Feature Article

Digital Multitoning Using Gray Level Separation

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Digital halftoning has, traditionally, been thought of as a bilevel quantization algorithm that converts continuous tone images to images composed exclusively of "on" and "off" pixels. With recent advancements in printing technologies, ink jet and laser printers are now capable of producing dots with more than two intensity levels. These advancements have led to halftoning research on multitoning algorithms or halftoning with more than two levels. An early example of multitoning is the Floyd and Steinberg error diffusion algorithm with an N-level quantizer replacing the conventional binary quantizer. A major problem associated with this approach is the introduction of unwanted texture near the intermediate gray levels in the printed image. A possible solution to this problem is the redistribution of the intermediate gray and black pixels near the printable gray levels. In this article, a novel multitoning algorithm is introduced that can control the amount of printable gray level pixels, represents a particular shade of gray, by using gray level transformation. In this method, the input image is decomposed into the printable gray scale images, by using a set of gray level transformation curves, each channel is halftoned using conventional bilevel error diffusion algorithm, in a correlated way, and the halftoned channels are then recombined to get the final halftoned image. For certain gray level curve specifications, elimination of the undesirable banding artifacts has been achieved, near the intermediate gray levels, in the output. The suggested method is mean preserving and has a very little computational overhead involved as compared to the conventional error diffusion.

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Introduction

Halftoning is a method of producing the illusion of continuous tone images using only a finite number of printable gray levels. The illusion of intermediate shades is produced by using an area operation in which the dots of printable gray levels are arranged in some specified order. Due to the inherent characteristic of the human visual system to record average gray level over the area, the human observer thus perceives intermediate tones. Conventionally, halftoning is considered as a simple "on" and "off" modulation technique, where the sensation of intermediate tones is created by the presence and absence of a pixel. However, image quality studies demonstrate that a few intermediate levels, in addition to the black pixels, can provide a significant reduction in contouring, giving better picture quality.[1] This general case of halftoning where some intermediate shades of gray, in addition to the black pixels, are used to create the illusion of intermediate tones, is referred to as multilevel halftoning.

In digital printing, devices capable of printing multiple shades of gray are becoming increasingly common, with the rapid technological developments in this field. A few examples of such devices are, the sub-pixel laser pulse modulation in the xerographic systems and the use of multiple inks in ink jet printers. Multilevel ink jet printing can be achieved in many ways such as: (1) by using multiple ink jet heads with different ink concentrations (multiple ink printing), (2) by using two or more orifices of different sizes (variable dot size printing), (3) by laying multiple droplets on the same location (color layering technology) or by combining the above methods. The first method affects only the dye concentration, the second method affects the dot size, and the third method affects both the dye concentration and the dot size. Typically, these systems are capable of rendering only a small number of levels (≤ 16) and are still far from the contone appearance, possible with 256 levels and above. Therefore halftoning is still needed to avoid quantization problem such as contouring. The halftoning techniques, applied to multilevel systems, are generally referred to as multilevel halftoning or multitoning. Today, significant research efforts focus on developing appropriate multilevel halftoning algorithms for the multilevel printing devices.

In 1976, Floyd and Steinberg introduced the revolutionary error diffusion algorithm[2] to produce visually pleasing random distribution of dots, characterized by Ulichney[3] as blue noise patterns. Since the human visual system is less sensitive to high frequency content

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(blue noise), the resulting halftone patterns are less visible to the human observer. Conventional error diffusion, for binary devices, can easily be extended to the multilevel case by replacing the thresholding stage by a multilevel quantizer.[4] Katoh and co-workers[5] first applied error diffusion to multilevel printing devices. The images halftoned, using this conventional multilevel halftoning algorithm, exhibit much lower quantization error as compared to the bilevel case, however, they suffer with banding artifacts in the regions of intermediate printable gray levels. The reason is that at the intermediate gray level there is no quantization error or contouring as we have the ink for that tonal level available. In the neighborhood of this zero error region, there is a sparse distribution of the black and white pixels, in an otherwise constant gray region. This appears as a banding artifact to the human observer.

A number of algorithms have been proposed to eliminate the above problems with multitones. All these methods focus on equalizing the distribution of mean square error (MSE) between the halftoned and the original image by distributing the MSE uniformly over all the gray levels. Ochi[6] has addressed the same problem by iterating the error diffusion with a layered structure to remove the contouring at midtones. This improvement is at the expense of increasing the deviation in the uniform density areas. Sugiura and Makita[7] developed a new multitoning algorithm, based on error diffusion, by adding noise to increase the deviation around the density, printed by the ink of intermediate gray level.

Recently, we have seen a newer and expanding role of stochastic screens in ink jet printing because of the implementation simplicity.[8] An input image value is thresholded by a corresponding screen value to turn the output pixel on or off. For multilevel ink jet printers, the method can be generalized,[9] by scaling the screen to a certain intermediate range, before thresholding. Miller and Smith[10] described another implementation of multilevel halftoning, in which a modularly addressed matrix is used to store pointers to a series of dither LUTs, instead of actual dither values. In this method the results of screening process for each of the possible input levels are precalculated and stored in these LUTs. A major advantage of this LUT based approach is that any conceivable dot growth pattern can be specified, hence smoother visual transition can be achieved at intermediate tones, getting rid of the banding artifacts.

