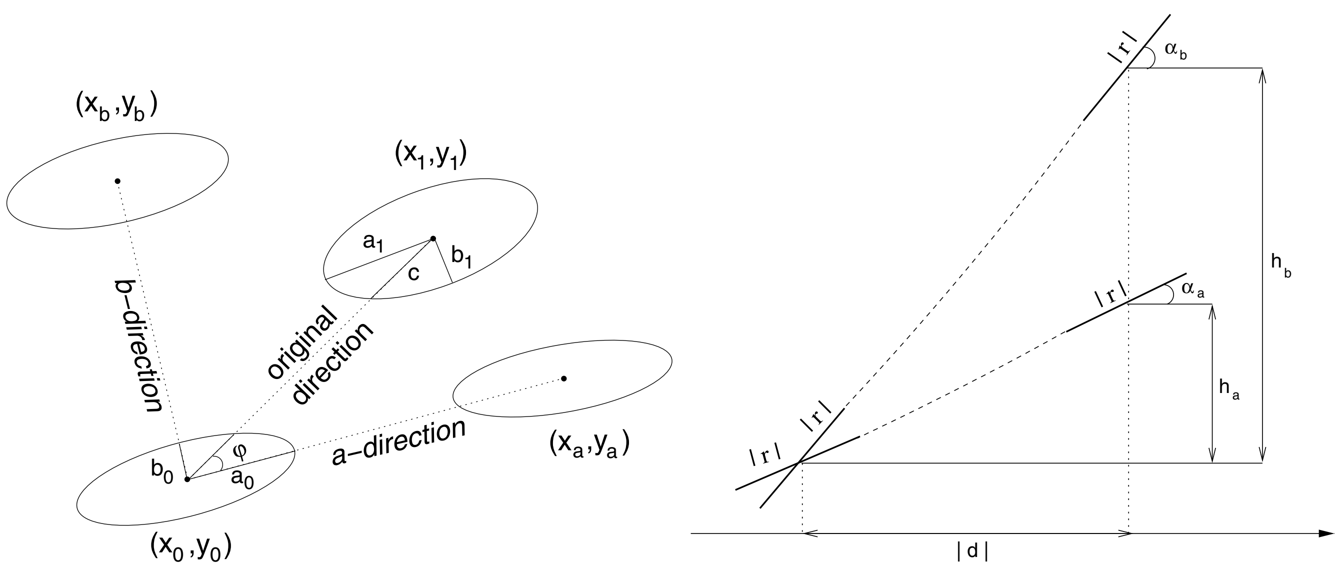

Figure 8. (a) The definitions of *a – direction* and *b – direction* on the *xy*-plane; and (b) the definition of heights in the three-dimensional model.

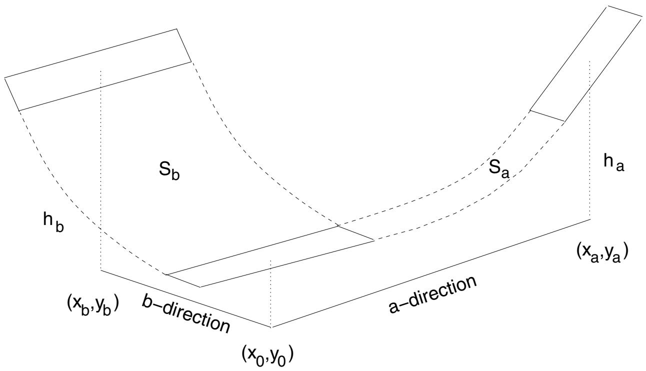

Figure 9. A 3D illustration of the S_a and S_b surfaces.

between the a-semiaxis and the line connecting the chromaticity points; see Fig. 8(a). Equation (8) is based on the equation of the ellipse parametrization

$$x = a \cdot \cos(\alpha),\ y = b \cdot \sin(\alpha) \tag{9}$$

where a and b are the lengths of the semiaxes and α is the rotation angle from the a-semiaxis.

The LMPP combine the calculated chromaticity differences from the surfaces which were oriented to *a-direction* and to *b-direction* in order to assess chromaticity difference in any direction. This leads to the problem that the chromaticity difference is never calculated in its original direction; see Fig. 8(a). The disadvantage is solved in LMCD using the ellipse parametrization before creation of the surface in order to determine the length of c-axis, which is the just-perceptible chromaticity difference on the original direction.

The Line Model with Chromaticity Difference Grid (LMCD)

The line model with chromaticity difference grid (LMCD) was originally named as the Computational Model for Chromaticity Differences.[12]

In the line model for each chromaticity difference a surface is created and the surface consists of a chromaticity difference grid, i.e., a grid is created to surround the first chromaticity point (x_0, y_0), denoted as a starting point. A chromaticity difference is calculated from the starting point to all other points in the grid. The total chromaticity difference is calculated by WDTOCS[8] summing up the chromaticity differences on the shortest path along the surface between the two chromaticity points (x_0, y_0) and (x_1, y_1). In this manner the local variance of ellipse parameters are taken into account.

The definition of a surface is based on the three parameters of an ellipse: 1-2) the lengths of a- and b-semiaxes and 3) the rotation angle θ from the x-axis. The lengths of the semiaxes define the just-perceptible chromaticity differences to the semiaxes directions on the corresponding point. The length of the just-perceptible chromaticity difference to any direction, c-axis can be calculated from the ellipse parameterization.

