# Spectral Redundancy in a Six-Ink Ink Jet Printer

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We demonstrate for the multi-ink printers investigated that there are many spectral reflectances that can be produced approximately by a single printer through a large variety of different ink combinations. This *spectral redundancy* was evaluated for a pair of six-ink ink jet CMYKGO printers. Through use of the lookup tables, density maps were built illustrating the six-dimensional distribution of redundancy throughout colorant space. A tolerance of 0.01 RMS showed none of the inks in our CMYKGO systems to be fully replaceable by combinations of the other inks. However, when the tolerance was relaxed to 0.02 RMS, the degrees of freedom for matching spectra in the systems fall to five because the five chromatic inks cover the entire spectral gamut without the need of the black ink. Systematic relationships among the inks are reported, detailing the likelihood that combinations of printer digital counts may be replaced by largely different ones while preserving spectral reflectance to within an RMS spectral reflectance factor tolerance.

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## Introduction

For multi-ink printers, it is found that approximate many-to-one relationships exist between some combinations of inks printed on paper and the measured spectral reflectances of the prints. This we call *spectral redundancy* or *spectrally stable ink variability*.[1] Since there are many sources of uncertainly within any printing and measurement system, two reflectances can only be said to match when they do so within a specified tolerance. Also, for every application there is a level of difference between spectral curves that can be considered negligible. Thus, tolerances may be set based on an error analysis of a system and may also be based on an application's specific requirements.

Color reproduction chains are already well known for their redundant aspects. For example, colorimetry can often be maintained when an original combination of printing inks is replaced by an appropriate alternative combination of printing inks. For four ink printers, methods built around colorimetric redundancy include GCR and UCR where black ink is swapped for some amounts of chromatic inks. Results of the current investigation illuminate a phenomenon with many analogies to the observations underlying successful gray replacement algorithms, but instead of holding appearance constant, it is shown that in many cases the more fundamental property of spectral reflectance may be maintained approximately while drastically modifying ink levels.

For these investigations, reflectance factor spectral root-mean-square (RMS) difference was the chosen method for determining a match between spectra. For many applications other metrics would be far more appropriate. The investigation of metrics for spectral reproduction applications is an ongoing area of research.[2]

Determining the presence of widespread spectral redundancy within a printing system is an important discovery in itself. For those developing means of efficient image processing for spectral color reproduction, it raises important cautions when considering the use of traditional color management building blocks such as multidimensional lookup tables. Problems include the fact that redundancy can manifest in inconsistencies between spectral space and ink space leading to interpolation errors.[3,4] Further, it enables spectral reproduction systems that manage imaging characteristics beyond spectra, such as minimizing total ink coverage or controlling the use of individual inks. This discovery may also point toward the development of criteria for design of inks in spectral reproduction systems and to the introduction of important new capabilities such as spectrally translucent watermarking.[1]

## Spectral Redundancy in a CMYKGO Printer

A six-ink ink jet printer was retrofitted to print with four standard process inks plus an orange and a green ink. The characterization process has been discussed in Ref. 5. During the investigation documented there, it was found that a six-dimensional characterization lookup-table (LUT) with five nodes per dimension ($5 \times 5 \times 5 \times 5 \times 5 \times 5$) was accurate for converting from fractional area coverage to estimated spectrum of a printed patch. The characterization LUT described the forward printer model.

In color processing applications, the inverse of the characterization model is typically more useful than the forward model. Inverting the characterization LUT

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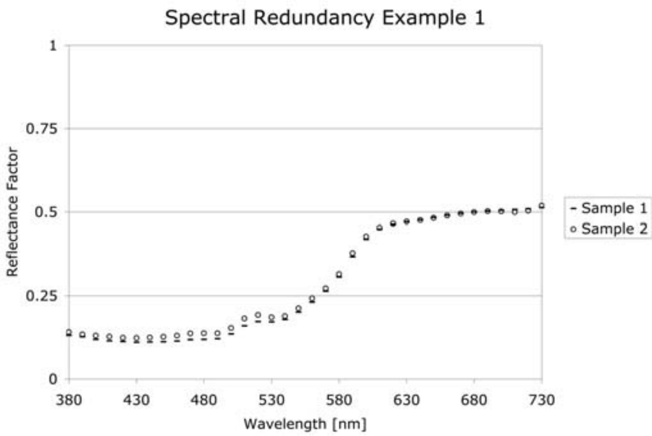


**Figure 1**. Measured spectral reflectances from samples 1 and 2.

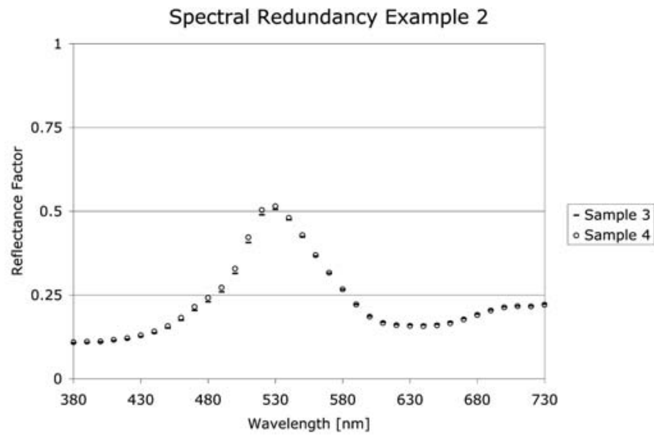


**Figure 2**. Measured spectral reflectances from samples 3 and 4.

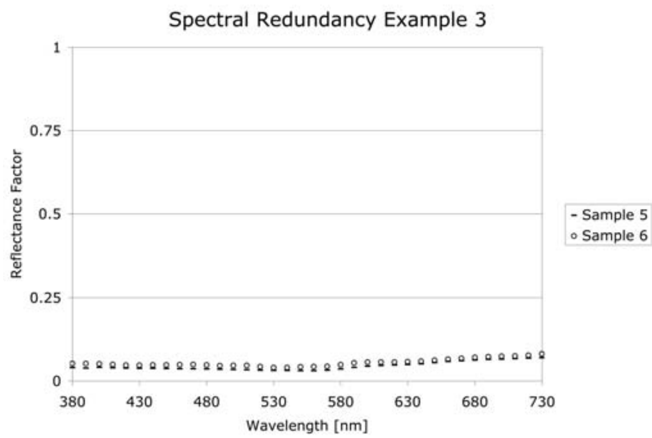


**Figure 3**. Measured spectral reflectances from samples 5 and .6.

**TABLE I. Ink Combinations for Figs. 1–3**

| Fig. | Sample | Fractional Ink Coverages | | | | | |
|---|---|---|---|---|---|---|---|
| | | C | M | Y | K | G | O |
| 1 | 1 | 0.00 | 0.50 | 0.75 | 0.50 | 0.00 | 0.50 |
| | 2 | 0.37 | 0.56 | 0.81 | 0.00 | 0.14 | 0.63 |
| 2 | 3 | 0.25 | 0.00 | 0.75 | 0.25 | 0.75 | 0.00 |
| | 4 | 0.22 | 0.05 | 0.76 | 0.17 | 0.78 | 0.02 |
| 3 | 5 | 0.25 | 1.00 | 0.50 | 1.00 | 0.00 | 0.00 |
| | 6 | 0.67 | 1.00 | 0.75 | 0.82 | 0.64 | 0.79 |

