# Estimation of Pigment Distribution from Spectral Transmittance of an Optical Micrograph

**Masaru Takeya**
*National Institute of Agrobiological Sciences, Tsukuba-city, Japan*

**Chawan Koopipat*, Hideki Sato*, Norimichi Tsumura▲*, Hideaki Haneishi▲*, and Yoichi Miyake✦***
** Graduate School of Science and Technology, Chiba University, Chiba-city, Japan*

Relative amounts of pigments on an overhead projector (OHP) sheet printed by laser printer can be estimated by using spectral transmittance at each pixel of an optical micrograph, and the multi-layered pigments also can be separated into each component. The transmittance spectra were estimated from multiband images by the Wiener estimation method. In the experiment, relative amounts of pigments printed by one or two kinds of color pigments were estimated, and overlapping magenta and yellow pigments were separated into each category. The result was accurate as compared with separation based on RGB values.

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## Introduction

Color of a printed image is usually predicted by the Neugebauer model, however this model is not accurate insofar as it assumes that each ink layer has the same thickness. Printed dots are randomly distributed, and optical dot gain is not considered by the model. In a real printed image, the thickness of ink is not equal at each point of the image. We can observe from an optical micrograph that the optical density is not uniform in the dot. In addition, many printing techniques do not use a random dot halftoning algorithm. The other problem is that when we print four color inks, the spread of one ink on the another ink layer is different from printing on bare paper because the upper ink layer is influenced by the unevenness on the lower ink layer.[1,2] The color prediction will be more accurate if we know the two-dimensional spatial distribution of ink in each layer of the print.

From the Neugebauer model, the color is predicted by using density or a colorimetric value; however when high image quality is concerned, the spectral reflectance is usually considered. Broadband reflectance measurements are inappropriate for Neugebauer model of a color printer because the real inks do not have constant reflectance.[3] Many investigations have attempted to extend the Neugebauer model to the spectral domain such as for colorant estimation of printing,[4,5] compensation for optical interactions,[6] and art printing reproduction.[7–9]

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✦ IS&T Fellow

▲ IS&T Member

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In this article, we propose a technique to estimate the two–dimensional distribution of pigment for each printed color. The relative amounts of pigments on each layer are estimated from total spectral transmittance, obtained by multiband imaging. For this estimation, some measured transmittance spectra are used as reference spectra. The Wiener estimation method[10,11] is used for estimating the total spectral transmittance per pixel from the multiband image. Multiple linear regression analysis is applied to extract relative amounts of pigment from the estimated transmittance spectra. Two-dimensional distributions of relative amounts of pigments printed by one or two kinds of color pigments are estimated, and the result is compared with those based on RGB values.

## Model of Total Spectral Transmittance in Layered Pigments

In case of pigment on a transparency, the negative logarithm of spectral transmittance is directly proportional to the concentration and to the thickness of the absorbing pigment, i.e., Beer's law. The relation is described at position (x,y) and wavelength λ as follows.

$$-\log T(x,y;\lambda) = \varepsilon(\lambda)c(x,y)d(x,y), \tag{1}$$

where $T(x,y;\lambda)$, $\varepsilon(\lambda)$, $c(x,y)$, and $d(x,y)$ denote the spectral transmittance, the characteristic function of the pigment, the concentration, and the thickness, respectively. The product $c(x,y)d(x,y)$ is proportional to the amount of pigment at position (x,y).

Transmission spectra of the pigment are measured by using a microspectroscope at several random points in the dot. The measured transmittance spectra are averaged at each wavelength in order to reduce the error of measurement. We call this average transmittance the reference transmittance. We assume that the position with reference transmittance in the image is a refer-

