Controlling the Gray Component with Pareto-Optimal Color Space Transformations

David Littlewood▲ and Ganesh Subbarayan*

University of Colorado at Boulder, Department of Mechanical Engineering, Boulder, Colorado

**Currently at Purdue University, School of Mechanical Engineering, West Lafayette, Indiana*

The Pareto-optimal approach to color management, presented previously by the authors, is further developed to allow for direct conversion into CMYK with complete user control over gray component replacement (GCR). The Pareto-optimal formulation unifies a number of strategies for transforming image data into CMYK: specification of arbitrary GCR, conversion using only chromatic inks, and conversion using at most two chromatic inks and black ink. Use of the black printer is analyzed in terms of extending the CMY gamut and replacing chromatic inks. The program NeuralColor is used to implement the Pareto-optimal formulation, providing data for in-depth analysis of the various conversion methods. NeuralColor accurately models the transformation from CIELAB to CMYK using artificial neural networks. Prints obtained using NeuralColor are accurate within 2 to 4 ΔE^*_{ab} across all levels of GCR.

Journal of Imaging Science and Technology 46: 533–542 (2002)

Introduction

An important and difficult problem in the printing and graphic arts industries is that of color management. Color management refers to the facilitation of color reproduction among various digital color imaging devices such as scanners, displays, and printers. The current work centers on color management for four color printing, focusing specifically on the practice of gray component replacement (GCR).

Various techniques have been proposed for conversion into printer color spaces.[1] Transformation into a printer color space typically requires conversion from a device-independent color space into the CMYK color space, where CMYK refer to the dot fractions of cyan, magenta, yellow, and black.[2] Difficulty stems from nonlinear ink mixing behavior, gamut mismatch, and the indeterminacy of a 3-space to 4-space conversion. Techniques for performing transformation into the CMYK color space include: interpolation methods, models based on optics and ink mixing, and regression models. Interpolation methods involve the creation of a look-up-table (LUT) from which output values can be calculated using a variety of interpolation algorithms.[3,4] Models based on optics and ink mixing include the Neugebauer equations, the Yule–Neilsen model, the Clapper–Yule model, the Kubelka–Munk theory, and the Beer–Bouguer law.[5] Regression models typically involve finding model parameters (such as polynomial coefficients) which minimize the difference between a numerical model and a set of characterization data.[6–8] Artificial neural networks (ANNs), which were utilized in this study, are one type of regression model that has been applied to color printing.[9–13]

The black printer is used in addition to cyan, magenta, and yellow due to a number of benefits. These benefits include: an increase in maximum obtainable density, improved stability throughout a press run, a reduction in cost as less expensive black ink is substituted for chromatic inks, and improved ink drying.[14,15] In terms of transformation from a device-independent color space into the CMYK color space, the addition of the black printer has two main effects: extension of the printer gamut, and the introduction of redundant solutions.

Figure 1 was created to illustrate the difference between the CMY and CMYK gamuts. Figure 1 contains both experimental data and model data; physically measured data are shown as circles, and data generated using the program NeuralColor are shown as solid points. The results show the gamut expansion that results from addition of a black printer and correlate well with the results of Nakamura and Sayanagi.[16]

There exist three regions in the CIELAB color space relevant to GCR in four color printing: the out-of-gamut region, the CMY gamut, and the region outside the CMY gamut but within the CMYK gamut. Colorimetrically accurate transformation from the out-of-gamut region is impossible and requires gamut mapping.[17,18] The conversion of out-of-gamut colors to the closest in-gamut

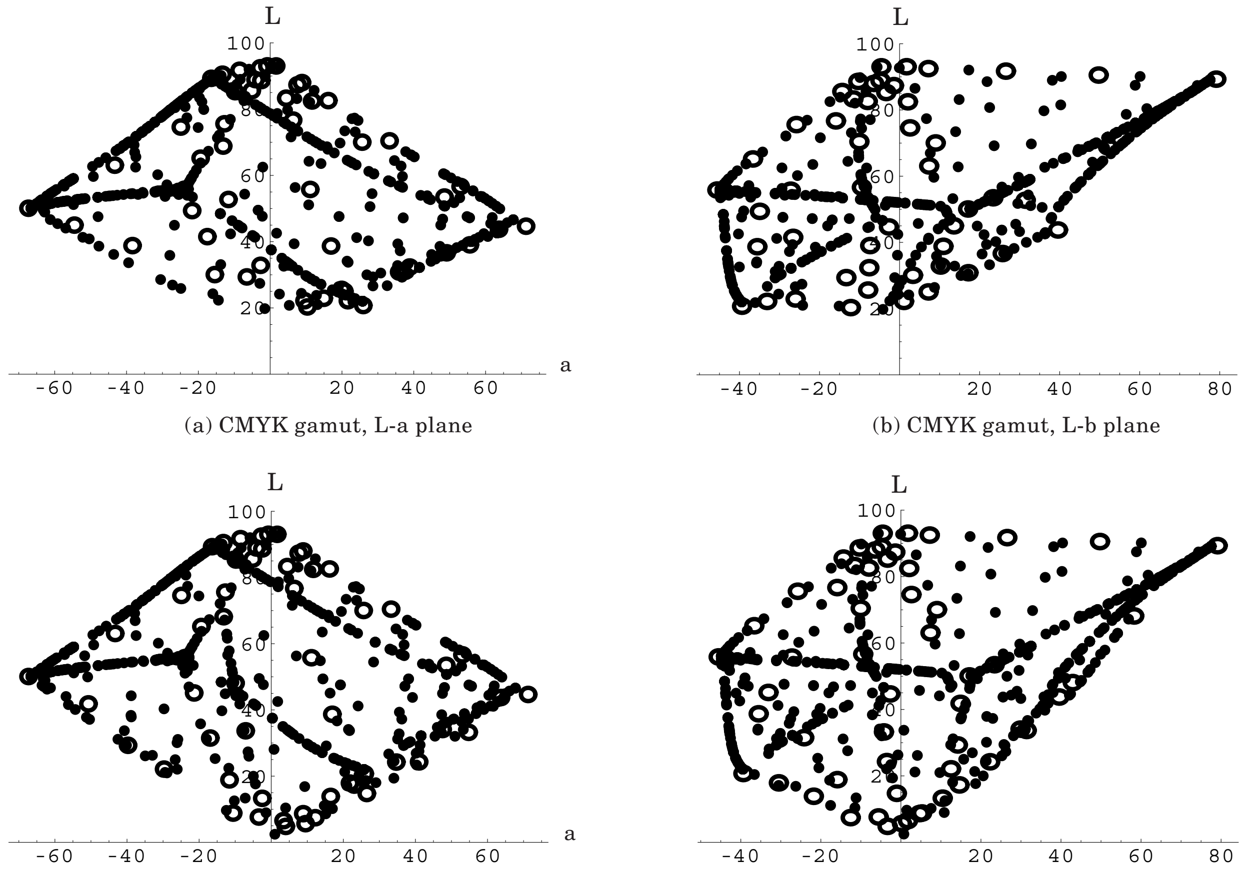
Original manuscript received October 3, 2001

▲ IS&T Member

littlewo@sprocket.colorado.edu; ganeshs@purdue.edu

Supplemental Materials—The Appendix to this article can be found in color on the IS&T website (www.imaging.org) for a period of no less than two years from the date of publication.