Yu and co-workers[11,12] introduced an over-modulation method, to achieve a smoother transition, at the intermediate output levels. A preprocessing step is added before the screening, where the input pixel value is checked to see whether it is inside a predetermined range of any intermediate output levels. If not, this pixel is passed to the screening stage, or else the overmodulation function is called to modify the input pixel value before passing it to the screen. This is a mean preserve process. With this simple method, the dot patterns around the intermediate output levels can be manipulated to achieve the desired halftone patterns. All these stochastic screening based methods trade off memory for faster execution. Error diffusion, on the other hand, trades off execution speed for a memory efficient implementation.

In this article, we introduce a novel multitoning technique, based on a simple modification to the conventional bilevel error diffusion algorithm. The proposed method is based on the decomposition of the original image into intermediate printable gray scale images, using some constrained modulation function. Each channel is then halftoned using bitonal error diffusion algorithm in a correlated fashion. Finally the halftoned channels are recombined, according to a recombination rule, to get the desired halftone.

In the Introduction, we discuss the conventional multilevel error diffusion algorithm, addressing the problem of banding artifacts. In this section, we also present an overview of earlier approaches, used to solve the banding problem based on stochastic screening methods and error diffusion. In the Multilevel Error Diffusion section, we will introduce the concept of gray level separation to achieve any conceivable dot growth pattern followed by the introduction of the new multilevel halftoning algorithm, based on the idea of gray level separation. In the Novel Multitoning Algorithm section, we present simulation results using the new method. For evaluating the results, we use the perceived mean square error (MSE) between the original and the halftoned image as a metric. As a measure of perception we incorporate the human visual system model proposed by Sullivan and co-workers.[13]

Multilevel Error Diffusion

Error diffusion is a nonlinear adaptive algorithm that uses the threshold error feedback to produce patterns with different spatial frequency contents, depending on the input image values. The process consists of thresholding the input pixels and the error computation. This error is passed to the neighboring pixels that have not been processed. Therefore, it forces total tone content to remain the same and attempts to localize the distribution of the tone levels. Appealing images, with high spatial frequencies, are reproduced using the Floyd–Steinberg error diffusion algorithm,[2] characterized by Ulichney[3] as blue noise patterns.

Multilevel error diffusion can be carried out by replacing the thresholding unit by a quantizer. The output value of the pixel, in this case, is not thresholded but quantized to the nearest value, thus, greatly reducing the error between the modified and the quantized pixel values. Mathematically, multilevel error diffusion can be expressed as:

$$x_a[m, n] = x[m, n] + \Sigma w_{k,l}.y_e[m-k, n-l] \quad (1)$$

$$y[m, n] = Q\{x_a[m, n]\} \quad (2)$$

$$y_e[m, n] = x_a[m, n] - y[m, n], \quad (3)$$

where $Q\{\bullet\}$ denotes the quantization operation, $x_a[m, n]$ is the adaptively adjusted input pixel value, and $y_e[m, n]$ is the error between the adjusted pixel value and the pixel printed at the output.

Equation 1 describes the modified input pixel; Eq. 2 provides the output image pixel after quantization; and Eq. 3 gives the error in quantization to be distributed to the neighboring pixels. The quantization process can be written, explicitly, as:

$$y[m, n] = q_1, \text{ for } x_a[m, n] < T_1 \quad (4)$$

$$y[m, n] = q_2, \text{ for } T_1 \le x_a[m, n] < T_2 \quad (5)$$

$$\vdots$$

$$y[m, n] = q_n, \text{ for } T_{n-1} \le x_a[m, n] \quad (6)$$

for an n-level device, where q_i is the i-th output tone level and T_i is the i-th threshold value. Figure 1 shows the schematic diagram of the conventional multilevel error diffusion algorithm.

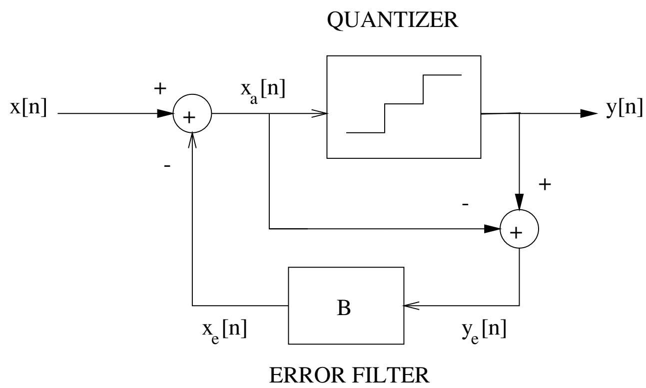


Figure 1. Multilevel error diffusion algorithm.

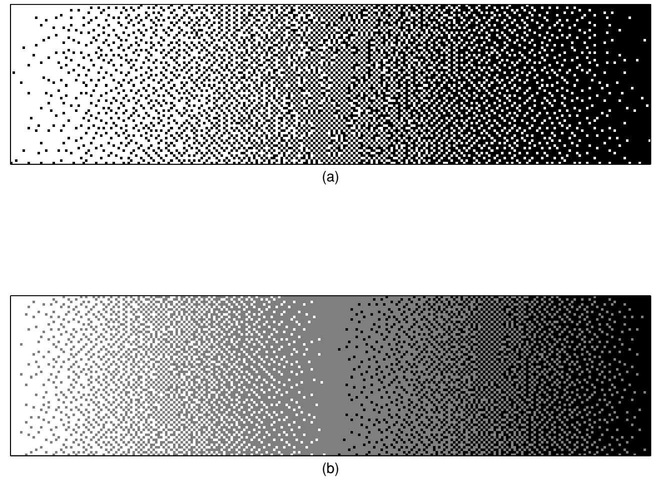


Figure 2. Halftoned *gray scale ramp* using (a) binary (b) trilevel halftoning. Banding can clearly be seen in the ramp generated using trilevel error diffusion around gray level 0.5.