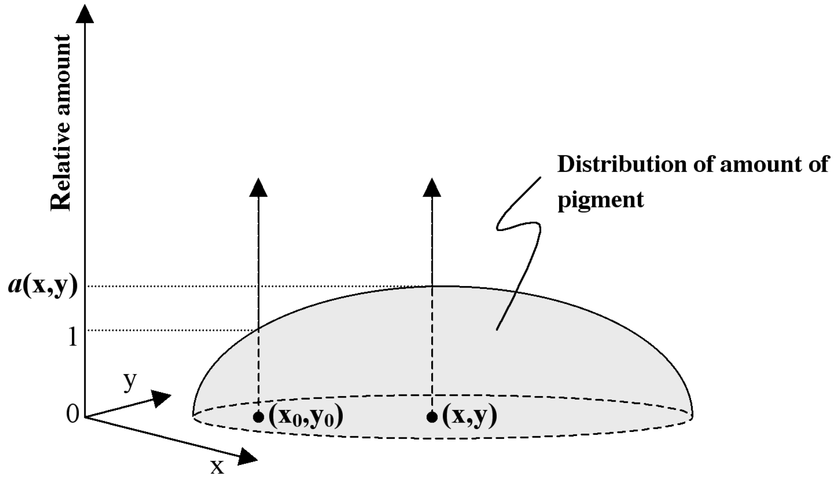



**Figure 1.** Schematic diagram of two–dimensional distribution of relative amount of pigment on transparency.

ence point ($x_0$,$y_0$) and that $c(x_0,y_0)d(x_0,y_0)$ is equal to unity. Equation (1) at a reference point becomes

$$-\log T(x_0,y_0;\lambda) = \varepsilon(\lambda). \tag{2}$$

Amount of pigment at an arbitrary position (x,y) relative to the reference point is expressed as $a$(x,y). Using the relative amount $a$(x,y), Eq. (1) becomes

$$-\log T(x,y;\lambda) = \varepsilon(\lambda)a(x,y). \tag{3}$$

Substituting Eq. (2) into Eq. (3), we can obtain following equation.

$$\log T(x,y;\lambda) = a(x,y)\log T(x_0,y_0;\lambda). \tag{4}$$

Figure 1 shows the schematic diagram of two–dimensional distribution of pigment on a transparency.

Equation (4) can be extended for the multi-layered pigment. When four kinds of color pigments, cyan, magenta, yellow, and black (C,M,Y,K) are layered at (x,y), the relationship between transmittance spectra and relative amounts is expressed as

$$\log T(\lambda) = a_c\log T_c(\lambda) + a_m\log T_m(\lambda) + a_y\log T_y(\lambda) + a_k\log T_k(\lambda), \tag{5}$$

where $T(\lambda)$ denotes total spectral transmittance at (x,y). $T_c(\lambda)$, $T_m(\lambda)$, $T_y(\lambda)$, and $T_k(\lambda)$ denote transmittance spectra at reference point ($x_0$,$y_0$) of C,M,Y,K, respectively. $a_c$, $a_m$, $a_y$, and $a_k$ denote relative amounts at (x,y) of C,M,Y,K, respectively. The coordinate ($x_0$,$y_0$) and (x,y) in the equation are not written for simplicity. Estimation method for relative amounts of pigments is shown in the next section.

## Linear Operation to Estimate Relative Amount of Pigment

Considering the measurement error with respect to sampling wavelengths, Eq. (5) becomes Eq. (6), (see below) where $\lambda_i$, $i$=1,2,…, $N$, is sampling wavelength, and $e_i$ . $i$=1,2,…, $N$, denotes the error.

Relative amounts $a_c$, $a_m$, $a_y$, and $a_k$ are determined in order to minimize a sum of squares error at each wavelength. The sum is represented as $G$, shown in Eq. (7). Taking the partial derivative with respect to $a_c$, $a_m$, $a_y$, $a_k$ and setting the result equal to zero we obtain the minimum of $G$. For example, the calculation with respect to $a_c$ is shown in Eq. (8).

$$\left.\begin{array}{r}\log T(\lambda_1) = a_c\log T_c(\lambda_1) + a_m\log T_m(\lambda_1) + a_y\log T_y(\lambda_1) + a_k\log T_k(\lambda_1) + e_1\\ \log T(\lambda_2) = a_c\log T_c(\lambda_2) + a_m\log T_m(\lambda_2) + a_y\log T_y(\lambda_2) + a_k\log T_k(\lambda_2) + e_2\\ \vdots\qquad\qquad\\ \log T(\lambda_N) = a_c\log T_c(\lambda_N) + a_m\log T_m(\lambda_N) + a_y\log T_y(\lambda_N) + a_k\log T_k(\lambda_N) + e_N\end{array}\right\} \tag{6}$$

$$\begin{aligned} G &= \sum_{i=1}^{N} e_i^2 \\ &= \sum_{i=1}^{N}\left(\log T(\lambda_i) - a_c\log T_c(\lambda_i) - a_m\log T_m(\lambda_i) - a_y\log T_y(\lambda_i) - a_k\log T_k(\lambda_i)\right)^2. \end{aligned} \tag{7}$$

$$2\sum_{i=1}^{N}\left(\log T(\lambda_i) - a_c\log T_c(\lambda_i) - a_m\log T_m(\lambda_i) - a_y\log T_y(\lambda_i) - a_k\log T_k(\lambda_i)\right)\log T_c(\lambda_i) = 0 \tag{8}$$