**TABLE II. RMS Spectral Reflectance Factor Differences for Figs. 1 through 3**

| Fig. | Samples | RMS Difference |
|---|---|---|
| 1 | 1 & 2 | 0.011 |
| 2 | 3 & 4 | 0.006 |
| 3 | 5 & 6 | 0.010 |

would allow one to transform from a requested reflectance spectrum to the fractional area coverages that would approximately produce that spectrum when printed. In the previous study, Powell's multidimensional successive-line-minimization method[6] was utilized to search the characterization LUT and return for a requested reflectance spectrum the CMYKGO fractional area coverages estimated to print the request with minimum spectral error. The iterative routine progressively guesses CMYKGO values and compares the interpolation of those values through the characterization LUT with the requested spectrum. Although applying such iterative optimization routines to the inversion of spectral lookup tables has only been attempted in recent years, the use of such routines for the inversion of colorimetric characterization LUTs is well known.

When inverting through the six-dimensional spectral characterization LUT there were some surprises. It was previously unpublished that during our prior investigation we found that by slightly modifying the inversion parameters, it was possible to produce a variety of different ink specifications that matched well the same goal reflectance. These inversion parameters were the seed values and the error tolerance. The *seed values* dictated where within CMYKGO fractional area space the routine should begin guessing ink combinations. The *error tolerance* indicated the precision required for a match.

Printing the chosen ink digital counts showed that the reflectances for the different ink specifications were well predicted and did indeed produce nearly identical reflectances. See Figs. 1, 2 and 3. Table I explains the ink combinations used to make the examples described in the figures. Table II shows the RMS spectral reflectance factor difference between the measurements from the sample pairs.

## System Precision Analysis

An experiment was conducted to determine the source of noise in the system. Included in this test were intra-instrument measurement variability, variation across the print medium, and printer variation across a page. The average and maximum root-mean-square (RMS) errors between measurements for a given patch were used as the evaluation metrics.

On one page, patches in the upper left, upper right, and central area were measured three times without replacement, and three times with replacement. The upper left group of patches was defined as the standard and referred to as Control Target 1. *Without replacement* was defined as measuring patches in the upper half of the page (22 rows and 30 columns) three times in a row without moving the paper. *With replacement*

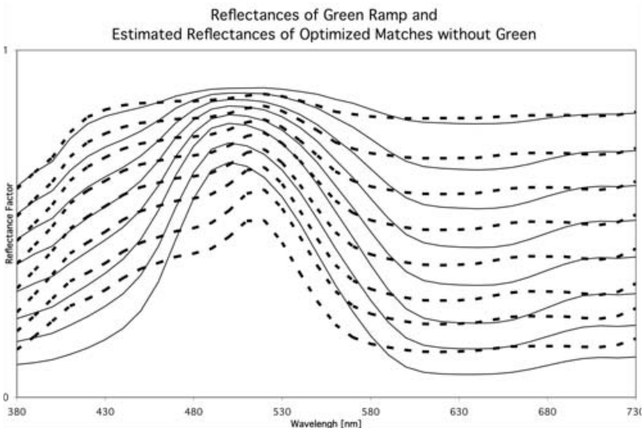


**Figure 4**. Linear independence of the Green ink. Solid lines: Green ramp; broken lines: estimated matches.

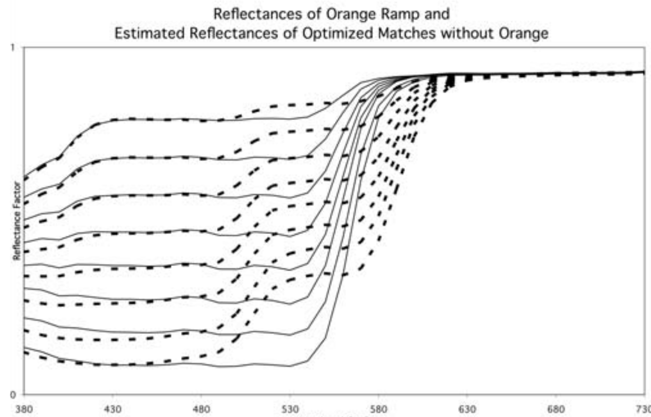


**Figure 5**. Linear independence of the Orange ink. Solid lines: Orange ramp; broken lines: estimated matches.

**TABLE III. Mean and Maximum RMS Error Between Standard Patches and Control Targets**

| Without Replacement | Measurement 1 | | Measurement 2 | | Measurement 3 | |
|---|---|---|---|---|---|---|
| | Mean | Max | Mean | Max | Mean | Max |
| Control Target 1 | Std. | Std. | 0.0019 | 0.0030 | 0.0024 | 0.0039 |
| Control Target 2 | 0.0046 | 0.0061 | 0.0047 | 0.0062 | 0.0047 | 0.0063 |
| Control Target 3 | 0.0038 | 0.0098 | 0.0042 | 0.0099 | 0.0044 | 0.0103 |
| **With Replacement** | **Measurement 1** | | **Measurement 2** | | **Measurement 3** | |
| | Mean | Max | Mean | Max | Mean | Max |
| Control Target 1 | 0.0024 | 0.0039 | 0.0019 | 0.0031 | 0.0025 | 0.0039 |
| Control Target 2 | 0.0047 | 0.0063 | 0.0047 | 0.0067 | 0.0046 | 0.0064 |
| Control Target 3 | 0.0044 | 0.0103 | 0.0043 | 0.0100 | 0.0045 | 0.0099 |

data was collected similarly except that the page was removed and realigned between each of the three measurements. Each of the three measurements was defined as Measurement 1, Measurement 2, and Measurement 3. With the 12 upper left patches of the first measurement defined as the standard, comparisons made between the upper right (Control Target 2) and central quality control patches (Control Target 3) for each measurement are shown in Table III.