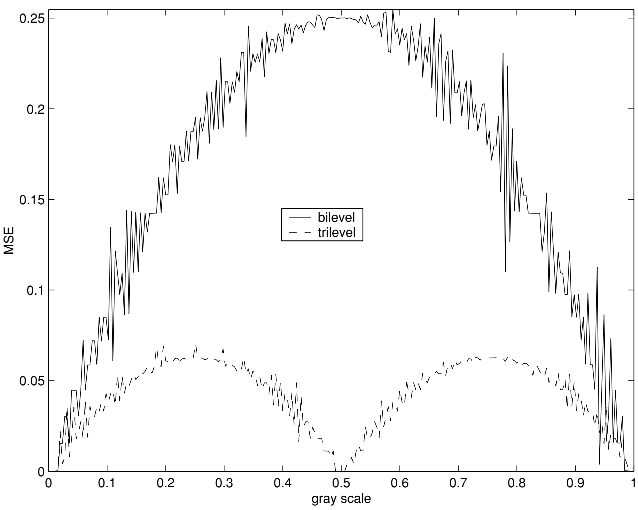


Figure 3. MSE for bilevel and multilevel halftoning.

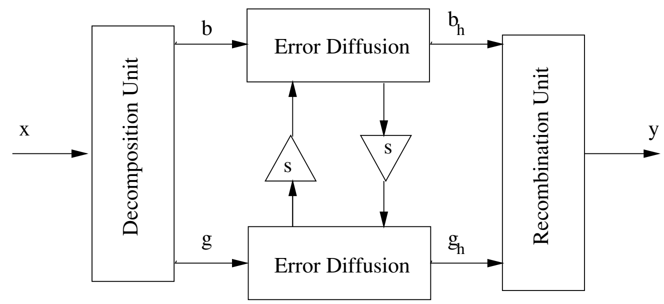


Figure 4. New halftoning algorithm.

Multilevel halftoning, using error diffusion, reduces the maximum error to be distributed to the neighboring pixels, from $(g_{max}–g_{min})/2$ for a bilevel device, to (*stepsize*)/2 if a uniform scale is used. This helps to localize the error distribution and reduce the graininess. Figure 2(a) illustrates the bilevel halftoned version of the *gray scale ramp* and Fig. 2(b) shows the multitoned version of the same ramp. Patches of each gray level are generated and multitoned using the conventional method, and the mean square error (MSE) is computed for each patch using the Human Visual System (HVS) model of Sullivan and co-workers.[13] Figure 3 shows the MSE vs. gray level for both bilevel and trilevel halftoning.

Significant reduction in the MSE can be observed in case of trilevel halftoning. However, for the trilevel curve, around the intermediate output level of 0.5, there is a distinct dip. This is because at the intermediate output states, there is no halftone error introduced because the ink for that particular level is available. Slightly away from these levels, there is a sparse distribution of minority pixel over a uniform background. This appears like a band to the human eye, resulting in high visual error in this region.

A proposed solution to this problem is to introduce dots of neighboring gray levels in the region where banding is dominant in order to achieve a more smooth visual transition. A number of methods, based on multilevel stochastic screens and multilevel error diffusion, have been proposed to get rid of the unwanted banding artifacts in multitones and, in general, to allow the specification of any possible dot growth pattern.[6,7,10,11]

Novel Multitoning Algorithm

Our method is based on the idea of gray level separation. The input image is decomposed into the printable gray scale images. The individual channels are then halftoned in a correlated way using conventional bilevel error diffusion and finally recombined to give the halftoned image.

Gray Level Decomposition

The idea behind the gray level separation is that we can represent a gray level of a continuous tone image by a linear combination of a finite number of intermediate gray levels. In multilevel halftoning, black and gray pixels along with the background pixels (white), in appropriate proportions, give the perception of a shade of gray. However, it is not well defined in what proportion the gray, black and white pixels must be present to represent a particular gray level of the input image. In fact, a wide range of characteristics can be rendered to the halftoned image using the redistribution of pixels under certain bounds.