The total chromaticity difference is represented as a multiple of a reference axis r, which is defined to be equal to the longest just-perceptible chromaticity difference in the data set used. The rest of the just-perceptible differences are parallel projections of the reference axis, which are obtained by vertical rotation of the reference axis. The height of each point of the surface can be calculated using both the distance d_i between the starting point (x_0, y_0) and a chromaticity point (x_i, y_i) in the chromaticity difference grid and the angle of the vertical rotation α_i in a right-angled triangle, see Figs. 10(a), 10(b) and 10(c).

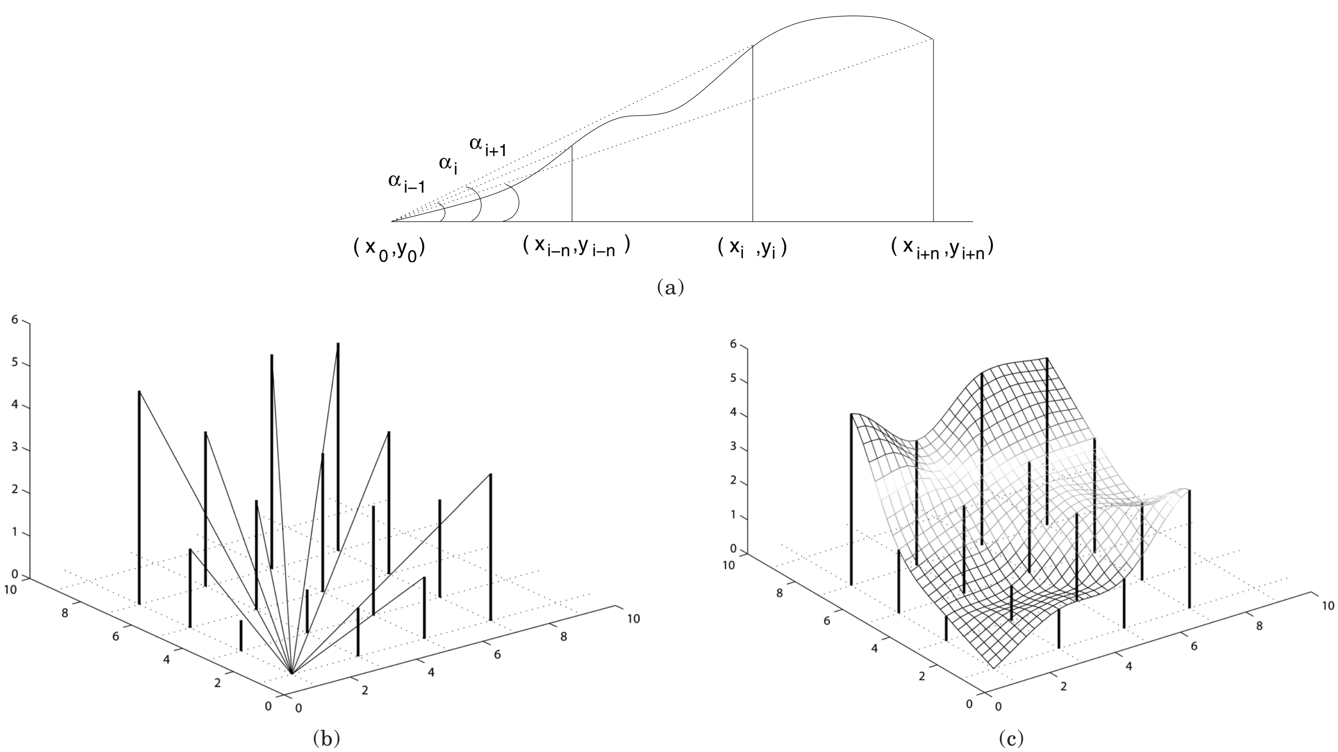

Figure 10. (a) The definitions of the heights of the each point in the chromaticity difference grid. The points (x_{i-n}, y_{i-n}) and (x_{i+n}, y_{i+n}) denote points, which are calculated before and after of the point (x_i, y_i), respectively; (b) an illustration of the chromaticity difference grid; and (c) the chromaticity-difference surface

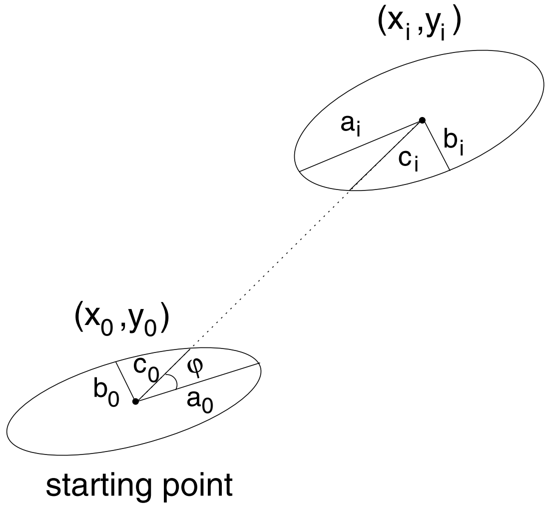

Figure 11. The visualization of the ellipse parameters.