Color Plate 3 is printed in the color plate section of this issue, pp. 552–556



(a) CMYK gamut, L-a plane (b) CMYK gamut, L-b plane (c) CMYK gamut, L-a plane (d) CMYK gamut, L-b plane

Figure 1. CMY and CMYK gamuts for a Tektronix® Phaser®740 printer. Physically measured data are shown as circles, and model data are shown as solid dots.

color is commonly termed clipping, and is often considered a preferable technique for gamut mapping. Transformation inside the CMY gamut is possible with many combinations of CMYK. One extreme solution in the CMY gamut uses only chromatic inks ($K = 0$). The other extreme solution in the CMY gamut uses only two chromatic inks plus black (min(CMY) = 0). Intermediate solutions can be described in terms of percent GCR, which is determined as the percentage of the achromatic portion of an image printed with black ink. Colorimetrically accurate transformation is also possible in the region outside the CMY gamut but within the CMYK gamut. In this region all four inks may be required for colorimetric match. Flexibility in the choice of solution does exist, however. Excluding the points that lie on the gamut surface itself, color transformation in this region is again bounded by two extreme solutions. The extreme solutions may be referred to as the maximum-black solution and minimum-black solution. It is noted that for any in-gamut color the minimum-black solution has either $K = 0$ or max(CMY) = 1, while the maximum-black solution has either $K = 1$ or min(CMY) = 0. A final subset of in-gamut colors that is of interest consists of colors that lie directly on the gamut surface. Such colors have a unique CMYK solution.

In terms of GCR, methods for determining CMYK values based on device-independent values may be divided into two categories: transformations with fixed GCR, and transformations which allow user-specified GCR. Fixed GCR methods are most common in practice (as in LUTs), however a number of attempts have been made to allow for flexible GCR.[7,19–22] The Pareto-optimal method presented here differs from the GCR methods cited above in that it generalizes a number of conversion objectives into a single methodology. It utilizes a single method for converting CIELAB to CMYK, finds CMY and K values simultaneously, and does not rely on interpolation between a set of GCR solutions. The Pareto-optimal approach allows for full user control over the gray component, including image reproduction across all levels of GCR, image reproduction with only chromatic inks, and image reproduction using at most two chromatic inks plus black. In addition, the Pareto-optimal approach does not impose any restrictions on the gamut of reproducible colors. The Pareto-optimal method can achieve any given GCR scheme with minimal loss in colorimetric accuracy.

The program NeuralColor was used to validate the Pareto-optimal method for GCR control. NeuralColor is an application of the Pareto-optimal approach that utilizes ANNs as transfer functions between the CMYK and CIELAB spaces.[23–26] Specifically, NeuralColor was used to characterize a Tektronix®Phaser®740 printer. The unification of various printing objectives in the Pareto-optimal scheme gives maximum user control over reproduction of the gray component.

Methods

As an introduction to Pareto-optimal methods for color management, consider the expression of color matching as an optimization problem. Given a set of functions $f_L(\mathrm{CMYK})$, $f_a(\mathrm{CMYK})$, and $f_b(\mathrm{CMYK})$ which predict L^*, a^*, and b^* values based on CMYK, a color match may be found by solving the optimization problem of Eq. (1)

minimize the function

$$f = \Delta E^*_{ab}(CMYK) = \sqrt{(L - f_L(CMYK))^2 + (a - f_a(CMYK))^2 + (b - f_b(CMYK))^2} \quad (1)$$

where $(L, a, b) = (L^*, a^*, b^*)_{\mathrm{input}}$

CMYK values which solve the optimization problem, Eq. (1) are suitable for colorimetric reproduction of the input CIELAB. The solution $f(\mathrm{CMYK}) = 0$ is found when the predicted output CIELAB values match exactly the input CIELAB values, which is possible for any in-gamut color. When converting out-of-gamut colors, solution of Eq. (1) yields the closest in-gamut color. Thus the optimization equation allows for colorimetrically accurate transformation inside the print-device gamut, and performs a clipping gamut mapping for out-of-gamut points.

Color Management as a Pareto-optimal Problem

The optimization problem given in Eq. (1) by Littlewood, Drakopoulos and Subbarayan, to include a number of competing print objectives.[24–26] Optimization among a set of competing objectives is termed Pareto-optimization. Equation (2) includes terms for ΔE^*_{ab}, total ink, chromatic ink, and black ink. Constraints are added to the Pareto-optimal problem to place bounds on total ink usage, black ink usage, and allowable ΔE^*_{ab}.

minimize the function

$$f = c_1 \Delta E^*_{ab}(CMYK) + c_2 I(CMYK) + c_3 \mathcal{C}(CMYK) + c_4 K \quad (2)$$

subject to the constraints

$$h_1 = \Delta E^*_{ab\ \max} - \Delta E^*_{ab} \geq 0$$
$$h_2 = I_{\max} - I \geq 0$$
$$h_3 = K_{\max} - K \geq 0$$
$$0 \leq c_i \leq 1$$

where

$$E^*_{ab} = \sqrt{(L - f_L(CMYK))^2 + (a - f_a(CMYK))^2 + (b - f_b(CMYK))^2}$$

$$I = C + M + Y + K$$
$$\mathcal{C} = C + M + Y.$$

Equation (2) allows for implementation of any number of print objectives through specification of the objective function parameters c_i and the constraint values $\Delta E^*_{ab\,\max}$, $I_{\max}$, and $K_{\max}$. Examples of possible print objectives are minimization of ΔE^*_{ab} with an upper bound on ink usage, and minimization of ink usage with an upper bound on ΔE^*_{ab}. In order to ensure consistent color reproduction and smooth transition between neighboring colors, it is important that optimization parameters be chosen such that multiple solutions for any given input color do not exist,[25] as was done throughout the current study. The following sections demonstrate how the gray component can be controlled through careful implementation of Pareto-optimization techniques.