Using vector–matrix notation, the equation with $a_c$, $a_m$, $a_y$, $a_k$ can be expressed as

$$\mathbf{X}\boldsymbol{a} = \mathbf{Y} \tag{9}$$

$$\mathbf{X} = \begin{pmatrix} \sum_{i=1}^{N} \log T_c(\lambda_i)^2 & \sum_{i=1}^{N} \log T_c(\lambda_i)\log T_m(\lambda_i) & \sum_{i=1}^{N} \log T_c(\lambda_i)\log T_y(\lambda_i) & \sum_{i=1}^{N} \log T_c(\lambda_i)\log T_k(\lambda_i) \\ \sum_{i=1}^{N} \log T_c(\lambda_i)\log T_m(\lambda_i) & \sum_{i=1}^{N} \log T_m(\lambda_i)^2 & \sum_{i=1}^{N} \log T_m(\lambda_i)\log T_y(\lambda_i) & \sum_{i=1}^{N} \log T_m(\lambda_i)\log T_k(\lambda_i) \\ \sum_{i=1}^{N} \log T_c(\lambda_i)\log T_y(\lambda_i) & \sum_{i=1}^{N} \log T_m(\lambda_i)\log T_y(\lambda_i) & \sum_{i=1}^{N} \log T_y(\lambda_i)^2 & \sum_{i=1}^{N} \log T_y(\lambda_i)\log T_k(\lambda_i) \\ \sum_{i=1}^{N} \log T_c(\lambda_i)\log T_k(\lambda_i) & \sum_{i=1}^{N} \log T_m(\lambda_i)\log T_k(\lambda_i) & \sum_{i=1}^{N} \log T_y(\lambda_i)\log T_k(\lambda_i) & \sum_{i=1}^{N} \log T_k(\lambda_i)^2 \end{pmatrix} \tag{10}$$

from which we get Eqs. (10) through (12).

$$\boldsymbol{a} = \begin{pmatrix} a_c \\ a_m \\ a_y \\ a_k \end{pmatrix} \tag{11}$$

$$\mathbf{Y} = \begin{pmatrix} \sum_{i=1}^{N} \log T(\lambda_i)\log T_c(\lambda_i) \\ \sum_{i=1}^{N} \log T(\lambda_i)\log T_m(\lambda_i) \\ \sum_{i=1}^{N} \log T(\lambda_i)\log T_y(\lambda_i) \\ \sum_{i=1}^{N} \log T(\lambda_i)\log T_k(\lambda_i) \end{pmatrix} \tag{12}$$

Let define matrix $\mathbf{K}$ and vector $\boldsymbol{t}$ as follows,

$$\mathbf{K} = \begin{bmatrix} \log T_c(\lambda_1) & \log T_m(\lambda_1) & \log T_y(\lambda_1) & \log T_k(\lambda_1) \\ \log T_c(\lambda_2) & \log T_m(\lambda_2) & \log T_y(\lambda_2) & \log T_k(\lambda_2) \\ \vdots & \vdots & \vdots & \vdots \\ \log T_c(\lambda_N) & \log T_m(\lambda_N) & \log T_y(\lambda_N) & \log T_k(\lambda_N) \end{bmatrix} \tag{13}$$

$$\boldsymbol{t} = \begin{bmatrix} \log T(\lambda_1) \\ \log T(\lambda_2) \\ \vdots \\ \log T(\lambda_N) \end{bmatrix} \tag{14}$$

