Fractal Analysis of Printed Structures

O. Zmeskal, M. Nezadal and M. Buchnicek

Institute of Physical and Applied Chemistry, Faculty of Chemistry, Brno University of Technology, Brno, Czech Republic

M. Ceppan▲

Department of Graphic Arts Technology and Applied Photochemistry, Faculty of Chemical and Food Technology, Slovak University of Technology, Bratislava, Slovakia

This article describes the basic concept of fractal analysis with relation to printed structures and introduces "intensity fractal function" as a tool of practical factor analysis of printed images. The properties of the "intensity fractal function" are demonstrated on samples of printed images.

Journal of Imaging Science and Technology 46: 453–456 (2002)

Introduction

Fractals are objects of rough or fragmented geometric shape that can be subdivided into parts each of which is (at least approximately) a reduced copy of the whole.[1] Fractals are very popular nowadays, because it seems they can help us to describe various aspects of nature more precisely than classical Euclidean mathematics.[2] The main and often the only one characteristic of fractals is "fractal dimension". Classical objects of Euclidean geometry such as a point, a line, a plane, or a cube have dimension equal to 0 (point), 1 (line), 2 (plane) and 3 (cube). This dimension is called "topological dimension". Another definition of fractal is that their dimension strictly exceeds topological dimension. It means, that fractals are objects with non-integer dimension (This is just an approximate definition and cannot be generally applied to all types of fractals). For example a planar curve with dimension equal to 1.26 is fractal and called a Koch curve; it is an object between a line and a plane.[3] Sounds strange? But nature is strange and cannot be approximated merely by squares and lines; fractals seem to be more appropriate.

The concept of fractals was found useful in various fields of science and technology.[4] There are also some attempts to use this concept in problems in imaging and printing. Trauzeddel and co-workers[5] studied the fractal dimension of halftone dots printed by various types of aluminium printing plates on papers of several grades. They found that the peripheral structure of all halftone dots under investigation was of fractal nature and the fractal dimension drastically increases with decrease in paper quality. They found a linear relationship between signal-to-noise ratio and the fractal dimension of printed dots. Tosch and co-workers[6] introduced the concept of fractal geometry as a mean for the study of inking capabilities of printing plate surfaces. Roudet-Rouis[7] extended this approach by using a new method of calculation of fractal dimension of printing plates surfaces, developed by Tricot,[8] and showed that the fractal analysis of the surface could reduce considerably the enormous number of classical parameters usually necessary for the description of surface.

The aim of this article is to introduce of the concept of "intensity fractal spectrum" to the study of printed structures and to elucidate its basic properties in relation to the parameters of paper surface and printed image.

Experimental and Computation

One of the most popular methods of fractal dimension determination is the Box-counting method[9–11] Dimensionality determined by this method is called the Box Dimension D_B. This method has a simple principle: a square mesh of various sizes r is superimposed over the object (Fig. 1). The number of mesh boxes $N(1/r)$ that contain part of the image are counted. The slope of the linear portion of a $\ln[N(1/r)]$, where $\ln N(1/r) = \ln K_B + D_B \ln (1/r)$, gives the fractal (box) dimension D_B (Fig. 3). Three fractal dimensions (counting black squares N_B, white squares N_W and squares on the black and white interface N_{BW}) can be acquired from every image: D_B, D_W, and D_{BW}: D_B and D_W characterize fractal properties of printed and nonprinted areas respectively, while D_{BW} characterizes properties of the interface between printed and nonprinted areas. The fractal dimension and spectrum D_{BW} is most significant for analysis of printed structures and will be used further.

Samples of prints differ according to the technology of printing and grade of paper. Three images of each sample were analyzed: image of a nonprinted area, image of a solid printed area and image of a printed edge. Images were acquired by system which consists of an

Original manuscript received December 10, 2001

▲ IS&T Member

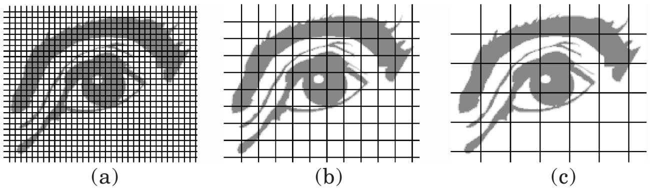
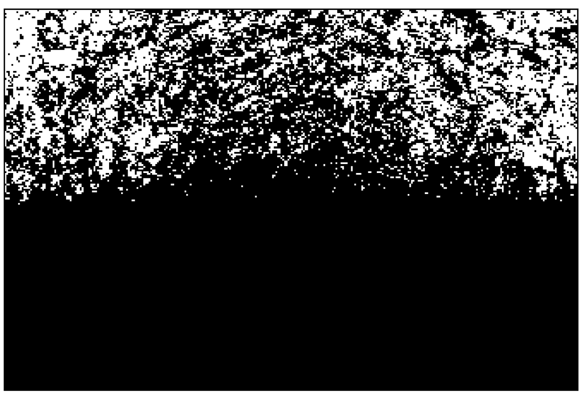