The smallest mean and maximum RMS errors were for measurements 1, 2, and 3 of Control Target 1 without and with replacement. Because the same control target was evaluated, this can be viewed as the precision of the instrument. Previous testing of the instrument within the laboratory has shown similar RMS error results when measuring BCRA Tiles over various time frames. The largest mean and maximum RMS errors of 0.0044 and 0.0103 respectively were found when comparing the standard target to Control Target 3 with replacement. These measurements included intra-instrument variability, paper variation, and printer variation.

## Spectral Independence of the Inks

When the observations of spectral redundancy within the printer as illustrated in Figs. 1, 2 and 3 came to light, an immediate question arose as to whether individual inks could be produced from combinations of the others. Given the presence of a Green and Orange, it was possible that the reflectance characteristics of one or both could be generated from some combination of the other inks.

To study this question, the spectra from ramps of each of the individual inks were matched as closely as possible by combinations of the other inks. Ramps consisted of single inks printed from 12.5% area coverage to 100% area coverage, stepped in increments of 12.5%. Results are found in Figs. 4 through 9. Table IV reports the RMS differences between the ramp spectral reflectances and the estimated reflectances from the inverted ink combinations.

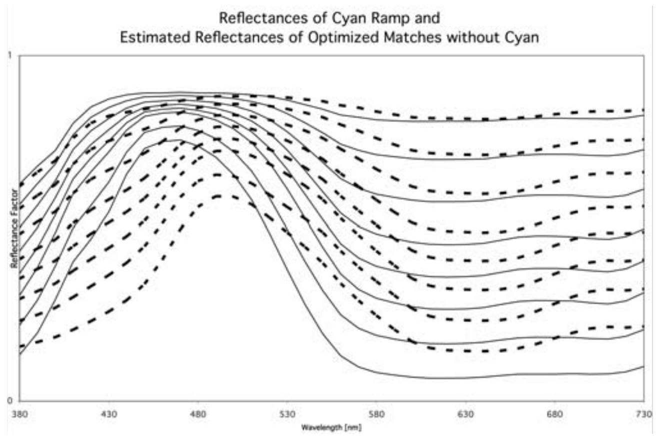


**Figure 6**. Linear independence of the Cyan ink. Solid lines: Cyan ramp; broken lines: estimated matches.

Analysis of the RMS data in Table IV shows that Green and Orange cannot be completely replaced by any combination of the other inks. For both at 100% area coverage RMS difference exceeds 0.10 as the other inks attempt to emulate the measured spectra. Table IV shows the story to be similar for the other chromatic inks as well. It is easy to see in Figs. 6 through 8 that the matches are overall quite poor.

Black, on the other hand, is shown to be much easier to emulate. A maximum RMS difference of 0.019 for matching the black ramp is reported in Table IV. This maximum error falls at 0.50 fractional area coverage. Figure 9 shows systematic differences between the combined matching inks and Black reflectances, especially in the low- and mid-wavelengths and in the mid-area coverages. The differences fall well within measurement error as area coverage becomes very small or very large. An RMS difference of 0.02 is twice the within-sheet repeatability error. If that were chosen as a spectral matching tolerance, Black might be considered as replaceable by the other inks. In the next section analysis goes be-