Each point in the chromaticity difference surface is defined as follows. Let C denote the chromaticity difference surface and (x_0, y_0) be the starting point. Each point i in the C is defined as

$$\mathcal{C}(i) = d_i \cdot \tan(\alpha_i) \tag{10}$$

where d_i is the planar difference of chromaticity points, defined as

$$d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2},$$

a_i is the angle of the vertical rotation, defined as

$$\alpha_i = \arccos\frac{c_i}{r},$$

(x_0, y_0) is the starting point, (x_i, y_i) is a chromaticity point in the chromaticity difference grid, r is the reference axis and

$$c_i = \sqrt{(a_i \cdot \cos(\beta))^2 + (b_i \cdot \sin(\beta))^2},$$

where

$$\tan\beta = \frac{a_0}{b_0} \cdot \tan(\varphi),$$

a_i and b_i are the corresponding ellipse semiaxes, a_0 and b_0 are the semiaxes of ellipse in the starting point (x_0, y_0) and φ is the angle between the a-semiaxis and the line connecting the chromaticity points. See Fig. 11 for geometry definitions.

The distances along the surface are calculated by the Weighted Distance Transform on Curved Space (WDTOCS). The total chromaticity difference is calculated as

$$\Delta E_c = \frac{D_s}{r} \tag{11}$$

where D_s is the shortest calculated distance between the two chromaticity points (x_0, y_0) and (x_1, y_1) along the surface and r is the reference axis.

a	b	c
d	e	f
g	h	k

Figure 12. The kernel for the WDTOCS calculation.

Calculating the Distance on a Curved Surface

The Weighted Distance Transform on Curved Space (WDTOCS) was applied to distance calculations along surfaces.[9] WDTOCS between two points is defned as the minimum of all possible paths linking those points. The calculated distance is an approximation of the shortest distance between two points along the surface, and the approximation error results from the discretization of the surface. In WDTOCS every sub-distance between neighboring pixels is Euclidean, but the whole distance between the two chromaticity points is not. WDTOCS produces distance function with the following three properties: symmetricity, positive definiteness and triangle inequality.[13] WDTOCS was originally created for gray level images, but it can be applied to digital surfaces as well.

The WDTOCS algorithm requires only two passes over the digital surface with a chosen kernel. In order to implement the WDTOCS algorithm, two surface models are needed: the original digital surface, and the second surface, which determines the region or regions over which the transform is calculated. The transform is performed on this surface. We have to select one point from the original digital surface as the starting point. From this point, after the calculation, the distances to all other chromaticity points are obtained.

The algorithm, which applies the WDTOCS, proceeds as follows. Let $\mathcal{G}(x)$ denote the original digital surface and let $\mathcal{F}(x)$ denote the binary surface which determines the region(s) in which the transform is calculated. $\mathcal{F}^*(x)$ means an already calculated value. $\mathcal{F}^*(e)$ denotes the new distance value of the point e in the surface $\mathcal{F}$. Let $N_4(e)$ denote the four neighbors of a point e. $G(e)$ denotes the surface value of the center point in the 3×3 kernel and $G(x_i)$ denotes the surface values of the points $x_i \in N_4(e)$. The kernel is depicted in Fig. 12.

1st Iteration: The first iteration round proceeds in the "direct video order" (from top to bottom, and from left to right) calculating $\mathcal{F}^*(e)$. The points marked with asterisk * hold already calculated distance values, while $\mathcal{F}(e)$ has the initial value, which is the maximum representative integer. The iteration proceeds as follows:

$$\mathcal{F}^*(e) = \min[\mathcal{F}(e), \min(da + \mathcal{F}^*(a), db + \mathcal{F}^*(b), dc + \mathcal{F}^*(c), dd + \mathcal{F}^*(d))] \tag{12}$$

where

$$da = \alpha\sqrt{(\mathcal{G}(e) - \mathcal{G}(a))^2 + \beta},\ db = \alpha\sqrt{(\mathcal{G}(e) - \mathcal{G}(b))^2 + \delta}$$

$$dc = \alpha\sqrt{(\mathcal{G}(e) - \mathcal{G}(c))^2 + \beta},\ dd = \alpha\sqrt{(\mathcal{G}(e) - \mathcal{G}(d))^2 + \delta}.$$

The parameter values $\alpha = 1$, $\beta = 2$ and $\delta = 1$ are the corresponding values in the WDTOCS definition.