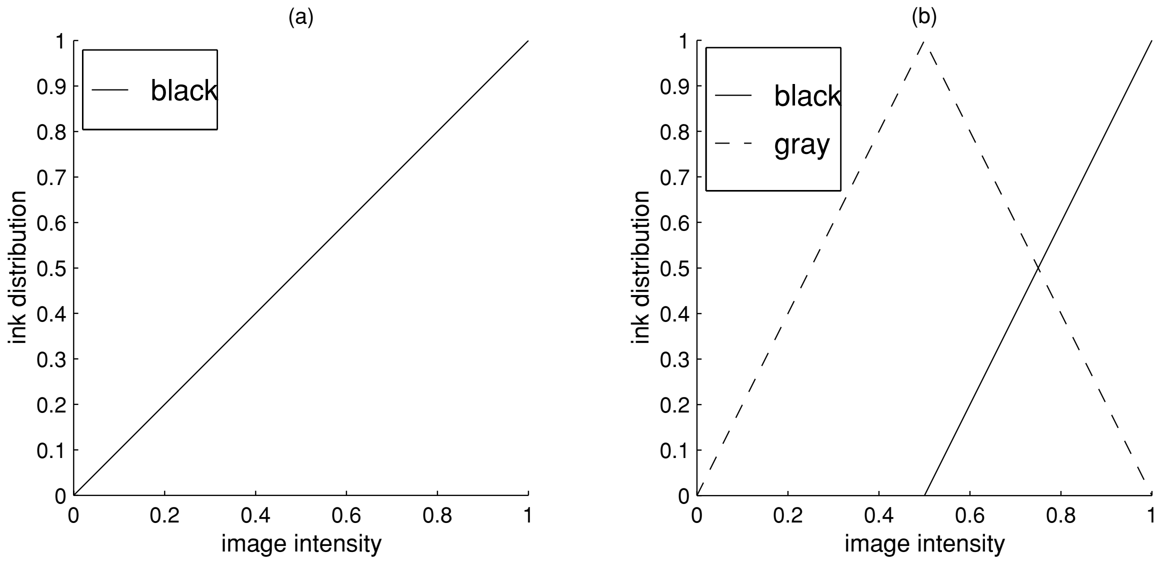


Figure 5. Distribution of ink in (a) bilevel (b) trilevel halftoning. In bilevel halftoning the only ink available is the black ink and the proportion of black pixels rises linearly with the image intensity. For the trilevel case, a 50% gray ink is also available and the resulting distribution of black and gray inks is as shown above.

Figures 5(a) and (b) show the distribution of printable gray levels in bilevel and trilevel halftoning using conventional error diffusion. Corresponding *gray scale ramps* are shown in Fig. 2(a) and (b) respectively. In case of trilevel halftoning, the gray level is symmetrically distributed about gray level 0.5 with no halftoning error at this point, resulting in banding. With the intent of eliminating the banding effect near gray level 0.5, the gray and black pixels have to be redistributed, yet preserving the mean value of the input gray level.

In case of trilevel halftoning, in addition to black, we have one more intermediate printable gray level. Hence the continuous tone (intensity) image can be represented by a linear combination of the black (B) and 50% gray (G) levels. This linear relationship between the continuous tone image and the printable gray levels can be expressed as:

$$xB = bB + gG,\ 0 \le x \le 1 \tag{7}$$

where x is the percentage gray level of the intensity image, b and g are the proportions of the black and gray levels, composing the input image. In fact, b and g are the factors controlling the distribution of the black and gray pixels in the halftoned image. B and G are the intensities of the black and gray inks. Assuming ideal inks, the relationship between the two ink intensities, for trilevel halftoning is $G = B/2$. Thus we have from Eq. 7:

$$x = b + \frac{g}{2}, \quad 0 \le x \le 1. \tag{8}$$

In general, for an M ary printing device, we have ($M - 2$) intermediate gray levels, in addition to the black level, and we have a more general relationship between the input image intensity and the constituent ink intensities, in the halftoned output. This relationship is given by the equation:

$$x = b + \sum_{i=1}^{M-2} a_i g_i, \quad 0 \le x \le 1 \tag{9}$$

where a_i is the proportion of ink of gray level g_i. A unique assignment for the a_is does not exist in Eq. 9. In fact, an infinite number of assignments of the a_is are possible.

Without loss of generality, from this point onwards, we focus on the trilevel case (setting $a_1 = 1/2$ and the rest of a_is = 0). From Eq. 8, a trivial bound on the selection of b and g is:

$$0 \le b + \frac{g}{2} \le 1. \tag{10}$$

Constraining the halftoning process, such that, no overlapping of the black and gray pixels is allowed, we have:

$$0 \le b,\ g \le 1. \tag{11}$$

Combined with the additional constraints, in Eq. 11, we have a tighter bound, given by:

$$0 \le b + g \le 1. \tag{12}$$

However, the above bound analysis is only valid for the processes where only a black or a gray pixel can be printed at a location, not both. This is not the case for uncorrelated halftoning where a black pixel may overlap a gray pixel.

For the case of uncorrelated halftoning, let b and g be the proportion of black and gray pixels, but there will be bg pixels where there is an overlap of black and gray pixels. So the effective number of gray pixels is $g(1 - b)$ and the resulting image intensity will be $Bb + Gg(1 - b)$, which is strictly less than $Bb + gG$. An example of such a printing device could be, the printer capable of printing multiple dots at a particular location so that one pixel gives 0.5 and two pixels at the same location give 1.0.