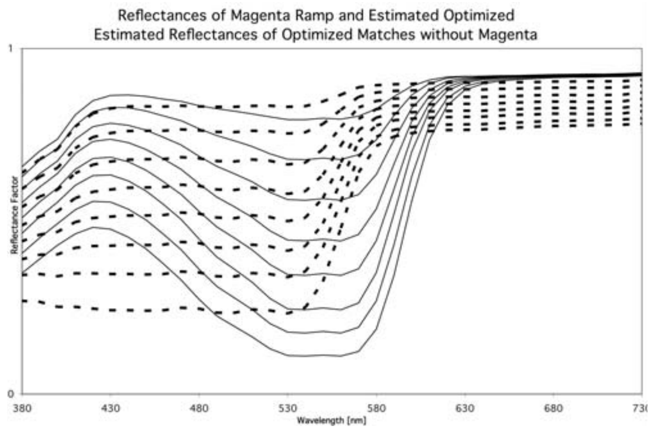


**Figure 7**. Linear independence of the Magenta ink. Solid lines: Magenta ramp; broken lines: estimated matches.

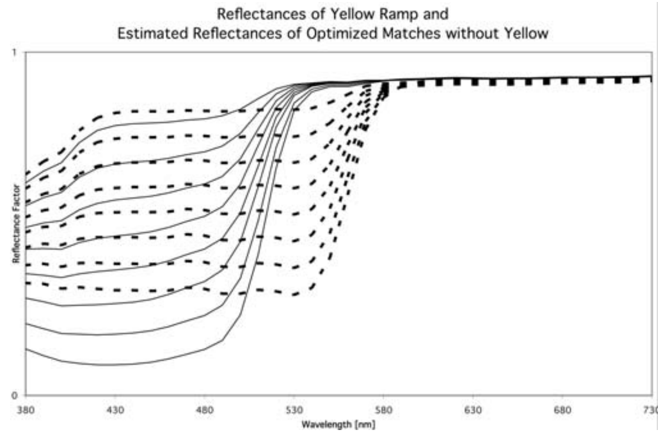


**Figure 8**. Linear independence of the Yellow ink. Solid lines: Yellow ramp; broken lines: estimated matches.

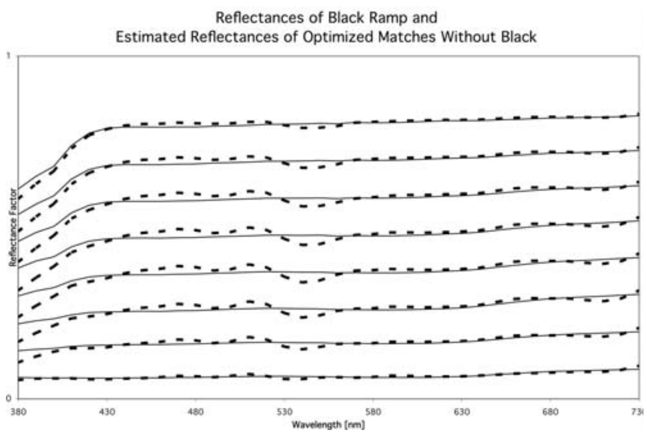


**Figure 9**. Linear independence of the Black ink. Solid lines: Black ramp; broken lines: estimated matches.

**TABLE IV. RMS Differences for Figs. 4 through 9**

| | Ramp | | | | | |
|---|---|---|---|---|---|---|
| Ramp Level | G | O | C | M | Y | K |
| 0.125 | 0.018 | 0.019 | 0.024 | 0.032 | 0.030 | 0.009 |
| 0.250 | 0.037 | 0.038 | 0.048 | 0.065 | 0.061 | 0.015 |
| 0.375 | 0.052 | 0.055 | 0.074 | 0.096 | 0.088 | 0.018 |
| 0.500 | 0.067 | 0.073 | 0.101 | 0.128 | 0.116 | 0.019 |
| 0.625 | 0.081 | 0.088 | 0.121 | 0.153 | 0.142 | 0.018 |
| 0.750 | 0.097 | 0.104 | 0.142 | 0.179 | 0.169 | 0.015 |
| 0.875 | 0.099 | 0.111 | 0.153 | 0.191 | 0.190 | 0.010 |
| 1.000 | 0.106 | 0.122 | 0.172 | 0.205 | 0.212 | 0.004 |

yond the pure color ramps and determines the redundancy of the inks in the presence of other inks.

## Experiment for Finding Spectral Redundancy in Colorant Space

A systematic approach to mapping out the density of spectral redundancy was undertaken. A second six-ink printer, with a different set of CMYKGO inks and substrate was characterized in a similar manner[5] as done for the experiments reported above. Two factorial samplings of the printer colorant space were analyzed to determine the distribution.

The first set included 729 printed samples consisting of all combinations of CMYKGO inks with fractional area coverages of 0.00, 0.50 and 1.00. The second set contained 4096 samples produced by all CMYKGO inks combinations from these fractional area coverages of 0.125, 0.275, 0.600, and 0.875. Combined, these two sets contain a total of 4825 samples.

To study spectral redundancy, the measured reflectance for each sample was fully probed. For each measured spectrum, the characterization LUT was inverted 1536 (6 × 256) times. This time the MATLAB function *fmincon()*,a constrained non-linear optimization routine that uses a Sequential Quadratic Programming method, was used as the optimization engine. The routine is fully described in Ref. 7. Again, each time a spectral reflectance was requested, the program iteratively searched the characterization LUT for a set of CMYKGO values that was estimated to print so that it delivered within tolerance the requested spectrum.

A limitation was imposed on the inversion routine. Each of the 1536 times the LUT was inverted, one of the CMYKGO values was held to a specific magnitude while the other five inks were allowed to vary to any level. In an 8-bit system, fractional area coverages are quantized to 256 digital equivalents. In successive optimizations, each ink was held to all 256 possible digital values for each spectrum. Ink levels returned by the inversion procdure for the other five inks were recorded producing a redundancy profile.

Figure 10 shows one example redundancy profile derived through the process. Here the original digital counts for the printed CMYKGO patch were respectively 7, 6, 39, 3, 179 and 5. Each curve in the figure is associated with one of the individual inks. The curve height at any point is the minimum RMS difference *fmincon()* could produce by varying the other five inks while holding the controlled ink to the x-axis value. Notice the plot minimum for each curve is found at or near the point where the x-axis is equal to the controlled ink's original digital count. Variation in the minimum with respect to the actual original value is due to a small amount of interpolation error in the characterization LUT and the fact that the inversion routine will return when the tolerance value is reached. As each plot moves away from an individual ink's original digital counts, RMS difference tends to rise due to the system's increased difficulty in matching original spectra with the fixed ink level of the controlled ink.

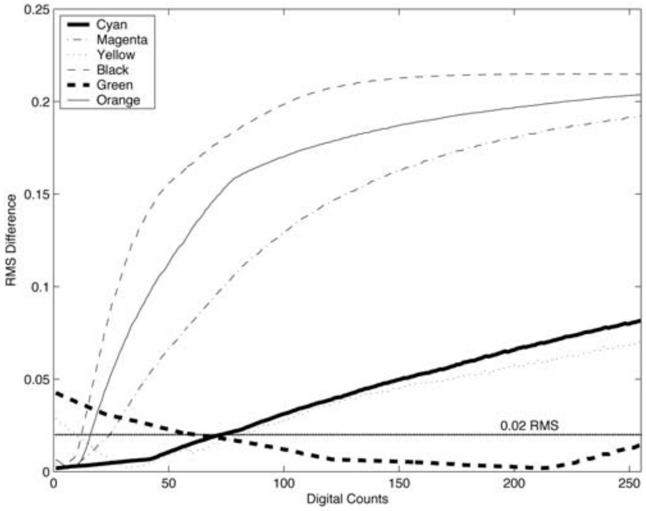


**Figure 10.** Error profile for reflectance from original digital counts C = 7, M = 6, Y = 39, K = 3, G = 179 and O = 5.

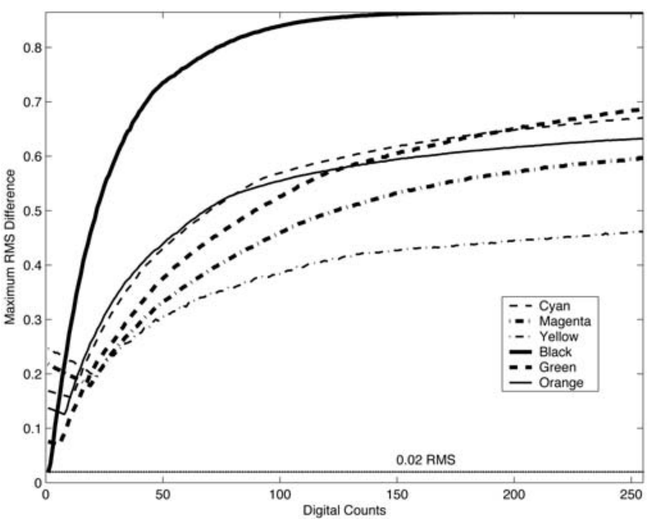


**Figure 12**. Maximum error differences for the dataset.

Figure 11 shows two reflectance spectra. The first is a measurement of a printed patch of the original digital counts of Fig. 10. The second is a measurement of a printed patch of the digital counts associated with the case where the Cyan curve crosses 0.02 RMS in Fig. 10. Those digital counts are CMYKGO 71, 0, 62, 0, 70 and 10, respectively. Although the resultant spectra are very close, the Euclidean distance in digit space is 112.16 digital counts. After printing and averaging the measurement of two samples, the RMS difference between the two is close to the prediction at 0.018.