TABLE I. Ellipse Data Sets

	Dataset	No. of ellipses
BFD-P	total	82
	BFD	41
	MMB	19
	VVVR	10
	CISCC	6
	CIE	5
	STROCKA	1
Rit-DuPont		19
Witt		6
CIE DE2000 dataset	total	107

TABLE II. Summary of the Chromaticity Difference Calculations from the MacAdam Dataset

	MM	LMPP	LMCD
Angles φ 0° and 90°			
arithmetic mean	0.257	1.0054	0.9976
standard deviation	0.180	0.0258	0.0070
Angles φ 22.5°, 45° and 67.5°			
arithmetic mean	0.276	1.0042	0.9991
standard deviation	0.139	0.0162	0.0098

2nd Iteration: The second iteration round proceeds in the "inverse video order" (from bottom to top, and from right to left) calculating $\mathcal{F}^*(e)$. The points marked with asterisk (*) hold already calculated distance values, while $\mathcal{F}(e)$ has a value obtained from application of Eq. (12). The second iteration proceeds as follows.

$$\mathcal{F}^*(e) = \min[\mathcal{F}(e), \min(df + \mathcal{F}^*(f), dg + \mathcal{F}^*(g), dh + \mathcal{F}^*(h), dk + \mathcal{F}^*(k))] \quad (13)$$

where

$$df = \alpha\sqrt{\left(\mathcal{G}(e)-\mathcal{G}(f)\right)^2+\delta},\ dg = \alpha\sqrt{\left(\mathcal{G}(e)-\mathcal{G}(g)\right)^2+\beta}$$

$$dh = \alpha\sqrt{\left(\mathcal{G}(e)-\mathcal{G}(h)\right)^2+\delta},\ dk = \alpha\sqrt{\left(\mathcal{G}(e)-\mathcal{G}(k)\right)^2+\beta}.$$

Again, $\alpha = 1$, $\beta = 2$ and $\delta = 1$ corresponding to the WDTOCS definition.

Ellipse Data Sets

In this work the ellipse data sets were MacAdam ellipses[1] and ellipses which are fitted from the visual color difference measurements used in deriving the CIE DE2000 color difference formula in the CIELAB color space,[6] in this article denoted as CIE DE2000 data set. The MacAdam data set was used in verification of the computational surface models because the interpolation of the MacAdam ellipses is straightforward: they do not overlap each other and they form a harmonic set.

The CIE DE2000 data set consists of several different studies. For deriving the CIE DE2000 color difference formula Luo et al.[14] combined four different color discrimination data sets: BFD-perceptibility,[14] RIT-DuPont,[15] Leeds,[16] and Witt[17] data sets to one single data set. In this work the Leeds data set was excluded since the fitted ellipse data set from the Leeds work was not available for the authors. The ellipses used are reported in Table I.

The CIE DE2000 data set consists of 107 ellipses, which lie in different illumination levels. The orientations and sizes of the ellipses do not a form harmonic set and the ellipses close in the illumination level are overlapping each other. This makes the interpolation of ellipse parameters problematic. The interpolation of the ellipses' parameters were made in the entire $L^*a^*b^*$ color space so the chromaticity differences could be calculated from the different illumination levels without merging all the ellipses to a plane, contrary to the MacAdam ellipses, which all lie in the same illumination level. Therefore in every point of the color space the ellipse parameters were defined. The interpolation of the ellipse parameters was made in the Matlab environment using the nearest-neighbor interpolation method.

The three data sets, from which the CIE DE2000 data set was derived, consist of different numbers of ellipses, which cause imbalance among the data sets in terms of their influence on the CIE DE2000 data set. For example, BFD-perceptibility data set consists of 82 chromaticity difference ellipses compared to 6 ellipses from the Witt data set. Between the Witt and the BFD-perceptibility data sets, the influence of the BFD-perceptibility data set is substantially greater on the CIE DE2000 data set. In this work the data sets are taken as they are without any weighting.

Experiments

The experiments were performed using two different data sets. First the models were tested with MacAdam ellipses to verify the computational surface models, and the results were contrasted with each other. In the second experiment measurements were made with the CIE DE2000 data set using LMCD and the obtained results were compared with the results from the CIE DE2000 color difference formula.

Experimental Results from the MacAdam Data Set

The results from the MacAdam data set show the influence of the model on measured chromaticity differences compared to the Euclidean distances on the (x,y)-plane. Table II collects the results from Figs. 13 and 14 in which the chromaticity differences are calculated from 25 MacAdam ellipses.

In Figs. 13 and 14 the chromaticity differences are calculated using different values for angle φ. In Fig. 13 angle φ was 0° and 90° and in Fig. 14 the chromaticity differences were calculated with angles φ angles of 22.5°, 45° and 67.5°. For the experiments the MacAdam ellipses are as enumerated in Fig. 1. Chromaticity differences are calculated from the center of the ellipse to the edge of the ellipse. The result as a difference between the two chromaticity points should be a constant value 1.0 for LMPP and LMCD, and it is then comparable to the standard deviation. For the mixing model (MM), the assessed chromaticity difference should be a constant value.

Table II presents results from the mixing model (MM), the line model with two pairs of planes (LMPP), and the line model with a chromaticity difference grid (LMCD) for chromaticity difference calculations from the same ellipses. The MM assessed chromaticity differences clearly more inaccurately than the other models. LMPP performed better, but the LMCD achieved considerably more accurate results than the others were able to achieve, see Figs. 13 and 14. When the angle φ was 0° or 90° the standard deviation of LMCD was one-fourth of the standard deviation of LMPP and when the angle