Solutions for Maximum and Minimum GCR

Several combinations of parameters in Eq. (2) yield a maximum-GCR solution. Setting $c_4 = -1$, $c_1 = c_2 = c_3 = 0$, $\Delta E^*_{ab\ \max} = 0$, $I_{\max} = 4$, and $K_{\max} = 1$ specifies maximization of black ink with zero allowable ΔE^*_{ab}. The constraints on total ink and black ink are effectively removed by setting the maximum allowable values equal to the maximum possible values. The Pareto-optimal equation is reduced to

$$\text{minimize the function} \quad f = -K \quad (3)$$
$$\text{subject to the constraint} \quad h_1 = 0 - \Delta E^*_{ab} \geq 0.$$

For in-gamut colors, the solution of Eq. (3) yields the maximum-GCR solution. For out-of-gamut colors, Equation (3) cannot be solved, in which case the clipping algorithm may be implemented by unconstrained minimization of ΔE^*_{ab}. This is the case whenever the constraint $\Delta E^*_{ab} = 0$ is violated. For the case of out-of-gamut colors mapped to the gamut surface, a unique solution exists and there is no leeway in determining the relative amounts of black and chromatic inks.

Two additional formulations that yield a maximum-GCR solution are minimization of total ink and minimization of chromatic ink. As in Eq. (3), these approaches require the constraint $\Delta E^*_{ab\ \max} = 0$. These formulations utilize the fact that the maximum-GCR solution has minimum total ink deposition and minimum chromatic ink deposition.

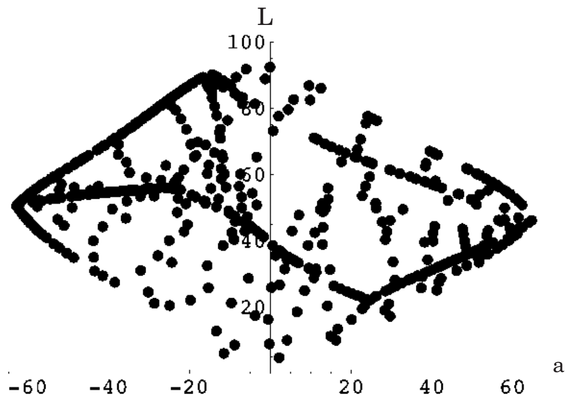
Minimum-GCR solutions can be obtained in a complementary fashion to the maximum-GCR solutions. Minimization of black ink with colorimetric match is obtained by reducing Eq. (2) to

$$\text{minimize the function} \quad f = K \quad (4)$$
$$\text{subject to the constraint} \quad h_1 = 0 - \Delta E^*_{ab} \geq 0:$$

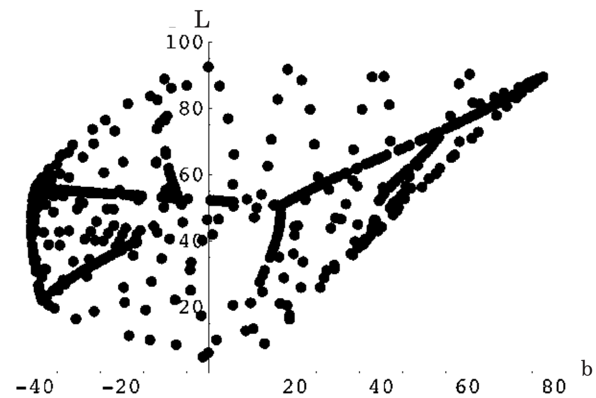
The maximization of total ink and the maximization of chromatic ink, when used with the constraint $\Delta E^*_{ab\,\max} = 0$, provide additional formulations for achieving minimum GCR.

An important characteristic of Pareto-optimal solutions for user-specified GCR can be noted at this time. Equation (4) specifies minimization of black ink. The constraint on ΔE^*_{ab}, however, allows this minimization to take place only under the condition of colorimetric match. Black ink is therefore reduced to zero only when to do so would not sacrifice colorimetric accuracy. When black ink is needed to extend the gamut of the print device, it is provided in the minimum amount possible. In this sense, Pareto-optimal formulations are able to distinguish between the two effects of the black printer: extension of the CMY gamut, and latitude in reproduction of the gray component.

A final method is available for maximizing GCR that does not allow for the use of black ink when required for colorimetric match. This is the pure chromatic solution. In this case, ΔE^*_{ab} is minimized with $K_{\max} = 0$. Colo-



(a) L – a plane



(b) L – b plane

Figure 2. The gamut of attainable colors using only two primaries plus black for any given color

rimetric error is introduced in this solution, as black ink is not available to extend the CMY gamut.

Two-Step Methods

Adjustments to the gray component in the maximum- and minimum-GCR solutions can be made with an additional processing step. Methods for reducing black from the maximum-GCR solution, and for reducing *min*(CMY) from the minimum-GCR solution, are presented below. The presentation of these methods makes use of the constraint-equation parameters G_i. The constraint equations were developed in such a way that the parameters G_i roughly correlate to percent GCR. Explicit use of the term percent GCR, however, is restricted to solutions that are bounded by both the 0%- and 100%-GCR solutions.

Beginning with the minimum-GCR solution, a second solution may be found in which *min*(CMY) is reduced by some fraction of its value in the 0%-GCR solution. The smallest dot fraction from among the chromatic inks is constrained equal to $I_{\min}^{0\%} \cdot (1 - G_1)$, where $I_{\min}^{0\%}$ is equal to the smallest of the cyan, magenta, and yellow inks for a given pixel in the 0%-GCR solution. G_1 represents the degree to which the chromatic inks are reduced, and falls between zero and one. The corresponding optimization problem is

$$\text{minimize the function} \quad f = \Delta E^*_{ab}(\text{CMYK}) \tag{5}$$
$$\text{subject to the constraint} \quad g_1 = min(\text{CMY}) - I_{\min}^{0\%} \cdot (1 - G_1) = 0$$

where

$$I_{\min}^{0\%} = \text{the value of } min(\text{CMY}) \text{ in the } 0\% - \text{GCR solution:}$$

Equation (5) effectively removes chromatic inks from the 0%-GCR solution and replaces them with black ink. By setting $G_1 = 1$, the smallest of the inks in the 0%-GCR solution is constrained to zero. The resulting solution is one that uses only two chromatic inks plus black for any given pixel. This is a valid solution for all points inside the CMY gamut and the majority of reproducible colors in general, but introduces error in the case of colors which require all four inks for colorimetric match. The range of colors requiring all four inks for colorimetric match is small, however, and in most cases significant error will not result. To confirm this fact, the program NeuralColor was used to display the gamut of colors obtainable using only two of the three primaries plus black, i.e., using either CMK, CYK, or MYK for any given color. The results are displayed in Fig. 2. The gamut obtained with two primaries plus black has very little apparent difference compared to the full CMYK gamut (Fig. 1), except near $L^* = 0$.

Equation (5) decreases chromatic ink from the 0%-GCR solution. It is also possible to reduce black from the 100%-GCR solution, forcing an increase in chromatic ink deposition to maintain colorimetric match. In this case, the black dot fraction is constrained to $K^{100\%} \cdot G_2$, where $K^{100\%}$ is the dot fraction found for black in the 100%-GCR solution and G_2 represents the percentage of black retained from the 100%-GCR solution. As black ink deposition approaches zero and the printer gamut is reduced to the CMY gamut, this formulation becomes prone to significant error.