Figure 12 is similar to Fig. 10 except it summarizes results for all 4825 nodes. Like Fig. 10, the x-axis indicates the digital value for the controlled ink. The y-axis for Fig. 12 shows the largest error value found at that controlled ink value from the entire set of 4825 spectra. Of particular interest are the values at a digital count of 0 because that is where there is no participation at all from the particular controlled ink. Significantly, Fig. 12 shows that for the entire dataset, Black can be held to 0 without introducing RMS spectral reflectance factor difference above 0.02. Thus, for an RMS tolerance of 0.02, Black is found to be completely spectrally redundant to the other five inks for this printer. This is stronger evidence than was provided earlier in the paper regarding the spectral redundancy of Black in this system because here we not only evaluated the Black ramp but also Black in a factorial sampling with all other inks throughout the printer's ink gamut.

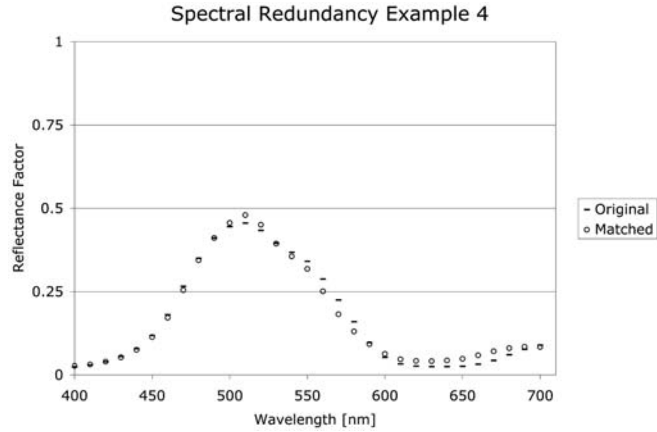


**Figure 11.** Measured spectral reflectances for Fig. 10's original CMYKGO and its match where Cyan crosses 0.02 RMS.

## Redundancy Results

The experiment showed how common it was for various ink combinations to match spectra from within the printer's gamut. Printing and measuring a factorial sampling of ink combinations produced a wide sampling of the printer's realizable spectra. For a requested spectrum taken from the printed set, the routine described in the previous section inverted the characterization LUT while forcing single inks to be held at specific levels. Stepping through the spectra and matching each of them while systematically holding every ink to every possible level produced the data. The effect of this approach was to allow the optimization routine to change five ink levels to any necessary extent for matching every spectrum while holding the sixth controlled ink to specific levels. For each spectrum, the six inks were controlled one at a time to every possible level. When spectral error associated with matches stayed low even as the controlled ink was held far from the original CMYKGO, there was said to be high levels of redundancy in the region of the original CMYKGO with respect to the controlled ink.

Figures were produced to exhibit the discovered spectral redundancy characteristics of the printer. These were built from the first set of spectra described in the previous section where fractional area coverages had all combinations of 0%, 50% and 100%. The figures illustrate the relative quantities of ink combinations that can match spectra in various regions of CMYKGO space. Because it is very difficult to present six-dimensional data on a two-dimensional printed sheet, the figures require interpretation. Figure 13 was designed to aid the understanding of Figs. 14 through 19.

Looking at the topmost part of Fig. 13, one can see a 3 × 3 block of patches. In that block the fractional area coverage for yellow from the original CMYKGO goes from 0% on the left, 50% in the middle and 100% on the

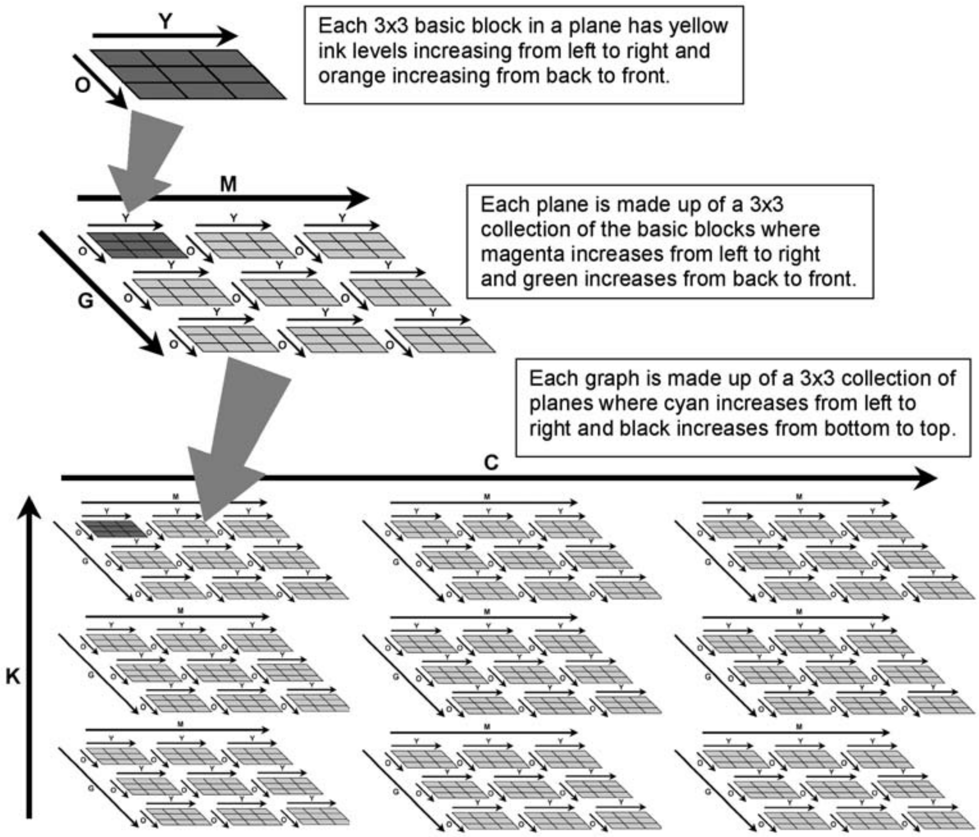


**Figure 13**. Description of how to interpret Figs. 14 through 19. The ink levels are indicated with respect to the original LUT node CMYKGO. The grayscale level indicates the area coverage range for error less than 0.02 RMS.