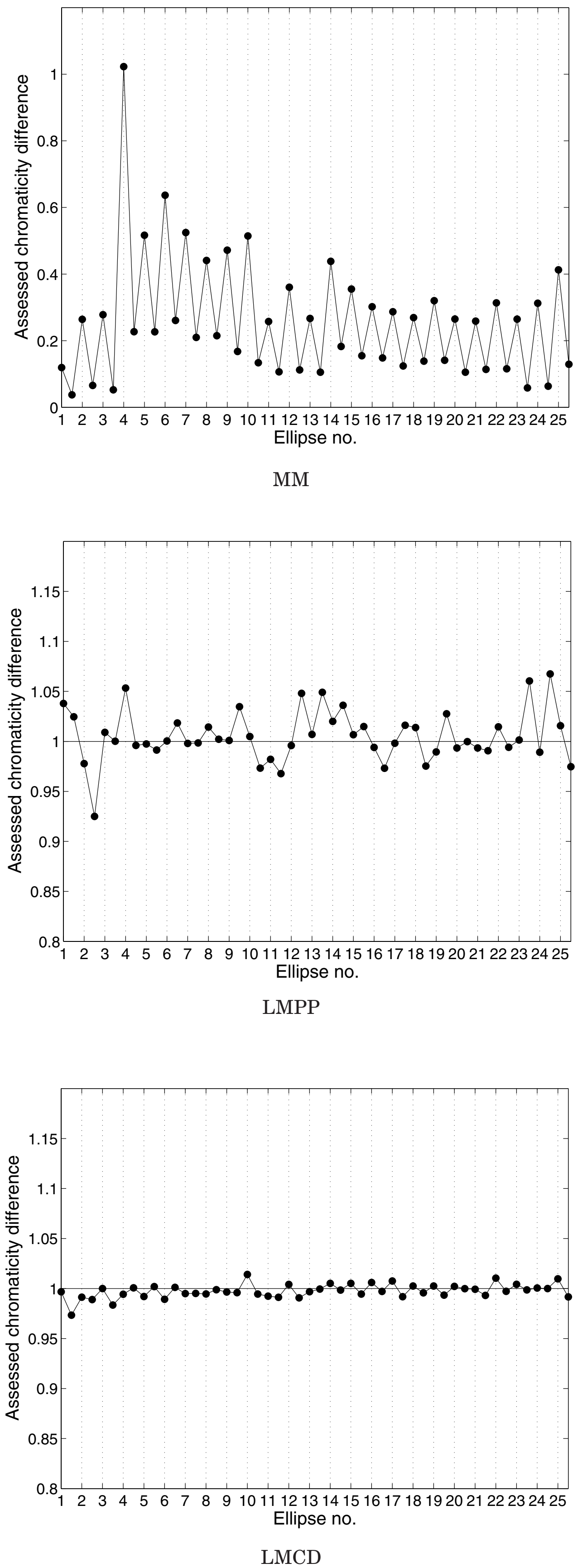

Figure 13. The assessed chromaticity differences from the MacAdam data set. Angle φ was 0° and 90°.

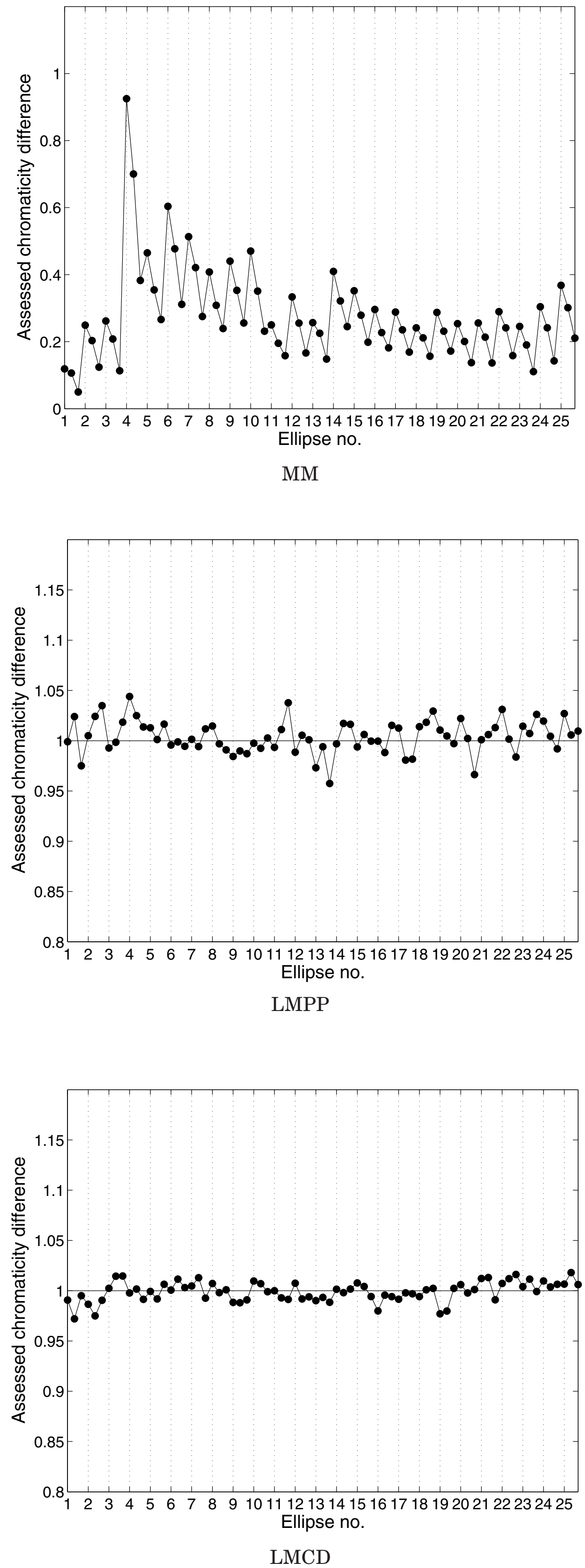

Figure 14. The assessed chromaticity differences from the MacAdam data set. Angle φ was 22.5°, 45° and 67.5°.