At this point, we present some simulation results with different conceivable gray level distributions, constrained by the bound defined in Eq. 12. Figures 6(a) and (b) show two new distribution curves, out of the

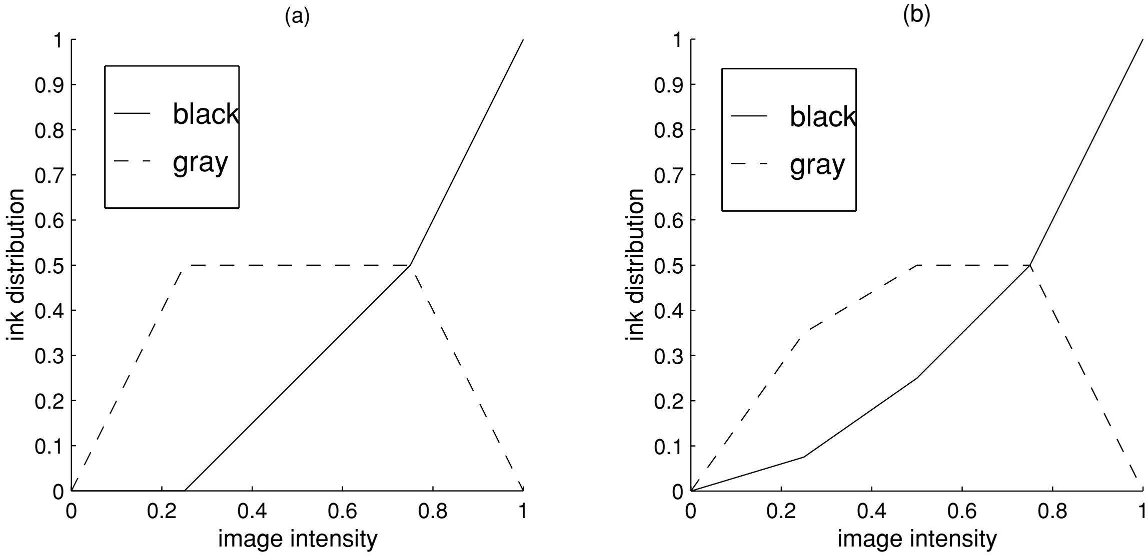


Figure 6. Gray level transform curves for two conceivable patterns. (a) shows a symmetrical distribution of gray ink around gray level 0.5 and (b) shows an asymmetrical distribution of gray ink.

many possible distributions. Using the gray level separation method proposed in this section, any conceivable dot growth pattern can be achieved, rendering different characteristics to the resulting halftones. Figure 7 shows the *gray scale ramps*, halftoned using the new multitoning technique, for 3 levels and the corresponding gray level distributions are shown in Fig. 8.

The result in Fig. 7(a) is similar to the result generated using the conventional multilevel error diffusion algorithm, the only difference being that, in case of the conventional multitoning method, the quantizer is responsible for the dot growth pattern and in our method, the dot growth pattern is determined using the gray level distributions in the separator (decomposition unit).

Figure 7(b) shows the result for halftoning using the proposed method, using a different gray level distribution. The peak of the g curve is flattened in this case, thus, demanding the introduction of black pixels in the flat region to maintain the gray level balance. This results in the new distribution of the b curve, as shown. This redistribution, in turn, results in the introduction of black and white pixels in the banding region. In the *gray scale ramp*, the black pixels extend to the right of the gray level 0.5 and vice versa for the white pixels, thus, eliminating the band.

In Figs. 7(c) and (d), the peak of the g curve has been further flattened to allow more penetration of the black and white pixels to either side of gray level 0.5. A limiting case would give a result, similar to the bilevel case, where the black and the white pixels extend to the ends, thus, eliminating the use of gray ink.

Figure 9 demonstrates the effect of introducing skewness in the gray level curves. Corresponding gray level distributions are shown in Fig. 10. It can be observed that with the increasing degree of skewness, a smoother transition of black and gray pixels is possible, across the band, but result in increased graininess at low gray levels. This is due to the premature introduction of black and white pixels in the distribution.

The above results can easily be extended to the case of N–level multitoning where the input image is decomposed into N possible printable gray scale images, using the gray level transform, under certain bounds. The decomposition method involves iterative application of the

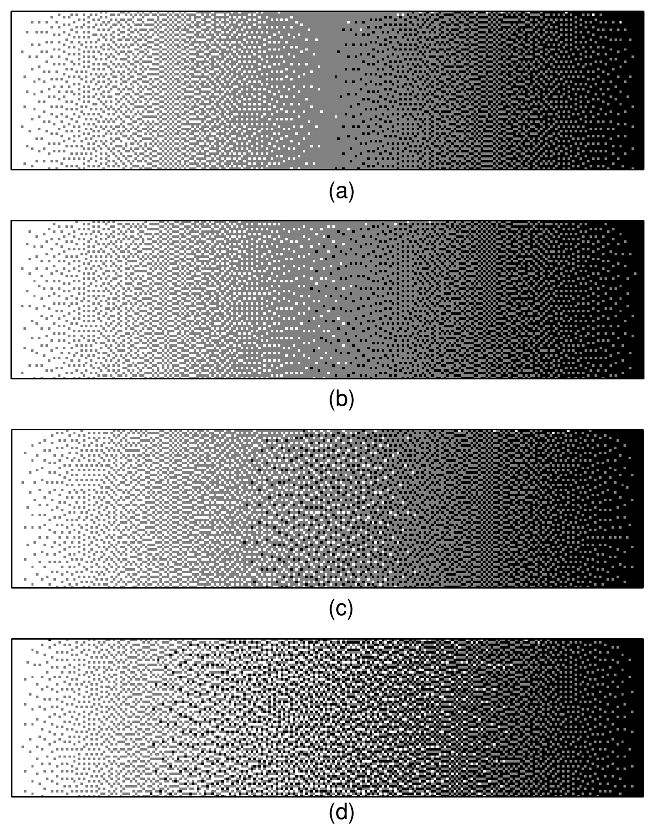


Figure 7. *Gray Scale Ramps* halftoned using the new multitoning technique with uniform intermediate gray level distributions.