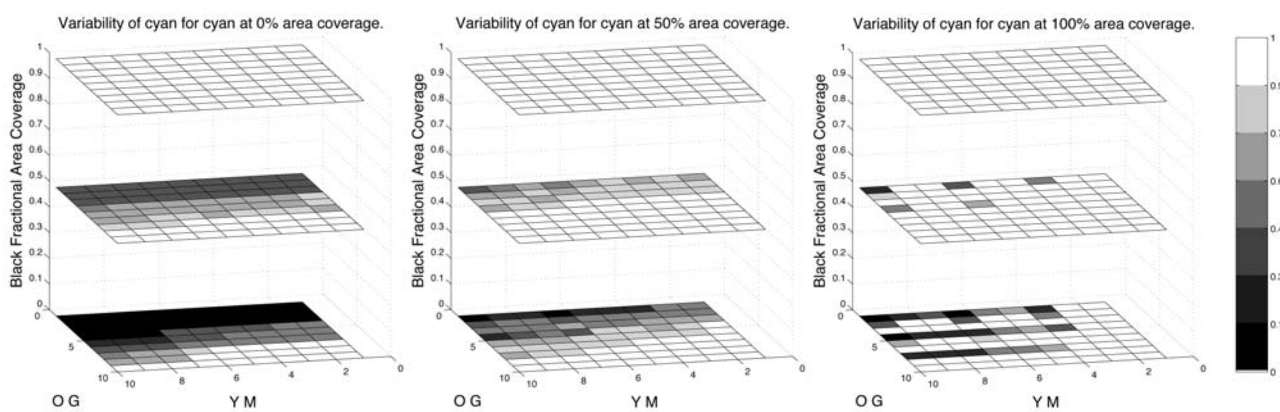


**Figure 14**. Cyan redundancy density map. Gray level indicates the range of Cyan area coverage range that can match original sample spectra at an RMS difference of 0.02 or less. See Fig. 13 for positional interpretation information.

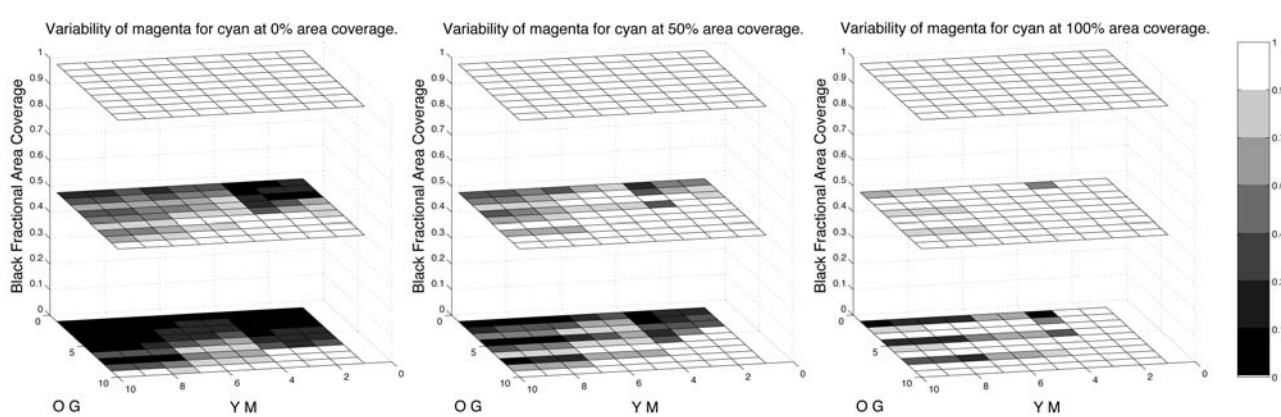


**Figure 15.** Magenta redundancy density map. Gray level indicates the range of magenta area coverage range that can match original sample spectra at an RMS difference of 0.02 or less. See Fig. 13 for positional interpretation information.

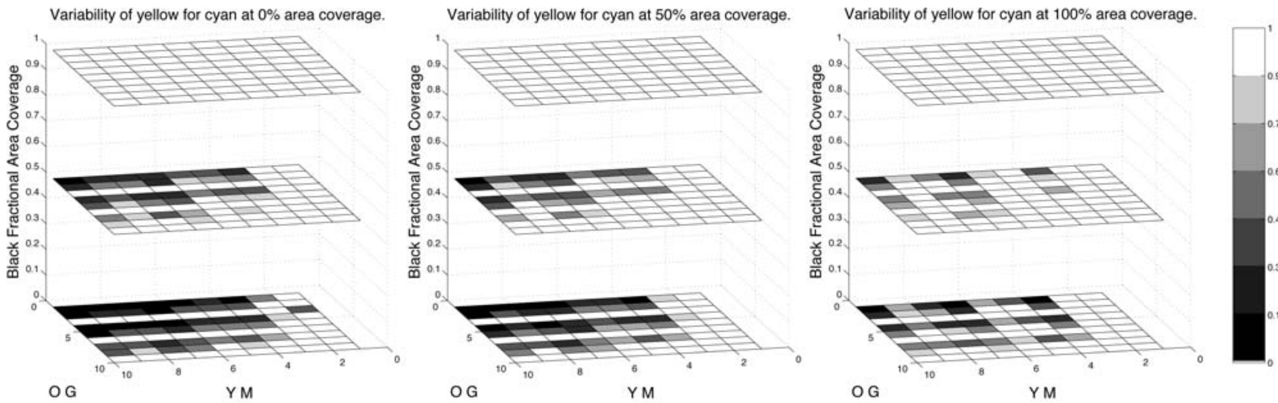


**Figure 16**. Yellow redundancy density map. Gray level indicates the range of Yellow area coverage range that can match original sample spectra at an RMS difference of 0.02 or less. See Fig. 13 for positional interpretation information.