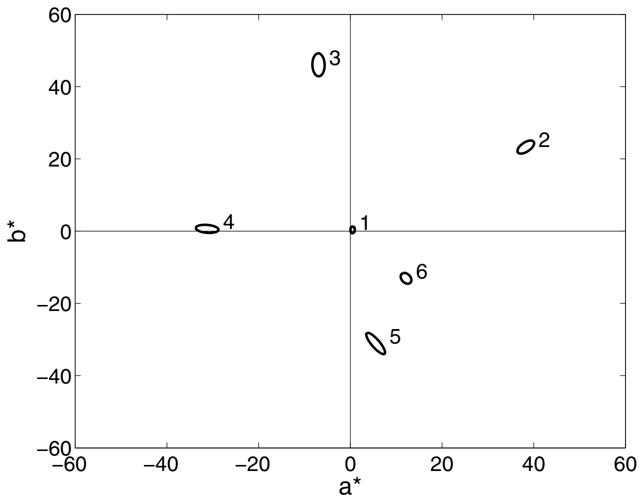

Figure 15. The numbers of the chosen ellipses from the CIE DE2000 data set.

TABLE III. Summary of the Chromaticity Difference Calculations from the CIE DE2000 Dataset

	LMCD		CIE DE2000		
Ellipse	Arithmetic mean	Standard deviation	Arithmetic mean	Standard deviation	Ratio of Arithmetic mean
Small scale					
all	1.036	0.099	0.863	0.106	0.83
1	1.004	0.095	0.793	0.093	0.79
2	1.025	0.117	0.769	0.090	0.75
3	1.023	0.049	0.955	0.084	0.93
4	1.048	0.052	0.863	0.140	0.82
5	1.095	0.165	0.864	0.031	0.79
6	1.020	0.068	0.931	0.026	0.91
Medium scale					
all	4.154		3.295		0.79
1	7.036	1.794	5.258	0.819	0.75
2	3.545	0.597	2.649	0.525	0.75
3	2.531	0.824	2.310	0.763	0.91
4	3.781	0.915	2.882	0.552	0.76
5	4.303	1.923	2.964	1.162	0.69
6	3.730	0.855	3.708	0.738	0.99

φ was variable the standard deviation of the LMCD was one-half.

Experimental Results from the CIE DE2000 Data Set

The second experiment deals with the chromaticity difference calculations with LMCD using CIE DE2000 data set. The achieved results are compared to chromaticity differences calculated by the CIE DE2000 color difference formula.[6] Previous experiments confirmed the performance of the LMCD, but now the interpolation of the ellipse parameters in the CIE DE2000 data set was more challenging.

The chromaticity differences were calculated from five different color centers, which are recommended by CIE[18]: gray (ellipse #1), red (ellipse #2), yellow (ellipse #3) green (ellipse #4) and blue (ellipse #5); see Fig. 15. In addition one ellipse was calculated from a purple center (ellipse #6), which has recently been added to the recommended color centers.[19] The assessed chromaticity differences from the former color centers were the small and medium scale differences.

The results are presented in Fig. 16, where the small scale chromaticity differences assessed by LMCD are marked with a black line and denoted as ΔE_c. The differences were calculated from the center of the ellipse to the edge of the ellipse. The calculated chromaticity difference should equal to 1.0, and it is then comparable to the just-perceptible chromaticity difference. The reference white in the calculations was $X_0 = 94.811$, $Y_0 = 100.000$, $Z_0 = 107.304$.

From Fig. 16 the assessed small scale chromaticity differences by the CIE DE2000 color difference formula can also be observed and the differences are marked with a wide grey line and denoted as ΔE_{00}. The color difference formula was used in chromaticity difference calculations excluding the illumination differences.

Medium scale chromaticity differences are presented in Fig. 17, calculated from the same color centers as in Fig. 16. The chromaticity differences were calculated from the color center to the distance of 5 CIELAB units to different directions. The calculated chromaticity difference should be in harmony with the ellipse size in the corresponding color center, that is in the gray center the assessed chromaticity differences should be larger than in the color centers further away from the achromatic center. Also the largest assessed differences in the color center should be perpendicular to the ellipse orientation in that center.

Table III collects the results from Fig. 16. In general the greatest difference between the LMCD and CIE DE2000 were in the calculated arithmetic mean. LMCD predicted chromaticity values close to unity, which was desirable, since the differences were calculated from the chromaticity difference ellipses. CIE DE2000 achieved an arithmetic mean of 0.86 which significantly differs from unity. The variances and standard deviations of the both models were of the same level. From Fig. 16 and Table III it can be seen that in the ellipses #3 and #6 the results from the CIE DE2000 and LMCD are closest to each other. These ellipses correspond to the yellow and purple centers, respectively.