Figure 1. The demonstration of Box-Counting method, a) mesh size 5 pixels, 446 squares needed to cover eye; b) mesh size 15, 69 squares needed; c) mesh size 26, 28 squares needed, resulting fractal dimension is equal to $D_B = 1.28 \pm 0.02$ with coefficient of correlation $R = 0.9762$.

(a)



(b)

Figure 2. Fractal structures of thresholded image of print edge: a) all pixels with $G_L \le 48$ were transformed to black, remaining pixels became white; b) all pixels with $G_L \le 186$ were transformed to black, remaining pixels became white.

optical microscope (JANTECH, Carl Zeiss Jena, 100× enlargement) and a digital camera (Electrim™ EDC-1000C, output resolution 751×488 pixels). Color digital images (RGB) were preprocessed to correct for noise and possible spatial nonuniformities of illumination, and transformed into 256 gray level images, using

$$I = 0.299\ R + 0.587\ G + 0.114\ B, \tag{1}$$

where I is the calculated intensity of the gray shade, R is the value of red channel, G the value of green channel and B is the value of blue channel of original color picture for an individual pixel.[12]

In this study the results for the following images of samples are presented:

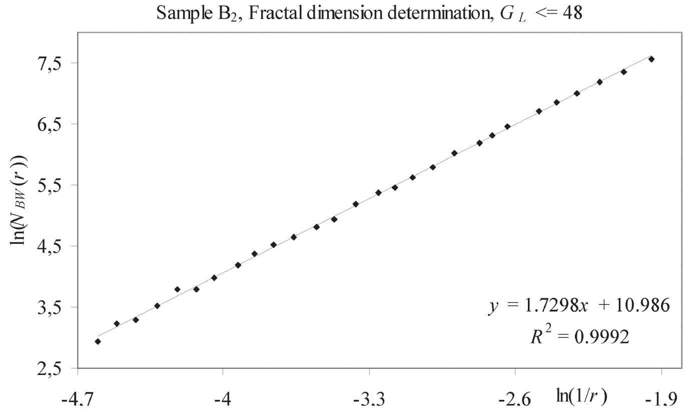


Figure 3. Determination of fractal dimension of fractal structure from Fig. 1a) $D_{BW} = 1{,}73 \pm 0.01$.

A_2	–	image of the edge of a razor blade
B_1, B_2, B_3	–	digital print technology, matte coated paper,
C_1, C_2, C_3	–	digital print technology, offset paper,
D_1, D_2, D_3	–	offset printing, matte coated paper,
E_1, E_2, E_3	–	offset printing, gloss coated paper,
F_1, F_2, F_3	–	offset printing, newsprint,
G_1, G_2, G_3	–	offset printing, offset paper.

Indexes refer to solid printed area (1), area of printed edge (2) and nonprinted area (3).

Every halftone image was transformed into 256 binary images (black-and-white) of fractal structure. Transformation was performed by sequential tresholding. Thresholding is a means for transformation of a gray level image into a binary black-and-white image. For certain gray level (G_L) darker shades were transformed into black and lighter shades were changed into white whereby the fractal structure arose (Fig. 2).

Varying G_L from 0 to 255, 256 binary images of fractal structure were obtained. Every one of this 256 images was subjected to fractal analysis and fractal dimension of included fractal structure was determined (Fig. 3). So, from each sample 3 × 256 = 768 images (751×488 pixels) were analyzed. This procedure took about one hour on a Pentium Celeron 350 MHz computer. Special software to perform this task was developed. It is called HarFA—Harmonic and Fractal image Analyser—and can be download free of charge from the site *http://www.fch.vutbr.cz/lectures/imagesci/harfa.htm*. Finally the values of fractal dimension were treated as a function of the thresholding condition (G_L). We have called this function "intensity fractal spectrum".