two level decomposition, discussed for trilevel halftoning, to hierarchically obtain the intermediate transform curves. However, an important constraint in this case would be to allow the penetration of pixels of the nearest neighbors only, meaning, for a gray level G_n, only pixels of gray level G_{n-1} and G_{n+1} shall be allowed to penetrate into it's vicinity. This is because the introduction of gray levels farther apart in the neighborhood

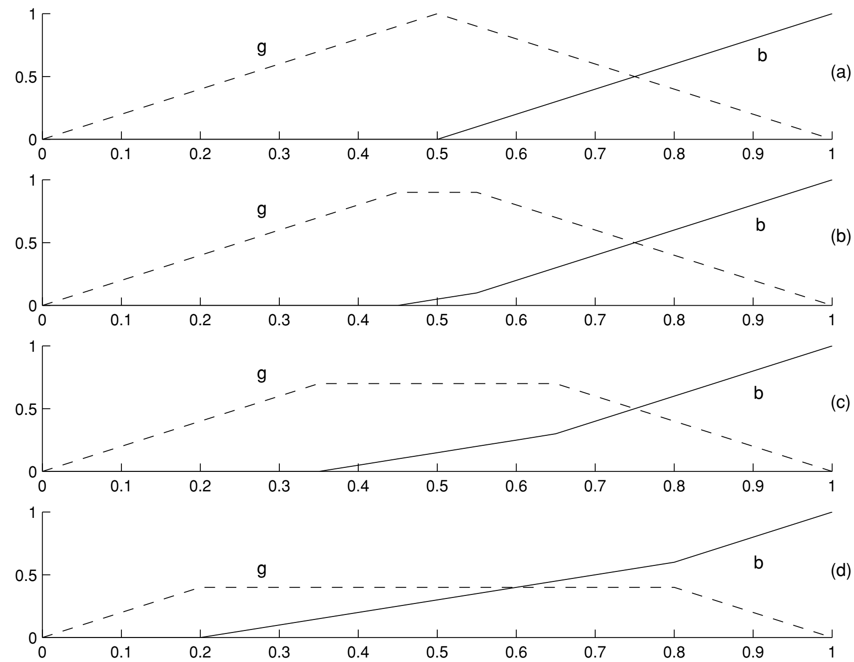


Figure 8. Gray Level distributions corresponding to the gray scale ramps shown in Fig. 7.

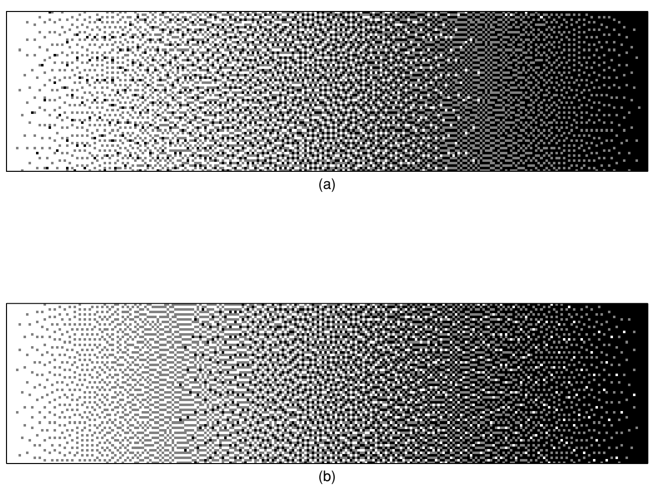


Figure 9. Gray Scale Ramps halftoned using the new multitoning technique with skewed or non-uniform intermediate gray level distributions.

results in the appearance of a strong visual error to the human eye in that region, due to increased graininess and hence increased Mean Square Error.

Figure 11 shows the *gray scale ramps*, halftoned using the new multitoning technique, for 4 and 5 gray levels and the corresponding gray level distributions (25% flattening) are shown in Fig. 12.

Correlated Halftoning Using Generalized Error Diffusion

The generalized error diffusion algorithm, first introduced by Lau,[14–16] was proposed for halftoning color images in a correlated fashion, allowing both the scalar and vector halftoning of color images. It has the provision for introducing a certain degree of correlation between the various color channels, thus regulating the overlapping of different color pixels. Figure 13 shows the simplified block diagram for the generalized error diffusion algorithm, without the sharpening and the hysterisis parameters, which are not of primary interest to our discussion.

In case of multilevel halftoning using the method introduced in this article, the gray scale image is decomposed into the constituent printable gray scale images. This gray level separation allows us to visualize the input image as an N–channel image. Each of the N channels can be halftoned using the conventional bilevel error diffusion algorithm, in a correlated fashion. The halftoning method used in our case is, Floyd Steinberg error diffusion, with 50% randomization of the error filter weights, to account for the inherent periodic texture, in error diffusion, at gray levels 0.25 and 0.75.