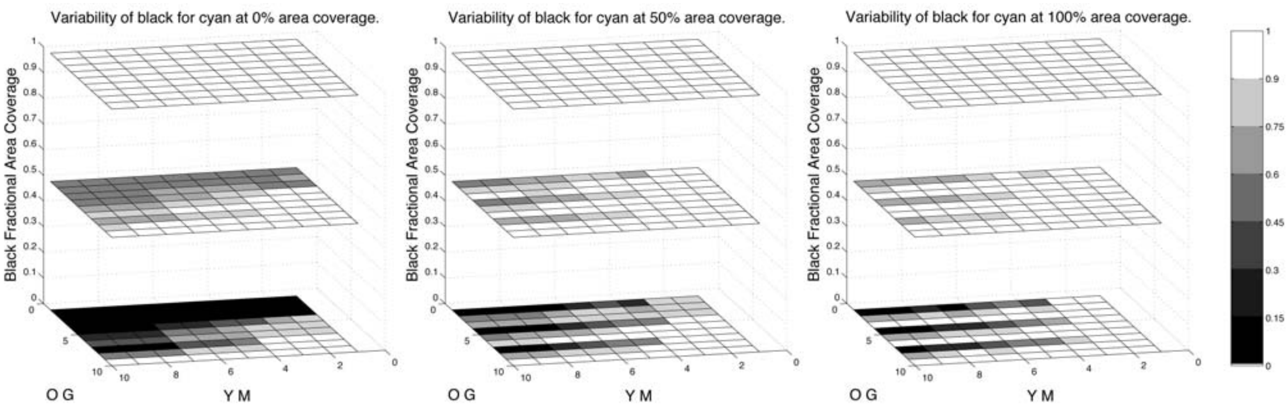


**Figure 17**. Black redundancy density map. Gray level indicates the range of Black area coverage range that can match original sample spectra at an RMS difference of 0.02 or less. See Fig. 13 for positional interpretation information.

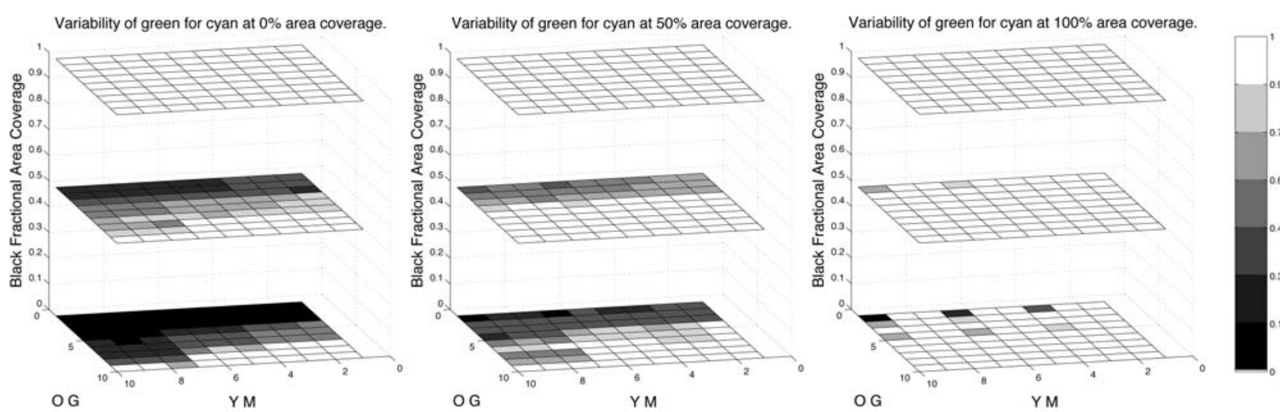


**Figure 18**. Green redundancy density map. Gray level indicates the range of Green area coverage range that can match original sample spectra at an RMS difference of 0.02 or less. See Fig. 13 for positional interpretation information.

right. Orange in these basic 3 × 3 blocks varies from 0% to 100% back to front. Larger planes consisting of 3 × 3 collections of these basic 3 × 3 blocks are shown in the next level down of Fig. 13. Magenta of the original CMYKGO combinations is at 0% for the leftmost row of basic 3 × 3 blocks, increasing to 50% for the middle row of 3 × 3 blocks and topping out at 100% for the rightmost row of 3 × 3 blocks. Green goes from 0% in the backmost row of 3 × 3 blocks in the planes to 50% for the middle row and 100% for the frontmost row. Finally, the bottom portion of Fig. 13 shows that 3 × 3 combinations of the planes make up the graphs as seen in Figs. 14 to 19. Black of the original CMYKGO is at 0% for the bottom row of planes, at 50% for the middle row of planes and 100% for the top row of planes. Cyan of the original CMYKGO is at 0% for the left column of planes, at 50% for the middle column of planes and at 100% for the right column of planes.

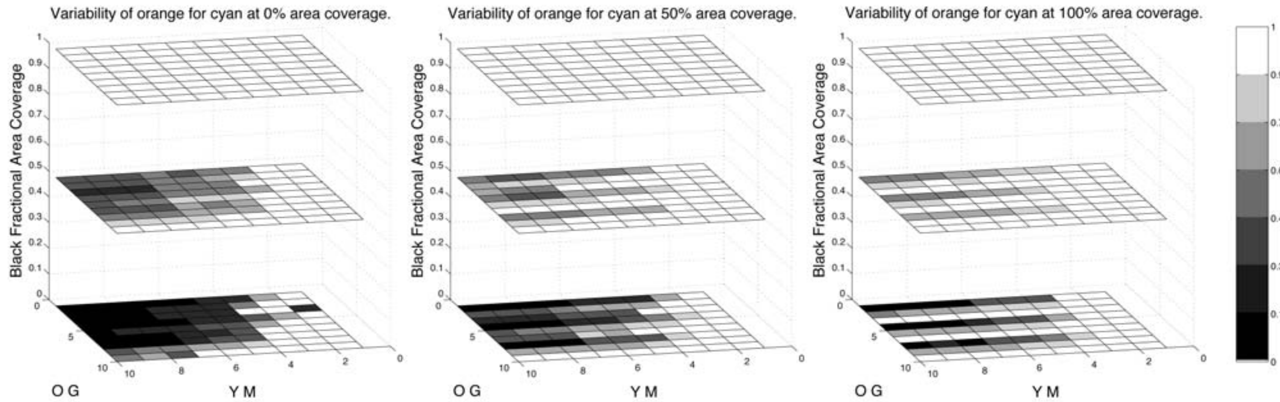


**Figure 19**. Orange redundancy density map. Gray level indicates the range of Orange area coverage range that can match original sample spectra at an RMS difference of 0.02 or less. See Fig. 13 for positional interpretation information.