Fractal Analysis of Nonprinted Areas

Typical intensity fractal spectrum of a nonprinted area is presented on Fig. 4. The spectrum of ideal white paper would be formed merely by zero values, i.e., no fractal structure arises for any gray level G_L. The position of the maximum in the spectrum P_W corresponds to whiteness of paper, higher P_W, denotes higher whiteness. The halfwidth of the peak $W_{0.5}$ corresponds to the smoothness of the paper surface structure. Smoother papers create a more homogenous image structure with fewer gray levels which exhibit fractal structure ($D_{BW} > 0$). Results of the measurements are summarized in Table I.

TABLE I. The Results of Fractal Analysis of Nonprinted Areas

Sample	P_W (G_L)	D_{BWmax}	$W_{0.5}$ (G_L)
B_3	187	1.9345	55
C_3	187	1.9183	54
D_3	192	1.8620	34
E_3	189	1.8671	37
F_3	147	1.8664	88
G_3	182	1.9064	58

TABLE II. The Results of Fractal Analysis of Printed Areas

Sample	P_B (G_L)	D_{BWmax}	$W_{0.5}$ (G_L)
B_1	47	1.7429	50
C_1	62	1.8917	81
D_1	59	1.9098	39
E_1	57	1.7942	31
F_1	66	1.9108	60
G_1	67	1.9225	84

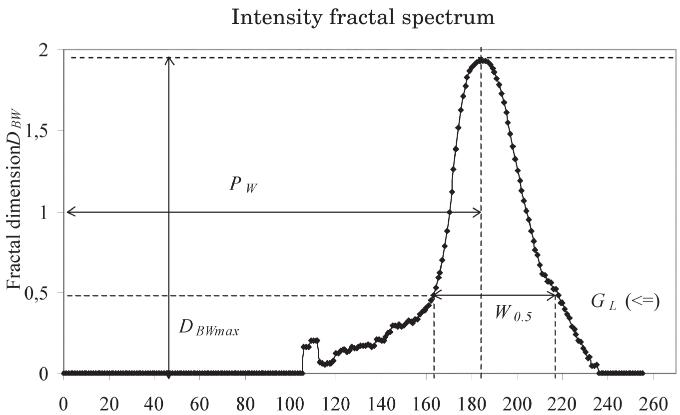


Figure 4. The typical intensity fractal spectrum of nonprinted paper.

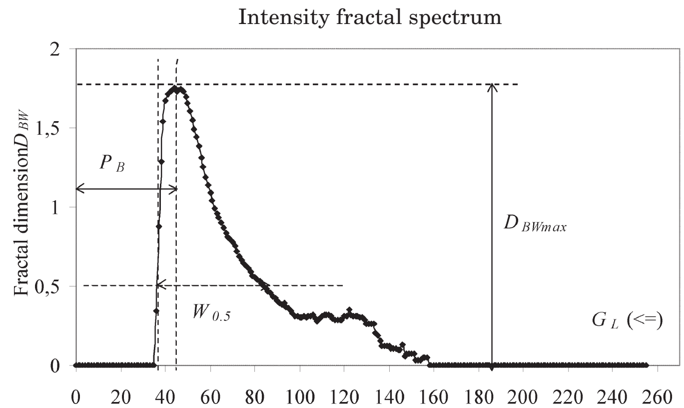


Figure 5. Typical fractal spectrum of printed paper.

The values obtained for the parameter P_W indicate that paper samples are of comparable whiteness; the most white is the sample D_3 (matte coated paper). The poorest sample with regard to whiteness is the sample of newspaper F_3. Threshold of fractal structure, G_L, of this sample is shifted about 40 units to the darker side in comparison with other samples. The values of halfwidth $W_{0.5}$ suggest that the more smooth papers are the gloss and matte coated papers printed by offset (E_3 and D_3). Offset paper of sample G_3 is in all parameters very similar to the sample C_3 (offset paper, digital print technology). We can claim that these papers are identical. On the other hand the two samples labeled as matte coated paper (B_3 and D_3) do not match. The highest halfwidth $W_{0.5}$ is comparable to that of the sample of newsprint (F_3).

Fractal Analysis of Printed Area

The fractal spectrum of an ideal solid printed area would be formed by zero values only, as there are no edges in this image. There is a maximum at the position P_B in the real spectrum of such image samples (Fig. 5). The following points can be deduced from these results. The position of the maximum P_B is the measure of dominant gray level. For the prints of darker shadows it is the position of maximum P_B shifted towards zero. The halfwidth of peak $W_{0.5}$ reflects coverage of the printing ink and the quality of printed paper (smoothness). A narrow peak indicates better ink coverage of the surface structure of paper and hence a lower number of gray levels G_L, which demonstrate the fractal dimension $D_{BW} > 0$. An ideal solid black printed area has only one shade of gray that $D_{BW} = 1$ and $G_L = 0$. Results of the measurements are summarized in Table II.