The specification of a ratio between the black dot fraction and the sum of the black dot fraction and the *min*(CMY) dot fraction can also be implemented. For this formulation, a constraint equation is developed from the expression

$$G_3 = \frac{K}{K + \min(CMY)}. \tag{6}$$

This ratio roughly equates to black over the total gray component. This formulation differs from the methods presented previously in that neither $K^{100\%}$ nor $I_{\min}^{0\%}$ appear in the constraint equation. It is necessary that the ink with the smallest dot fraction be known, but the value of this dot fraction is not required.

Bounding Solutions by the 0%-GCR and 100%-GCR Solutions

In the framework of the two-step approaches presented above, the final solution is bounded by solutions corresponding to $G_i = 0$ and $G_i = 1$. In all cases, at least one of these extreme solutions restricts the gamut of printable colors. A gamut restriction occurs when the solution is constrained to using only two chromatic inks plus black, and also when the solution is constrained to using only the chromatic inks. The following formulations bound the solution set by the true 0%- and 100%-GCR solutions, thus allowing for colorimetric match throughout the solution set.

If the 100%-GCR and 0%-GCR solutions are known, then an arbitrary GCR solution may be obtained by specifying either *min*(CMY) or black to fall between the upper and lower bounds provided by these solutions. In the case of specifying a value for *min*(CMY), percent GCR may be specified in terms of the following parameter

$$G_4 = \frac{I_{\min}^{0\%} - \min(CMY)}{I_{\min}^{0\%} - I_{\min}^{100\%}} \tag{7}$$

where

$I_{\min}^{0\%}$ = the value of *min*(CMY) in the 0% – GCR solution

$I_{\min}^{100\%}$ = the value of *min*(CMY) in the 100% – GCR solution.

Setting G_4 equal to zero in Eq. (7) forces *min*(CMY) to be equal to its value in the 0%-GCR solution. Likewise, setting G_4 equal to one yields a solution in which *min*(CMY) is equal to its value in the 100%-GCR solution. Arbitrary GCR levels can be obtained by setting G_4 to any value between zero and one. The corresponding optimization problem is

$$\begin{aligned} &\text{minimize the function} \quad f = \Delta E^*_{ab}(\text{CMYK}) \qquad (8)\\ &\text{subject to the constraint } g_1 = min(\text{CMY}) - I_{\min}^{0\%} + G_4 \bullet (I_{\min}^{0\%} - I_{\min}^{100\%}) = 0. \end{aligned}$$

In an analogous fashion, black may be specified to fall between its value in the 0%- and 100%-GCR solutions. Here, GCR is specified in terms of the parameter G_5,

$$G_5 = \frac{K - K^{0\%}}{K^{100\%} - K^{0\%}}. \tag{9}$$

Implementation

The preceding section presented a variety of methods for controlling the gray component. The Pareto-optimal problem given in Eq. (2), when expanded to include the various constraint equations for GCR control, unifies these procedures into a single methodology. The computer program NeuralColor is an implementation of this methodology. NeuralColor uses ANNs for the transfer functions f_L, f_a, and f_b, and solves optimization problems using the sequential quadratic programming routine NLPQL, developed by K. Schittkowski.[27] From a user's perspective, NeuralColor takes as input values for the objective-function parameters c_i, values specifying the constraint equations, and a CIELAB TIFF file. A converted CMYK TIFF file is returned as output, along with a log file containing conversion data, such as ink usage. Details regarding NeuralColor, including the optimization routine NLPQL and the ANNs f_L, f_a, and f_b, are given in Ref. 26.

The Pareto-optimal approach may be integrated into printing workflows in several ways. The most direct approach is to convert individual images into CMYK, as was done in this study using NeuralColor. In this case, each pixel is converted into CMYK individually, and consistency is enforced through carefully selected optimization parameters that do not allow for multiple CMYK solutions.[24,25] The Pareto-optimal method may also be integrated into an ICC-based workflow through creation of ICC profiles corresponding to distinct GCR schemes.[28] In the case of NeuralColor, LUTs for conversion into the printer color space can be generated by inputting an evenly spaced grid that spans the entire CIELAB space. LUTs for CMYK to CIELAB conversion can be created by direct application of the ANNs f_L, f_a, and f_b, which output L^*, a^*, and b^* values as a functions of CMYK. Unless file size is a major concern, highly dense LUTs are recommended to ensure only a negligible loss of accuracy in comparison to direct application of the Pareto-optimal approach. The creation of ICC profiles has the advantage of allowing for real-time color space conversion. Converting images directly using the Pareto-optimal approach is more time-consuming; image conversions using NeuralColor take on the order of ten minutes for for images containing fifty thousand pixels.[24,25] The Pareto-optimal formulations presented in the current work correspond to a colorimetric rendering intent, however the Pareto-optimal approach could be appended with additional methods for handling out-of-gamut colors that fulfill other rendering intents. Regardless of whether the Pareto-optimal approach is implemented directly or through the creation of LUTs, it has the advantage over many alternative methods of processing images with any number of conversion objectives based on a single print-device characterization.

Test Images

To evaluate the performance of NeuralColor, a CIELAB test image was designed that contains colors in both the CMY and CMYK gamuts. The image consists of 27 colored patches that span three lightness levels and a variety of hues. At lightness levels of both 20 and 80, nine patches are specified with a^* and b^* set equal to all combinations of –10, 0, and 10. At a lightness level of 50, nine patches are specified with a^* and b^* equal to all combinations of –20, 0, and 20. The selected test colors are all within the printer gamut and unsaturated, and hence all have a gray component. The majority of test colors with a lightness level of 20 required black ink for colorimetric match.

The test image was converted into CMYK using NeuralColor with each of the following intents:

- Maximum GCR by maximizing black ink, minimizing total ink, and minimizing chromatic ink with ΔE^*_{ab} constrained to zero;
- Minimum GCR by minimizing black ink, maximizing total ink, and maximizing chromatic ink with ΔE^*_{ab} constrained to zero;
- Pure chromatic-ink solution by minimizing ΔE^*_{ab} with black constrained to zero;
- Chromatic ink reduction of 25%, 50%, 75%, and 100% from the minimum-GCR solution;
- Black ink reduction of 25%, 50%, 75%, and 100% from the maximum-GCR solution;
- Constraints of .25, .50, and .75 placed on the ratio of black ink to black ink plus *min*(CMY);
- Fixing the black dot fraction between the minimum- and maximum-GCR solutions at levels of 0%, 25%, 50%, 75%, and 100%;
- Fixing the smallest chromatic dot fraction between the maximum- and minimum-GCR solutions at levels of 0%, 25%, 50%, 75%, and 100%.

For the purpose of qualitative evaluation, four pictorial images were converted with intents of maximum GCR, minimum GCR, and 50% GCR. Results for the first of these images are presented in the Results section. *Conversion results for the remaining images are presented as a* ***Supplemental Materials Appendix****, available at the IS&T website, (www.imaging.org).*

TABLE I. Average ΔE^*_{ab} values for maximum-GCR solutions.