Using matrix $\mathbf{K}$ and vector $\boldsymbol{t}$, $\mathbf{X}$ and $\mathbf{Y}$ are represented as follows,

$$\mathbf{X} = \mathbf{K}^T\mathbf{K} \tag{15}$$

$$\mathbf{Y} = \mathbf{K}^T\boldsymbol{t}\ , \tag{16}$$

where $\mathbf{K}^T$ represents a transposed matrix for $\mathbf{K}$. Substituting Eqs. (15) and (16) into Eq. (9), we can obtain the following equation,

$$\mathbf{K}^T\mathbf{K}\boldsymbol{a} = \mathbf{K}^T\boldsymbol{t}\ . \tag{17}$$

If $\mathbf{K}^T\mathbf{K}$ is a nonsingular matrix, relative amount $\boldsymbol{a}$ is given by the following equation,

$$\boldsymbol{a} = [\mathbf{K}^T\mathbf{K}]^{-1}\mathbf{K}^T\boldsymbol{t}. \tag{18}$$

## Experiment

Color laser printer (Phaser 550J, Tektronix) and the C,M,Y,K pigment were used to prepare the sample for microscopy. A single printed dot of each pigment on an overhead projector (OHP) sheet and overlapping magenta and yellow pigments were cut about 1 cm × 1 cm for microscopy. The spectral transmittance at the reference point of each pigment is shown in Fig. 2. Transmittance spectra of the micrographs of the OHP sheets were estimated from the multiband image using a Wiener estimation based method.[10]

The matrix $\mathbf{K}$ was calculated from Eq. (13), and vector $\boldsymbol{t}$ was calculated from Eq. (14) using the estimated transmittance spectra of the micrograph. Therefore, relative amount, $\boldsymbol{a}$, of pigment was estimated from Eq. (18) using the $\mathbf{K}$ and $\boldsymbol{t}$ values thus obtained. Figures 3 and 4 show the results of the estimation of the relative amount of cyan and black pigment, respectively. Figures 3(a) and 4(a) show the images of pigment, which are reproduced as RGB values from spectral transmittance at each position. The size of pigment is 10 ~ 15 µm in diameter. Figures 3(b) and 4(b) represent the two–dimensional distribution of the amount of each pigment. In Figs. 3(b) and 4(b), the points on the axis for relative amount and the pixel coordinates correspond to the estimated relative amount of pigment and the position (x,y) on micrograph taken by the multiband CCD camera, respectively.

Figure 5 shows the reproduced image with magenta and yellow pigment as an example of a mixed pigment image. In Fig. 5, the upper left and the lower right areas in the image are printed with only magenta and yellow pigment, respectively, and the center area is layered with magenta and yellow pigment. Figure 6 shows monochromatic pigment images derived by Eq. (18) based on transmittance spectra, and Figure 7 shows