In order to avoid the overlapping of black and gray pixels, in the output (halftoned) image, the thresholding stages for each channel are conditioned such that the channel with highest accumulated error value $x_{a,i}[m, n]$ will contribute a pixel in the final halftoned image, provided the value is greater than the threshold. The decision rule for the quantizer can be mathematically stated as follows:

$$y_i[m,n] = \begin{cases} 1 & x_{a,i}[m,n] > x_{a,j}[m,n] \ \& \ x_{a,i}[m,n] > 0, \ i \neq j \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

Multilevel Halftoning Using Gray Level Separation

In the new multitoning method, we first apply the gray level transform to the input image, to split it up into the black and gray channels, using any conceivable ink distribution that is within the bounds specified in Eq. 12. This gray level separation allows us to visualize the input image as a composition of multiple channels (two

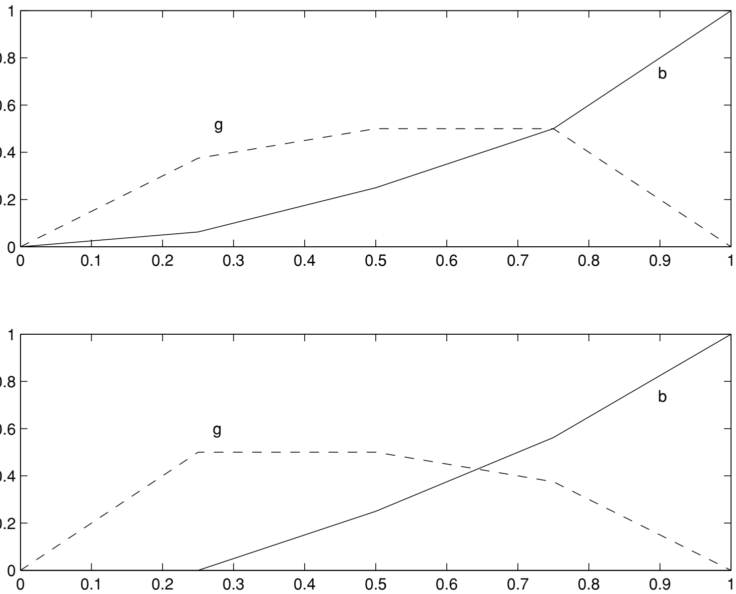


Figure 10. Gray Level distributions corresponding to the gray scale ramps shown in Fig. 9.

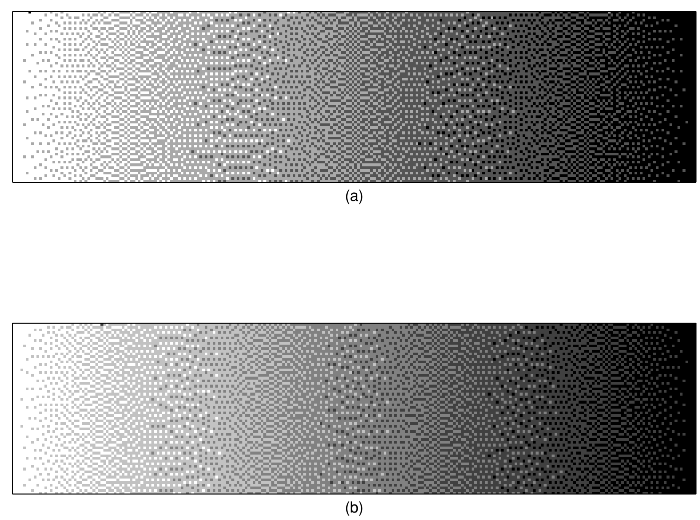


Figure 11. Gray Scale Ramps halftoned using the new multitoning technique for 4 and 5 gray levels with 25% flattening.

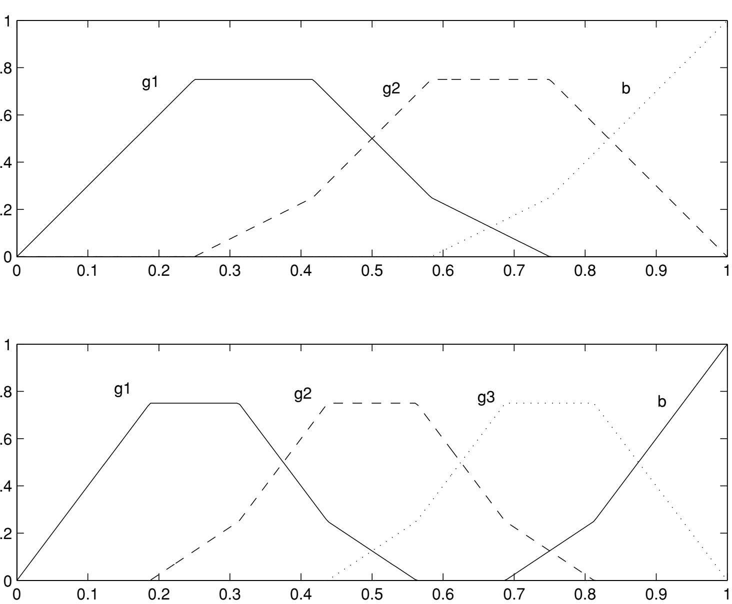


Figure 12. Gray Level distributions corresponding to the gray scale ramps shown in Fig. 11.