Method	L=20	L=50	L=80	Average
max(K)	2.9	2.9	1.9	2.5
min(C+M+Y+K)	2.9	2.9	2.0	2.6
min(C+M+Y)	2.8	2.6	2.1	2.5

TABLE II. Average ink usage for maximum-GCR solutions.

Method	L=20	L=50	L=80	Average
max(K)	1.750	1.072	0.425	1.082
min(C+M+Y+K)	1.750	1.072	0.425	1.082
min(C+M+Y)	1.750	1.072	0.425	1.082

TABLE III. Average ΔE^*_{ab} values for minimum-GCR solutions.

Method	L=20	L=50	L=80	Average
min(K)	5.8	2.4	2.5	3.5
max(C+M+Y+K)	5.4	2.3	2.4	3.4
max(C+M+Y)	5.8	2.3	2.4	3.5
min(ΔE^*_{ab}),k=0	9.0	2.3	2.5	4.6

TABLE IV. Average ink usage for minimum-GCR solutions.

Method	L=20	L=50	L=80	Average
min(K)	2.844	1.641	0.563	1.683
max(C+M+Y+K)	2.848	1.641	0.563	1.684
max(C+M+Y)	2.846	1.641	0.563	1.683
min(ΔE^*_{ab}),k=0	2.747	1.640	0.563	1.650

Results

The program NeuralColor was used to solve for each of the rendering intents listed above. Test images were then printed and measured for colorimetric accuracy. Three copies of each image were printed, and the corresponding results were then averaged. Based on initial experiments, averaging over three prints corresponds to a 95% confidence interval of $\pm 1.09\Delta E_{ab}$. The following sections present results for the one-, two-, and three-step approaches for GCR control.

Evaluation of Maximum-GCR and Minimum-GCR Solutions

NeuralColor successfully converted the 27-color test image using each of the three schemes for obtaining maximum GCR: maximization of black ink, minimization of total ink, and minimization of chromatic ink, all with the constraint $\Delta E^*_{ab\ \max} = 0$. For every test color, either the dot fraction for black was found to be one, or the smallest dot fraction from among the chromatic inks was found to be zero. This is consistent with the definition of maximum GCR given earlier. All three schemes yielded nearly identical CMYK images. ΔE^*_{ab} data are presented in Table I for each of the maximum-GCR schemes. Average ΔE^*_{ab} values are given for the test points at L = 20, L = 50, and L = 80, in addition to the overall average ΔE^*_{ab}. Ink usage data for the maximum-GCR solutions are presented in Table II. Ink usage is presented as the sum of the CMYK dot fractions averaged across the 27 test colors. Results for each of the three lightness levels are also presented.

NeuralColor was also used to convert the 27-color test image for minimum GCR by solving Pareto-optimal problems for minimization of black ink, maximization of total ink, and maximization of chromatic ink with ΔE^*_{ab} constrained to zero. A solution was also found for minimization of ΔE^*_{ab} with zero allowable black ink. ΔE^*_{ab} and ink-usage results for these conversion schemes are presented in Tables III and IV.

The solutions for minimum GCR were somewhat less consistent and less accurate than the solutions for maximum GCR. When compared to the solutions for maximum GCR, it is evident that the increase in error occurred most predominantly in the patches at the L = 20 lightness level. As is described in the **Discussion and Conclusions** section, this outcome is a function of the ANNs used by NeuralColor, and not a result of the Pareto-optimal formulation itself. The conversions obtained by minimizing black ink and maximizing total ink with ΔE^*_{ab} constrained to zero failed to find a minimum-GCR solution for one or more colors. The solution for minimization of black ink with zero allowable ΔE^*_{ab} yielded a solution for CIELAB = (20,–10,10) in which neither a chromatic dot fraction was one nor the black dot fraction zero, which contradicts the definition of minimum GCR. In the solution for this color, however, the cyan dot fraction was merely one ink level off from a dot fraction of one*. The image converted by maximization of total ink failed more dramatically in finding a minimum-GCR solution. Three color patches, all with a lightness value of L = 20, were converted such that neither a chromatic dot fraction was one nor the black dot fraction zero. In each case, the solutions differed from a minimum-GCR solution by a number of ink levels. The conversion based on minimization of ΔE^*_{ab} with zero allowable black ink departed significantly from the first three solutions for minimum GCR. The solution for minimization of ΔE^*_{ab} with zero allowable black ink did succeed in finding a CMYK solution with a black dot fraction of zero for every patch, and in that sense achieved a minimum-GCR solution. Average colorimetric error rose to 9.0 ΔE^*_{ab} for color patches at the lightness level of L = 20, however, which brought the overall average ΔE^*_{ab} up to approximately 4.6 ΔE^*_{ab}.

Evaluation of Two-Step Methods

Maximum- and minimum-GCR solutions were modified by reducing black from the maximum GCR solution, and by reducing the chromatic inks from the minimum-GCR solution. The initial 100%- and 0%-GCR solutions were obtained by minimizing chromatic ink with an allowable ΔE^*_{ab} of zero and by maximizing chromatic ink with an allowable ΔE^*_{ab} of zero, respectively.

In the case of reducing *min*(CMY) from the minimum-GCR solution, secondary solutions were found for G_1 = 0, 0.25, 0.5, 0.75, and 1, where setting G_1 = 0 yields the initial minimum-GCR solution. Figure 3 presents ink usage as *min*(CMY) is reduced from its full value in the 0%-GCR solution to a value of zero. Table V presents the corresponding average ΔE^*_{ab} results. The solutions in this case are bounded by the 0%-GCR solution and a solution in which every color is produced using only two chromatic inks plus black.

Black ink was reduced from the 100%-GCR solution through specification of the parameter G_2. Note that setting G_2 = 1 yields the initial maximum-GCR solution,

* Images were stored in a TIFF format using 8 bits per channel, allowing for 256 distinct levels per ink.

TABLE V. Average ΔE^*_{ab} values for reduction of min(CMY) from the 0%-GCR solution.

G_1	L=20	L=50	L=80	Average
0.00	5.8	2.3	2.4	3.5
0.25	4.0	2.7	2.5	3.1
0.50	3.2	3.3	2.9	3.1
0.75	2.8	2.6	2.7	2.7
1.00	3.4	2.6	2.8	2.9

TABLE VI. Average ΔE^*_{ab} values for reduction of black from the 100%-GCR solution.