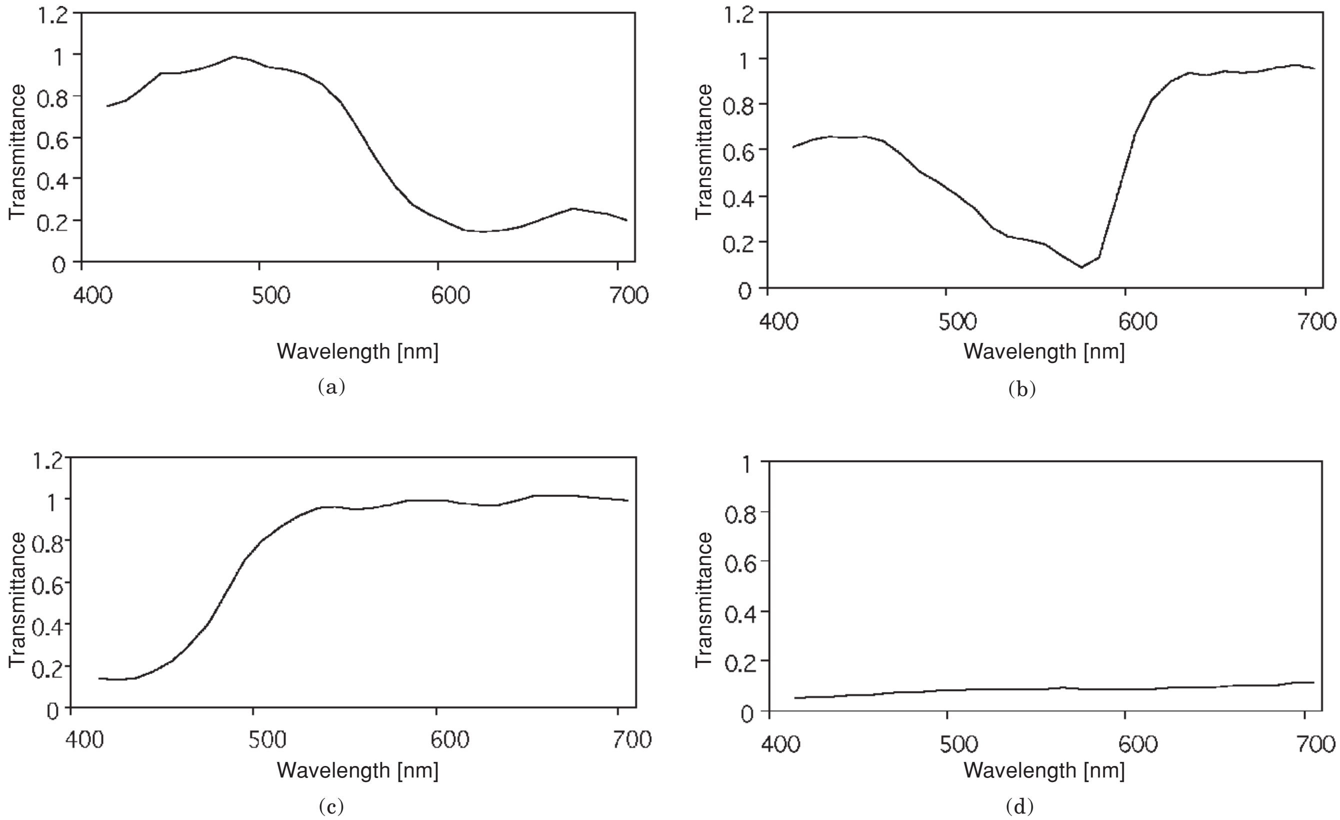



**Figure 2.** Spectral transmittance at reference point for each pigment; (a) cyan, (b) magenta, (c) yellow, (d) black.

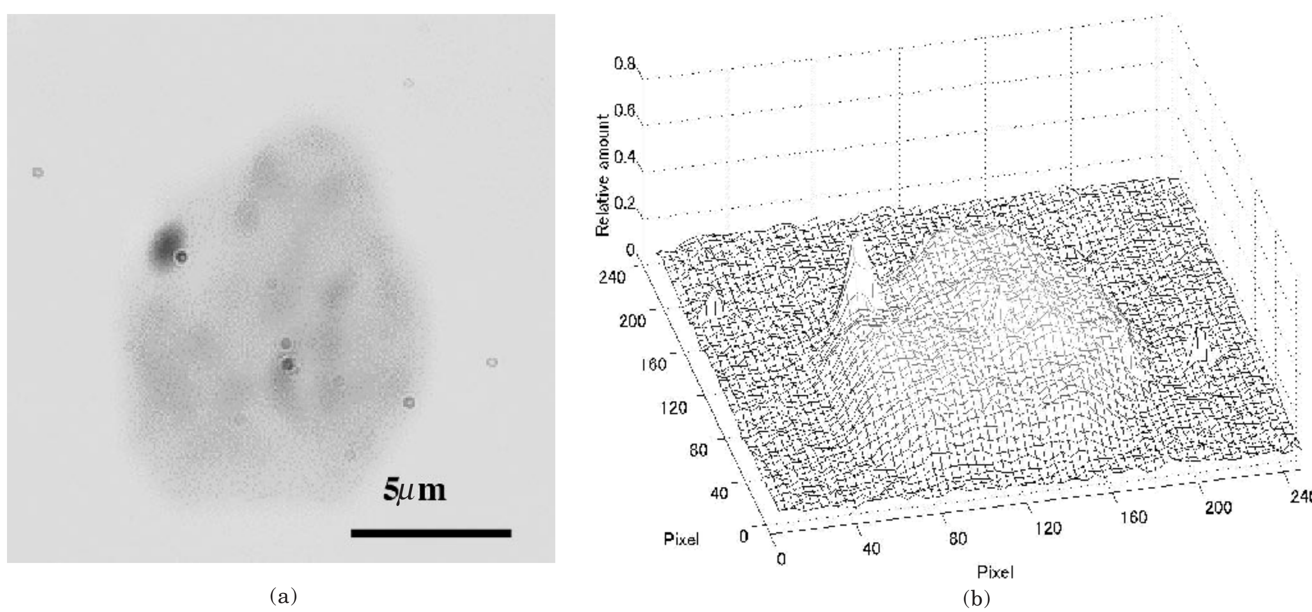
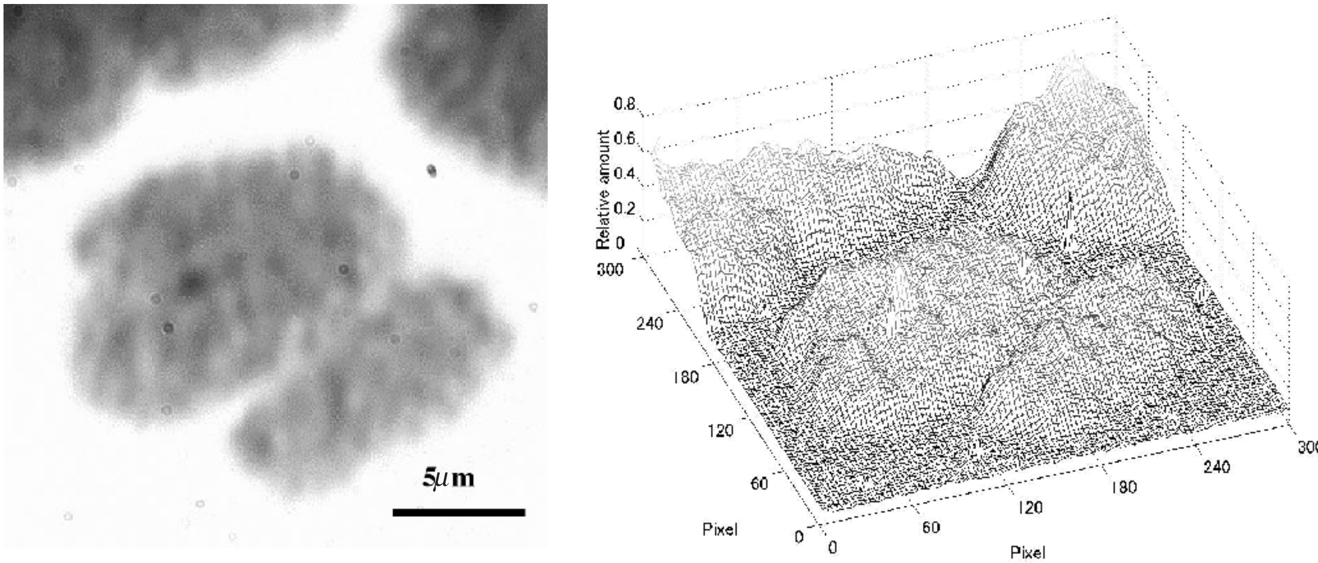



**Figure 3.** (a) Reproduced image of cyan pigment, (b) two–dimensional distribution of estimated relative amount of cyan pigment. *Supplemental Materials—Figure can be found in color on the IS&T website (www.imaging.org ) for a period of no less than two years from the date of publication.*




(a) (b)

**Figure 4.** (a) Reproduced image of black pigment, (b) two–dimensional distribution of estimated relative amount of black pigment. *Supplemental Materials—Figure can be found in color on the IS&T website (www.imaging.org) for a period of no less than two years from the date of publication.*