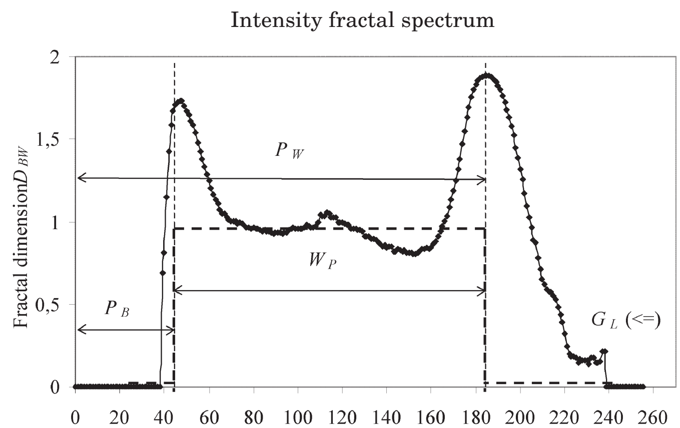


Figure 6. The typical fractal spectrum of printed edge.

Fractal Analysis of Printed Edge

Analyzing printed edge we could suppose a superposition of spectrum of solid printed (Fig. 5) and nonprinted areas (Fig. 4). In fact a spectrum such as shown in Fig. 6 is obtained. The superposition is satisfied in the range of peaks, but in the edge range (interspace W_P) a deviation occurs.

The ideal edge should have a fractal dimension equal to topological dimension of the line ($D = E = 1$). Values of fractal dimension less than 1 indicate discontinuity of the border structure. The spectrum of an ideal printed edge, printed on the same paper (maximum $G_L = 186$) and by the same printing ink (maximum $G_L = 48$) is plotted for comparison. Peaks in the spectrum correspond to the gradual filling of areas (printed or nonprinted paper) by thresholded pixels. The distance between peaks, $W_P = P_W - P_B$, where P_W and P_B are positions of

TABLE III. The Results of Fractal Analysis of Printed Edge

Sample	P_B	P_W	W_P
A_2	38	180	142
B_2	48	186	138
C_2	61	182	121
D_2	66	188	122
E_2	57	183	126
F_2	72	146	74
G_2	73	183	110

maximum of "black" and "white" peak, is relative to the contrast of printed and nonprinted areas. Results are summarized in Table III.

According to the parameter W_p the contrast of printed and nonprinted areas among the samples is quite similar. The only difference is the sample of newsprint F_2, with a significantly lower value of W_p.

Conclusion

We have demonstrated that fractal analysis of printed samples and paper provides complex information on the samples studied. In some regards this information similar to the analytical information obtained by means of conventional methods for evaluation of the quality of print, as surveyed elsewhere,[13] e.g., whiteness of paper. But fractal analysis provides new information wherein some ways is not comparable to results for conventional methods. The whole character of the fractal spectrum allows many nontraditional parameters to be derived, e.g., the smoothness of paper surface. The method is accurate enough to be used as an analytical tool to distinguish between grades of paper and the technology of print (Fig.7). It is up to the users of fractal analysis how they will exploit the terrific potential of this analytical tool.

Acknowledgment. This study was partly supported by Slovak Grant Agency VEGA.

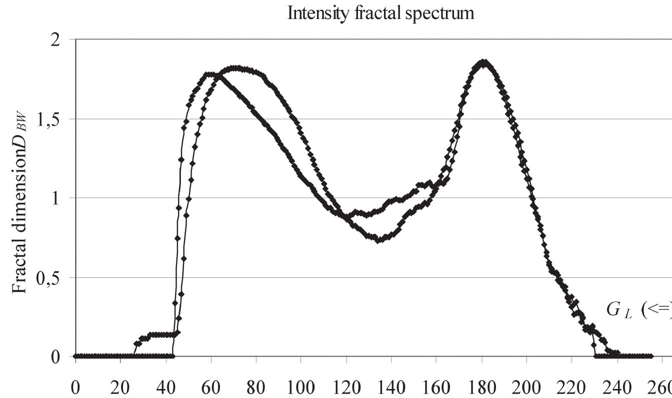


Figure 7. Comparison of fractal spectra of C2 sample (digital technology of print, offset paper) and G2 (offset, offset paper). Peaks in paper area ($G_L > 165$) are almost the same.

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