G_2	L=20	L=50	L=80	Average
0.00	9.9	2.2	2.6	4.9
0.25	7.3	2.7	2.4	4.1
0.50	4.2	3.1	2.6	3.3
0.75	3.1	2.8	2.9	2.9
1.00	2.8	2.6	2.1	2.5

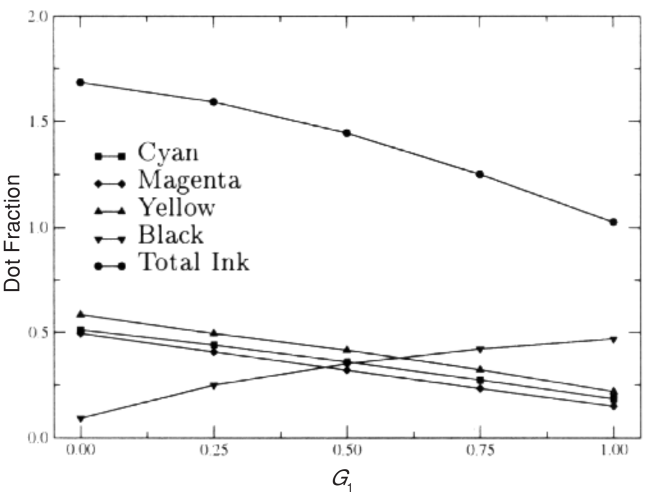


Figure 3. Average ink usage for reduction of min(CMY) from the 0%-GCR solution.

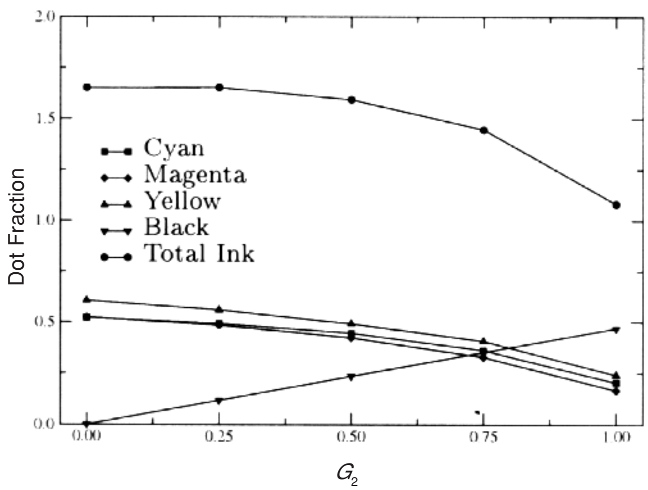


Figure 4. Average ink usage for reduction of black from the 100%-GCR solution.

and setting G_2 = 0 yields the solution that uses only chromatic inks. Figure 4 shows the resulting ink usage when black is reduced over the range of possible G_2 values. Table VI presents the colorimetric errors which resulted from the various values assigned to G_2.

The final two step method is the placement of a constraint on the ratio of the black dot fraction to the sum of the smallest chromatic dot fraction and the black dot fraction. The parameter G_3 controls this ratio.

Figure 5 presents ink usage for G_3 values of 0, 0.25, 0.5, 0.75, and 1. The measured colorimetric errors for these prints are given in Table VII.

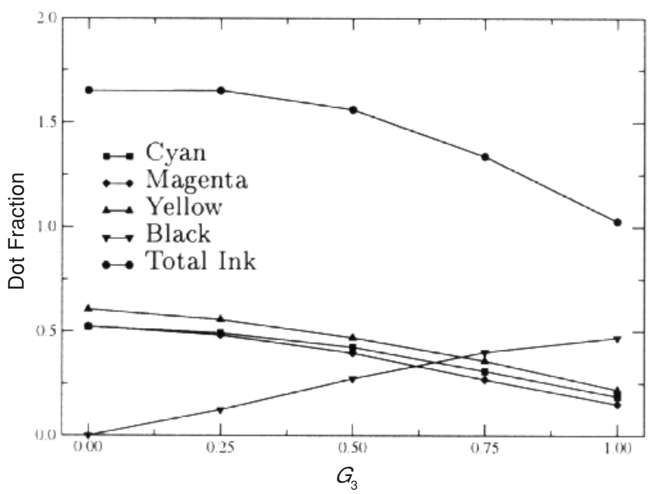


Figure 5. Average ink usage for Specification of a ratio between black and the sum of black and *min*(CMY).

Evaluation of Methods for Bounding Solutions with the 0%-GCR and 100%-GCR Solutions

The methods for bounding solutions with the 0%- and 100%-GCR solutions were implemented using the 0%-GCR solution obtained by maximizing chromatic ink with zero allowable ΔE^*_{ab}, and the 100%-GCR solution obtained by minimizing chromatic ink with zero allowable ΔE^*_{ab}. Arbitrary GCR was specified both in terms of reducing black ink from the maximum-GCR solution and in terms of reducing *min*(CMY) from the minimum-GCR solution. The 27-color test image was converted using both of these schemes for GCR levels of 0%, 25%, 50%, 75%, and 100%.

Results presented in this section were not acquired simultaneously with the results in the preceding sections. Due to printer instability, ΔE^*_{ab} data from this section should not be compared directly to that of the previous sections. The solutions for maximum GCR, for instance, yielded exactly the same CMYK values as the solutions for maximum GCR presented earlier, but the prints of these images differed by approximately one ΔE^*_{ab} unit. This small shift in printer characteristics is consistent with fluctuations previously recorded for the Tektronix®Phaser®740 used in this study.

Solutions bounded by the 0%- and 100%-GCR solutions were specified with the constraint-equation parameters G_4 and G_5. G_4 specifies the size of the smallest chromatic dot fraction relative to its size in the 0%- and 100%-GCR solutions. Ink usage as a function of G_4 is presented in Fig. 6. The corresponding ΔE^*_{ab} data are presented in Table VIII. Data resulting from the specification of G_5 are given in Fig. 7 and Table IX. The parameter G_5 is analogous to G_4, but differs in the fact that it specifies a black dot fraction as opposed to a chromatic dot fraction.

TABLE VII. Average ΔE^*_{ab} values for Specification of a ratio between black and the sum of black and min(CMY).

G_3	L=20	L=50	L=80	Average
0.00	9.1	2.3	2.9	4.7
0.25	7.1	2.9	2.2	4.1
0.50	3.4	3.6	2.6	3.2
0.75	3.3	2.6	2.5	2.8
1.00	3.4	2.4	2.5	2.8
0.00	0.25	0.50	0.75	1.00

TABLE VIII. Average ΔE^*_{ab} values for constraining min(CMY) between the 0%- and 100%-GCR solutions.

G_4	L=20	L=50	L=80	Average
0.00	8.1	2.7	3.7	4.8
0.25	4.8	3.0	3.3	3.7
0.50	3.9	3.5	3.3	3.5
0.75	3.4	3.3	3.8	3.5
1.00	3.7	3.4	3.8	3.6
0.00	0.25	0.50	0.75	1.00

TABLE IX. Average ΔE^*_{ab} values for constraining K between the 0%- and 100%-GCR solutions.

G_5	L=20	L=50	L=80	Average
0.00	8.1	2.7	3.7	4.8
0.25	5.4	2.7	3.1	3.7
0.50	4.9	3.3	3.6	3.9
0.75	3.6	3.3	3.9	3.6
1.00	3.7	3.4	3.8	3.6

TABLE X. Ink usage for conversions of the statue image.