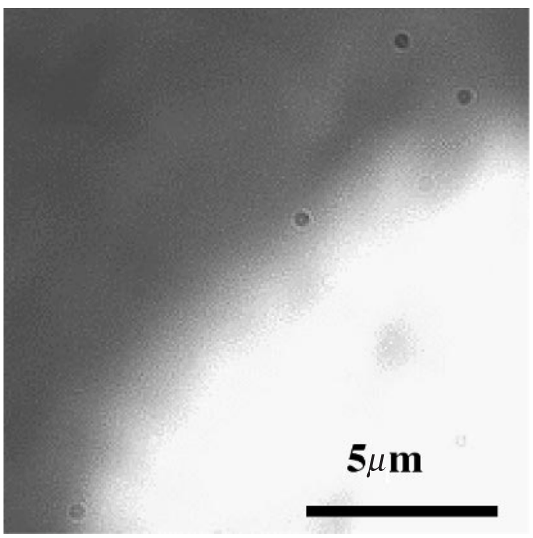
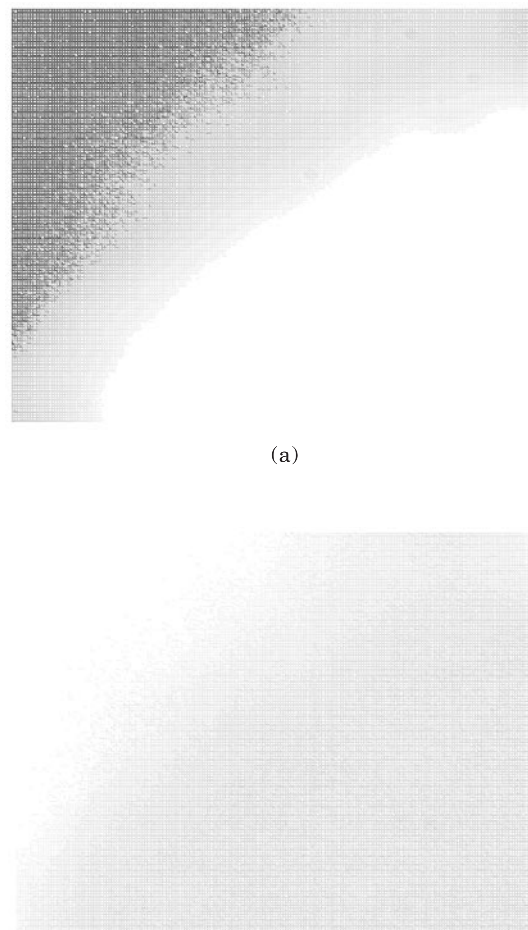



**Figure 5.** Reproduced image with magenta and yellow pigment. upper left area: printed only magenta pigment, down right area: printed only yellow pigment, center area: printed both pigments. *Supplemental Materials—Figure can be found in color on the IS&T website (www.imaging.org) for a period of no less than two years from the date of publication.*

the separated monochromatic pigment images based on RGB values. There is a difference between the images based on transmittance spectra and those based on RGB values.

## Conclusion and Discussion

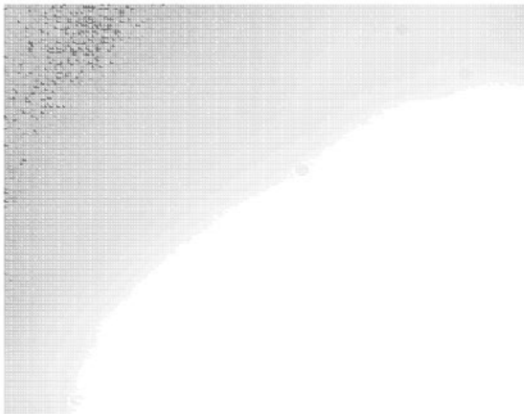
A method to estimate relative amount of printed pigment in optical micrograph has been proposed. Beer's law is applied to pigment printed on transparency substrate. The relative amounts of pigments at all pixels were calculated from the estimated spectral transmittance at each pixel and the measured spectral transmittance at reference point by a linear operation. The



(a)

(b)

**Figure 6.** Divided pigment images based on transmittance spectra, corresponding density to relative amount, (a) magenta, (b) yellow. *Supplemental Materials—Figure can be found in color on the IS&T website (www.imaging.org) for a period of no less than two years from the date of publication.*



(a)



(b)

**Figure 7.** Divided pigment images based on RGB values, corresponding density to relative amount, (a) magenta, (b) yellow pigment. *Supplemental Materials—Figure can be found in color on the IS&T website (www.imaging.org) for a period of no less than two years from the date of publication.*

method was applied to micrographs of transparencies with one or two kinds of pigments. Two–dimensional distributions of the estimated relative amounts were drawn. The image with overlapped pigments was separated into two component images.

Our technique can separate each layer image from a multilayered image, therefore it can be used in the printer model, because dot area and dot profile from multicolor printed image are obtained. This will give an accurate prediction of color reflectance by the spectral Neugebauer model.

In this article, the pigment printed on transparency was studied to evaluate the dynamic range of measurement. We intend to apply this method for estimating the relative amount of pigment from a four color printed image on paper.

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