Table III also presents the summary of the results from Fig. 17. CIE DE2000 achieved slightly smaller standard deviation than LMCD. For both models there were two color centers, gray and blue where the standard deviations were largest. In the gray center the arithmetic mean for LMCD was 7.036, which is in harmony with ellipse size in that color center. In the blue center, the ratio of the lengths of the ellipse semiaxes is significantly larger. For LMCD it seems that when the ellipse is small or narrow, performance in the medium scale chromaticity difference regime is less accurate. Table III also gives the ratios of arithmetic means for small and medium scale differences. The ratio is achieved by dividing the calculated arithmetic mean for CIE DE2000 by the arithmetic mean of LMCD for the corresponding scale. The ratios for small and medium scale chromaticity difference are reasonably close to each others.

Conclusions

New computational surface models for chromaticity differences are defined. The models are based on the surfaces, which are defined by the chromaticity difference ellipse data set and the two chromaticity points from which the chromaticity difference is about to be calculated. The distances are calculated by the Weighted Distance Transform on Curved Space. The surface varied

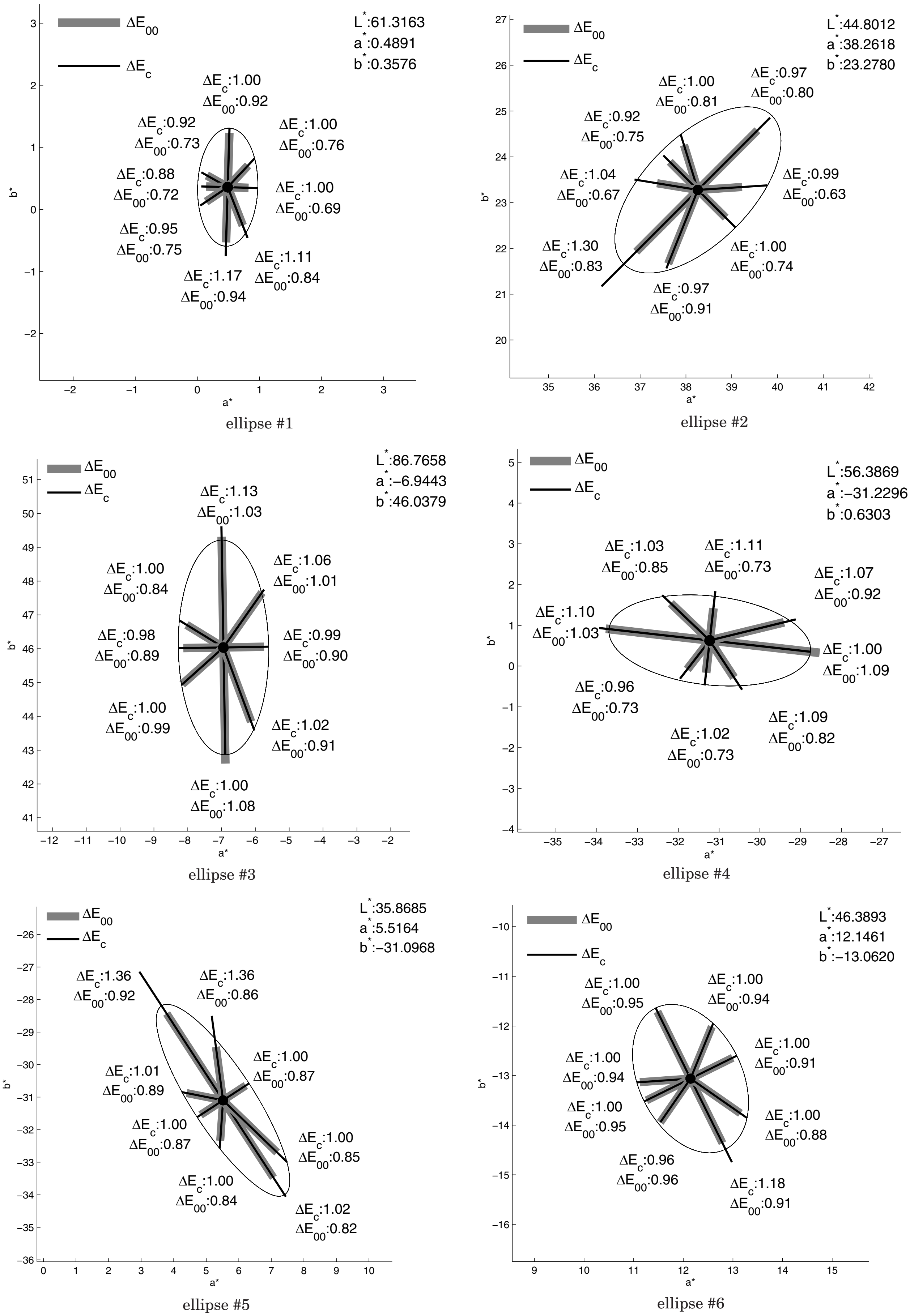

Figure 16. The ellipses showing the relative chromaticity differences assessed by this work and CIE DE2000 color difference formula (marked with wide black line and grey line and denoted as ΔE_c and ΔE_{00}, respectively).