Percent GCR	C	M	Y	K	C+M+Y	Total Ink
0	0.78	0.48	0.57	0.41	1.83	2.24
50	0.75	0.41	0.54	0.52	1.70	2.22
100	0.69	0.28	0.47	0.63	1.44	2.07

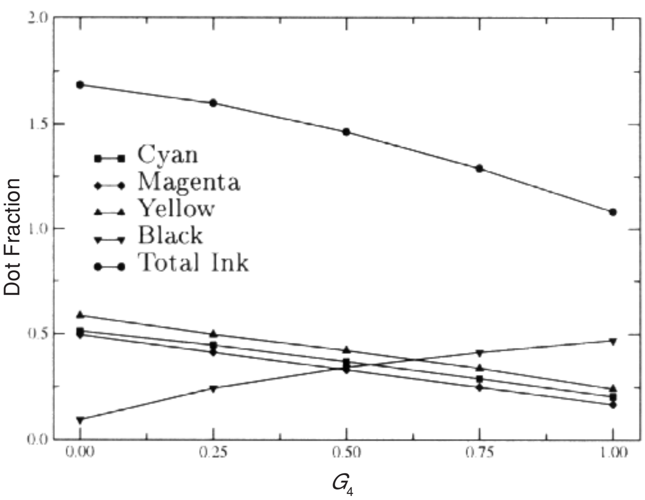


Figure 6. Average ink usage for constraining min(CMY) between the 0%- and 100%-GCR solutions.

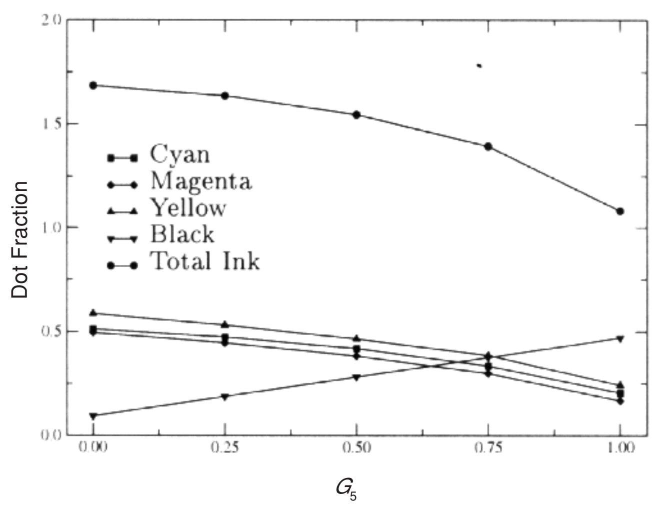


Figure 7. Average ink usage for constraining *K* between the 0%- and 100%-GCR solutions.

Evaluation of Pictorial Image Conversions

Four pictorial images were converted with the objectives of 0%, 50%, and 100% GCR. The 0%-GCR solutions were obtained by maximizing chromatic ink with ΔE^*_{ab} constrained to zero, and the 100%-GCR solutions were obtained by minimizing chromatic ink with ΔE^*_{ab} constrained to zero. The 50%-GCR solutions were obtained by fixing the black dot fraction to be halfway between its values in the 0%- and 100%-GCR solutions. **Color Plate 3 (pp. 554–556)** contains prints obtained by converting one of the four images (the statue image). These prints provide an illustration of the effects of GCR on image reproduction. Ink usage statistics for the statue image are given in Table X. The difference in chromatic ink deposition between the 0%-GCR and 100%-GCR prints is 21.4%. Overall ink deposition differs by 8.2%.

Discussion and Conclusions

The successful conversion of CIELAB images into the CMYK color space validates the Pareto-optimal formulation of color management as a means to control the gray component. The resulting data allow a number of conclusions to be drawn about the behavior of Pareto-optimal solutions, and of the program NeuralColor in particular.

The solutions for 100%-GCR were the most accurate solution methods overall, with a reproduction error of approximately 2.5 ΔE^*_{ab} for the 27 color test image. Substantially less ink was required for the 100%-GCR solution than was required for the 0%-GCR solution, as would be expected; reduction in ink usage is one of the primary advantages of GCR. The sum of the cyan, magenta, yellow, and black dot fractions for conversion of the 27-color test image with the objective of 0%-GCR was approximately 55% larger than ink usage for the 100%-GCR solutions.

Overall colorimetric error across the 27 color test image was slightly greater for the minimum-GCR solutions than for the maximum-GCR solutions, particularly in the low-lightness region. At a lightness level of L = 50, however, the minimum-GCR solutions gave slightly smaller errors. These inconsistencies are a result of the ANNs used to model the CMYK to CIELAB relationship, and are not an inherent characteristic of the Pareto-optimal formulations. It is difficult to determine the exact cause of these discrepancies within the scope of this study. It is speculated that peculiarities of the ANN training set may have allowed the program NeuralColor to be more accurate at certain levels of GCR in different regions of the printer gamut. The nonlinearity of color printing is also likely be a function of GCR and location in the printer gamut, making accurate color prediction more difficult in certain gamut regions. These are complex issues which require study in their own right in order to be resolved.

The schemes for modifying either the 0%- or 100%-GCR solutions succeeded in reducing the black dot fraction from the maximum-GCR solution, and in reducing the minimum chromatic dot fraction from the minimum-GCR solution. In these cases, the solution behavior was consistent with GCR methodology: reductions in chromatic ink usage were offset with an increase in black ink usage, and reductions in black ink usage resulted in increased use of chromatic ink. The two step methods all suffered from a similar shortcoming, however. As a substantial fraction of the black ink was removed from the 100%-GCR solution, the gamut of attainable color became limited. This yielded significant errors, as was the case when black was simply constrained to zero. Extreme reduction of the minimum chromatic ink from the 0%-GCR solution also leads to a restricted gamut, but in this case the gamut is restricted to all colors producible by two chromatic inks and black. This restriction is much less severe than restriction to the CMY gamut, and for this reason the approach of reducing the minimum chromatic ink from the 0%-GCR solution is preferable to the technique of reducing black from the 100%-GCR solution. The approach of fixing the ratio between the black dot fraction and the sum of the smallest chromatic dot fraction and the black dot fraction restricts the solution at both ends of the solution set. However, this approach does have the advantage that neither the 0%-GCR or 100%-GCR solutions is required; identification of the smallest chromatic dot fraction is all that is necessary.

The solution methods in which the 0%- and 100%-GCR solutions are used as bounds represent the best possible control over GCR. These schemes allow for true specification of arbitrary GCR without sacrificing colorimetric accuracy.