Each Fig. 14 to 19 is associated with one of the controlled inks. The gray level at each patch in the graph indicates how widely the controlled ink will vary while optimized spectra stay within a 0.02 RMS spectral difference from the original CMYKGO. As explained above, the original CMYKGO can be calculated from the patch's position in the graph. Bottom to top on each graph indicates the amount of Black for the original CMYKGO; left to right on the graph indicates the amount of cyan in the original CMYKGO; front to back on the planes indicates the amount of green in the original CMYKGO, left to right on the planes indicates the amount of magenta in the original CMYKGO; front to back in the subplanar 3 × 3 blocks of patches indicates the amount of Orange in the original CMYKGO; and, left to right in the 3 × 3 blocks of patches indicates the amount of yellow in the original CMYKGO.

The lighter the gray level, the further the controlled ink can move from the original CMYKGO and still be part of a an ink combination that has spectral reflectance within the RMS tolerance of 0.02 spectral difference. The darker the patch, the harder it is for different ink combinations to spectrally match the original CMYKGO.

For several examples, Fig. 14 will be examined. Figure 14 is associated with controlling the cyan level while trying to match the spectra produced by the original CMYKGO's. The lower backmost leftmost patch is associated with the spectrum produced by Cyan = Magenta = Yellow = Black = Green = Orange = 0. In other words, that point is associated with the spectrum of paper alone. The gray level for that patch in the graph is very dark. This shows that it is very hard to vary cyan's ink levels and match the spectral reflectance of paper, which should not be surprising.

Looking at the patch associated with an original CMYKGO of Cyan = 50%, Magenta = 0%, Yellow = 50%, Black = 0%, Green = 50% and Orange = 0% we see that the patch is a medium gray indicating that Cyan can vary up to approximately 50% area coverage while still matching the original CMYKGO spectrum to within 0.02 RMS spectral difference. That patch is located on the bottom of the graph (Black = 0%), in the middle column of the graph (Cyan = 50%), in the fourth row going forward (Green = 50%, Orange = 0%), and in the second row left to right (Magenta = 0%, Yellow = 50%).

The trends shown in Figs. 14 through 19 are indicative of the full 4825 dataset. The data displayed in the figures represent the spectra associated with 0% and 100% ink coverages, and thus come from the gamut edge.

**TABLE V. Summary of Figs. 14 through 19**

Figure **F** shows to an RMS tolerance of 0.02, that ink combinations exist allowing for any level of ink $\mathbf{I_1}$ for matching spectra where ink(s) $\mathbf{I_2}$ are elevated.

| F | $I_1$ | $I_2$ |
|---|---|---|
| 14 | Cyan | Black |
| | | Cyan & Orange |
| | | Green & Orange |
| | | Cyan & Magenta & Yellow |
| 15 | Magenta | Black |
| | | Cyan & Orange |
| | | Green & Orange |
| | | Cyan & Magenta & Yellow |
| 16 | Yellow | Black |
| | | Orange |
| | | Magenta & Yellow |
| 17 | Black | Black |
| | | Cyan & Orange |
| | | Green & Orange |
| | | Cyan & Magenta & Yellow |
| 18 | Green | Black |
| | | Cyan |
| | | Green & Orange |
| 19 | Orange | Black |
| | | Cyan & Orange |
| | | Magenta & Yellow |
| | | Magenta & Green |

Table V summarizes the important features of these density maps. Wherever the gray scale is light, there is a large amount of redundancy in the area of the original CMYKGO with respect to the controlled ink. Figure 14 is thus summarized in the first rows of Table V. As one can see by glancing at Fig. 14, when the original CMYKGO had a high level of black, there is a large amount of redundancy with respect to cyan, regardless of the level of any of the other inks in the original CMYKGO. The same can be said for when cyan and orange were both high in the original CMYKGO, as can be seen in the white stripes on the bottom plane of the rightmost graph. Table V further shows that spectra produced by high levels of green and orange also are highly redundant with respect to cyan, as indicated by the white rows in the front of the bottom level of the graphs. Further, Table V includes the spectra produced by the three-way combination of high levels of cyan, magenta and yellow as being redundant with respect to cyan. This is found in the white patches of Fig. 14 along the rightmost border. Similar interpretations of the other figures are available in Table V.

## Discussion and Conclusions

Figures 14 through 19 and Table V illustrate the fact that for the printers investigated throughout ink space there are many situations in which the same spectrum can be approximately matched by a multitude of ink combinations. Table V summarizes observations of systematic relationships in these figures. The table shows that when ink levels are high for an original $\mathbf{I}_2$ ink combination, the inks in the $\mathbf{I}_1$ column can be swapped in or out to any desired level, and ink combinations exist to match reflectance to within a 0.02 RMS spectral reflectance factor difference.

Analyses of Fig. 9 and Table III showed that within an RMS spectral reflectance factor tolerance of 0.02, combinations of the other five inks could match spectral reflectances of the pure black ink ramp in the evaluated printer. Further, Fig. 12 provides similar evidence for spectra of black printed in combination with other inks. Thus, if spectral RMS were chosen as a spectral matching metric and the tolerance set at 0.02 RMS difference, then black ink in our systems no longer provides an additional degree of spectral matching freedom. At a tolerance level of 0.01 RMS spectral difference, that conclusion does not hold.

One must be careful to choose spectral metrics and tolerances appropriate to their application. Spectral RMS difference may not be adequate to evaluate particular systems. A tolerance of 0.02 RMS difference may, likewise, be too high for particular applications. The same approaches described in this paper, may be used for any chosen metric and any set of tolerances.

Even when analysis shows an ink to be spectrally unnecessary for spectral matching, there might still be sound reasons to use the ink within a system. For example, some approaches to spectral reproduction break a six-ink system into logical sets of multiple four-ink systems,[8,9] in which it may continue to be advantageous to maintain certain inks in the system such as black. Also, there might be reasons similar to the common ones cited for justifying adding black to a CMY colorimetric reproduction system, such as cost of colorant and stability of the reproduction process.

The colorimetric analogy, though, eventually breaks down for the spectral situation. For a CMY printer, black will also increase the colorimetric gamut in the darks. It should be emphasized that, within an RMS 0.02 tolerance, the black inks in the evaluated six-ink systems added no increase to the spectral gamut. They are, instead, completely redundant.

In summary, there is much spectral redundancy within the evaluated CMYKGO six-ink ink jet systems. Investigations were able to map out the density of it throughout the ink space. Future systems can be designed to avoid or enhance spectral redundancy, based on application needs. By applying spectrally stable ink variability analysis to a printing system, spectral difference metrics can be evaluated. Further, new capabilities can be exploited such as spectrally translucent watermarking of images.

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