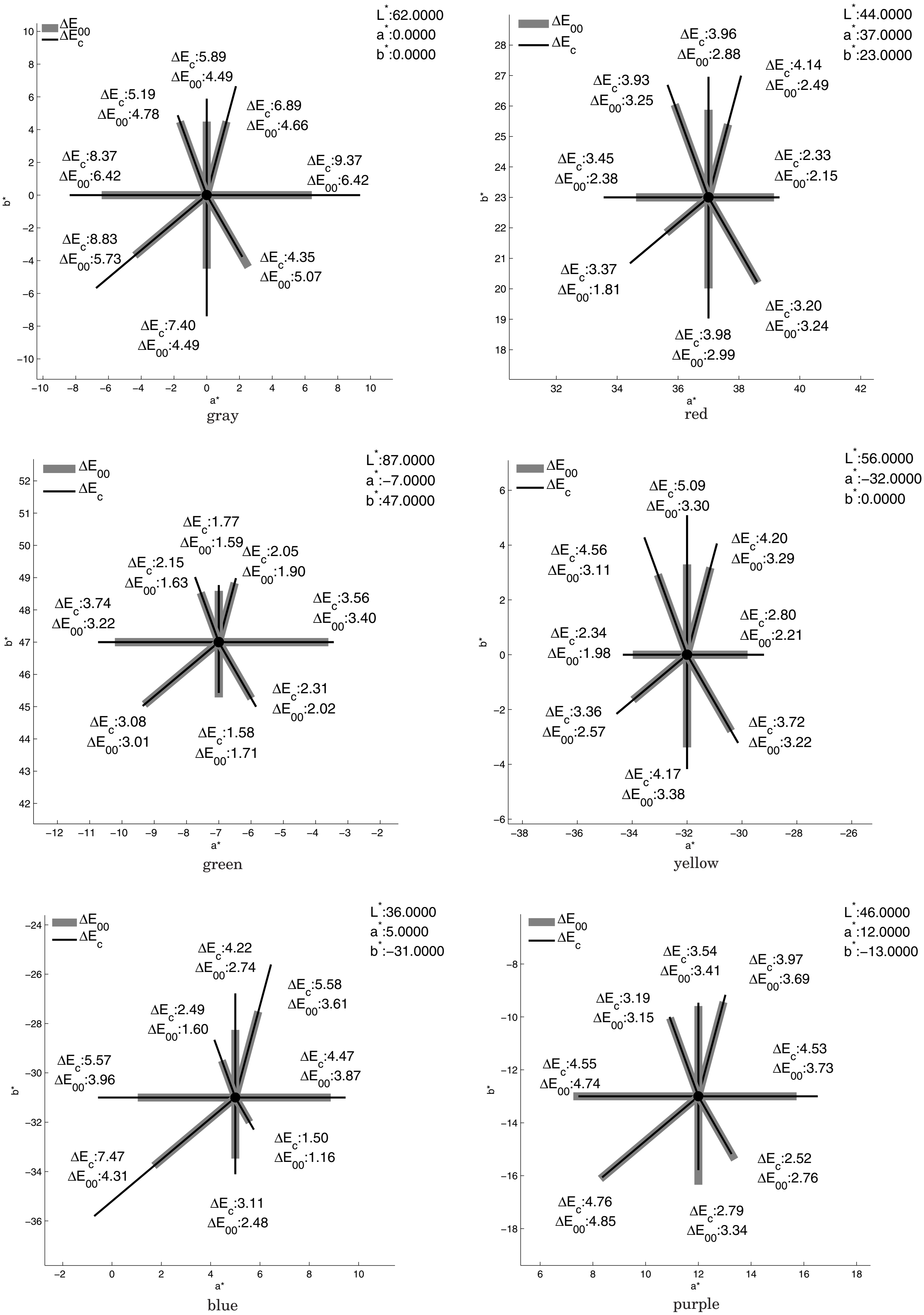

Figure 17. The results showing the medium scale chromaticity differences assessed by this work and CIE DE2000 color difference formula (marked with wide black line and grey line and denoted as ΔE_c and ΔE_{00}, respectively).

according to the two chromaticities whose difference was under consideration.

The results achieved were promising. The accuracy of the calculations from the MacAdam data set improved remarkably between models, and the latest model could overcome the disadvantages of the previous ones. The experimental results validated the computational surface models for chromaticity differences in the vicinity of the chromaticity difference ellipses found from the literature.

The results show that the new model could assess chromaticity differences reasonably well compared to CIE DE2000. The good performance was maintained when calculations changed from small to medium scale chromaticity differences. There are some occasions, where the CIE DE2000 color difference formula gave results notably dissimilar compared to LMCD. These occasions have to be investigated more specifically, because the CIE DE2000 formula is tested and adjusted to small and medium scale color differences.

The method can be applied to any set of planar ellipses. The projection principle is not fixed to the MacAdam ellipses or the CIE DE2000 data set; these were just used to illustrate the method. Thus, it can be expected that the method will also produce better results with other planar ellipse data available in the literature.

The results obtained were encouraging. With further study of interpolation methods and data sets better results may be achieved. Also the development of a color difference model will be an essential part of future work.

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