The major strengths of Pareto-optimal formulations for GCR control and of the program NeuralColor can be summarized as follows:

- Direct conversion into the CMYK color space is achieved over the full range of GCR;
- Solutions for arbitrary GCR can be bounded by the minimum- and maximum-GCR solutions. This allows for image reproduction with the guarantee that the full CMYK gamut is available under any specified level of GCR;
- Conversion for maximum GCR is achieved by maximization of black ink, minimization of total ink, or minimization of chromatic ink with ΔE^*_{ab} constrained to zero;
- Conversion for minimum GCR is achieved by minimization of black ink, maximization of total ink, or maximization of chromatic ink with ΔE^*_{ab} constrained to zero;
- Systematic reduction of chromatic ink from the minimum-GCR solution forces the substitution of black ink for the reduced chromatic inks. In the extreme case of total removal of the smallest chromatic dot fraction, the solution is obtained in which every color is reproduced by at most two primaries plus black;
- Black ink can be reduced from the maximum-GCR solution, which forces increased deposition of chromatic ink. Black ink can be removed from the reproduction entirely, in which case the image is reproduced using only the chromatic inks.

The Pareto-optimal approach for controlling GCR generalizes multiple GCR-control schemes into a single methodology. The freedom provided by the Pareto-optimal approach to color management offers powerful flexibility in the field of image reproduction.

Acknowledgment. The authors are thankful for support from the National Science Foundation under grant ECS 9734349.

References

1. H. R. Kang, Printer-related color processing techniques, *Proc. SPIE* **2413**, 410–419 (1995).
2. M. C. Stone, W. B. Cowan and J. C. Beatty, Color gamut mapping and the printing of digital color images, *ACM Transactions on Graphics* **7**(4), 249–292 (1988).
3. J. M. Kasson, Tetrahedral interpolation algorithm accuracy, *Proc. SPIE* **2170**, 24–35 (1994).
4. J. M. Kasson, S. I. Nin, W. Plouffe, and J. L. Hafner, Performing color space conversions with three-dimensional linear interpolation, *J. Electronic Imaging* **4**(3),226–250 (1995).
5. H. R. Kang, Applications of color mixing models to electronic printing, *J. Electronic Imaging* **3**(3), 276–287 (1994).
6. G. Vachon, Modeling the mixing behavior of inks with polynomials, *Color Res. Appl.* **13**(1), 46–49 (1988).
7. M. C. Mongeon, Image transformation into device dependent color printer description using 4th-order polynomial regression and object oriented programming development of image processing modules, *Proc. SPIE* **2658**, SPIE Press, Billingham, WA, 341–352 (1996).
8. Y. Azuma and M. Kaji, Evaluation of an algebraic technique for colorimetric calibration of a printing system, *J. Imaging Sci. Technol.* **45**(2), 93–99 (2001).
9. H. R. Kang and P. G. Anderson, Neural network applications to the color scanner and printer calibrations, *J. Electronic Imaging* **1**(2), 125–135 (1992).
10. G. Marcu and K. Iwata, RGB-YMCK color conversion by application of the neural networks, *Color Imaging Conference: Transforms and Trasportability of Color*, IS&T, Springfield, VA, 1993, pp. 27–32.
11. S. Abe and G. Marcu, A neural network approach for RGB to YMCK color conversion, *IEEE Region 10's Ninth Annual International Conference*, IEEE Press, Los Alamitos, CA 1994, pp. 6–9.
12. Y. Arai, Y. Nakano and T. Iga, A method of transformation from CIE *L*a*b** to CMY value by a three-layered neural network, *IS&T/SID Color Imaging Conference: Transforms & Transportability of Color*, IS&T, Springfield, VA, 1993, pp. 41–44.
13. S. Tominaga, Color control using neural networks and its application, *Proc. SPIE* **2658**, SPIE Press, Bellingham, WA, 253–260 (1996).
14. J. A. C. Yule, *Principles of Color Reproduction,* John Wiley and Sons, New York, NY, 1967, pp. 282–304.
15. J. W. Birkenshaw, M. Scott-Taggart and K. T. Tritton, The black printer, *TAGA Proc.,* 403–429 (1986).
16. C. Nakamura and K. Sayanagi, Gray component replacement by the neugebauer equations, *Proc. SPIE* **1184,** 50–63 (1989).
17. R. S. Gentile, E. Walowit and J. P. Allebach, A comparison of techniques for color gamut mismatch compensation, *J. Imaging Technol.* **16**(5), 176–181 (1990).
18. J. Morovic and M. R. Lou, The fundamentals of gamut mapping: A survey, *J. Imaging Sci. Technol.* **45**(3), 283–290 (2001).
19. P. Hung, Smooth colorimetric calibration technique utilizing the entire color gamut of CMYK printers, *J. Electronic Imag.* **3**(4), 415–424 (1994).
20. M. Tsukada and J. Tajima, New algorithm for UCR using direct color mapping, *Proc. SPIE* **2413,** 365–374 (1995).
21. H. Ogatsu, K. Murai, and S. Kita, A flexible GCR based on CIE *L*a*b**, *Proc. SPIE* **2414**, 123–133 (1995).
22. R. Balasubramanian and R. Eschbach, Reducing multi-separation color moire by a variable under-color removal and gray component replacement strategy, *J. Imaging Sci. Technol.* **45**(2),152–165 (2001).

23. P. A. Drakopoulos, *Color printer characterization using optimization theory and artificial neural networks*, MS Thesis, University of Colorado Department of Mechanical Engineering, Boulder, Colorado, 1997.
24. D. Littlewood, P. Drakopoulos and G. Subbarayan, Pareto-optimal formulations for cost versus colorimetric accuracy trade-offs in printer color management, December 1999, Technical Report no. 1999-01, Department of Mechanical Engineering, University of Colorado, Boulder, 427 UCB, Boulder, CO, 80309.
25. D. Littlewood, P. Drakopoulos and G. Subbarayan, Pareto-optimal formulations for cost versus colorimetric accuracy trade-offs in printer color management, *ACM Transactions on Graphics* 21(2) 132–174 (2002).
26. D. Littlewood, Printer Color Management Using Pareto-Optimization and Efficient Calibration Techniques, PhD Thesis, University of Colorado Department of Mechanical Engineering, Boulder, Colorado, 2001.
27. K. Schittkowski, NLPQL: A FORTRAN subroutine solving constrained nonlinear programming problems, *Annals of Operations Research* **5**, 485–500 (1985/6).
28. ICC, Specification ICC, 1:1998-09 file format for color profiles, http://www.color.org/ 1998.

(a) (b)

Color Plate 3. Prints of statue image with 0%, 50%, and 100% GCR. *(Littlewood and Subbarayan, pp. 533–542).*

(c)

(d)

Color Plate 3. (con't) Prints of statue image with 0%, 50%, and 100% GCR. *(Littlewood and Subbarayan, pp. 533–542).*

(e)

Color Plate 3. (con't) Prints of statue image with 0%, 50%, and 100% GCR. *(Littlewood and Subbarayan, pp. 533–542).*