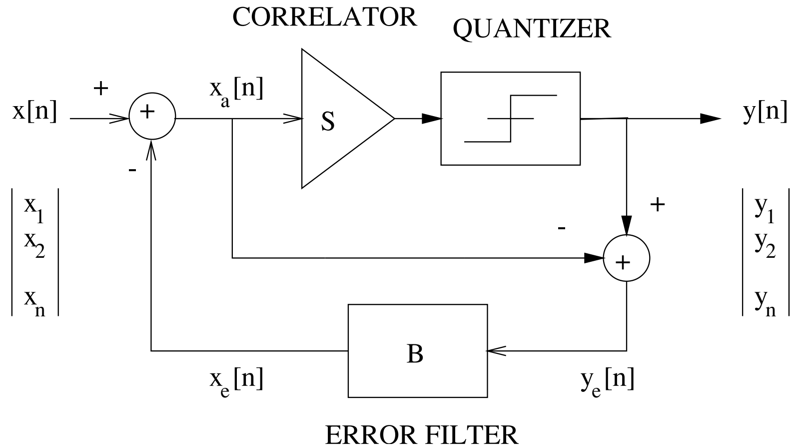


Figure 13. Error Diffusion Algorithm with the provision of correlated quantization. *S* is the correlation factor that controls the threshold of the bilevel quantizer.

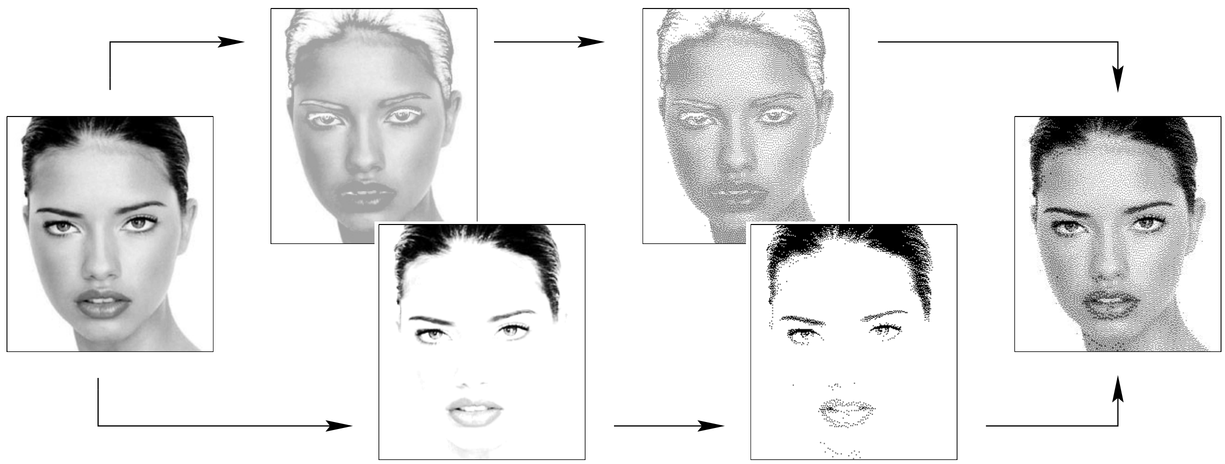


Figure 14. Image *Adrian* halftoned using the proposed halftoning method, with the intermediate processing results.

channels in case of trilevel halftoning where only black and gray inks are available). The gray and black channels are halftoned using conventional Floyd–Steinberg error diffusion, with the constraint that none of the pixels of the two channels overlap each other, thus preserving the mean gray level. Overlapping of the pixels results in a shift in the gray level, requiring gray level correction as a post processing step. No overlapping of the gray and black channel pixels is ensured by conditioning the quantizer operation in accordance with Eq. 13. The block diagram for the new multitoning method is shown in Fig. 4.

Finally, the halftoned channels have to be recombined to give the final halftoned image. Let b_h and g_h be the two independently halftoned channels that have to be recombined to form the final halftoned output. The recombination equation for trilevel halftoning is as follows:

$$y = b_h + \frac{g_h}{2}. \qquad (14)$$

Figure 14 shows the intermediate processing results for our new halftoning method. Image *Adrian* is decomposed into the gray and the black channels, each channel is halftoned in accordance with the quantization rule defined in Eq. 14 and finally the two halftoned channels are recombined to give the final halftoned output with desired characteristics.

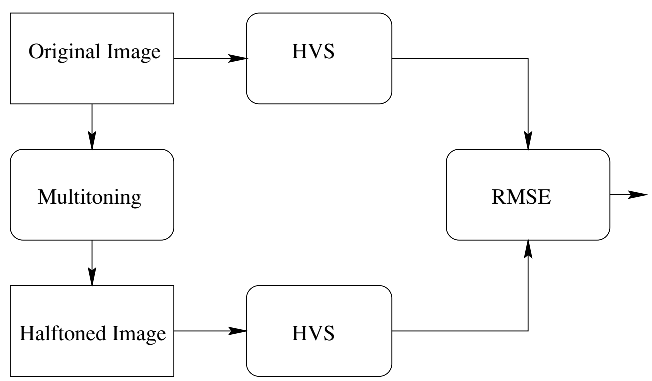


Figure 15. Evaluation of Halftones using Human Visual System model.

Experimental Results and Discussion

All experiments have been performed on the *gray scale ramp* and image, *Adrian*, using the proposed multilevel halftoning method, based on gray level separation. The error filter weight used are the Floyd–Steinberg weights

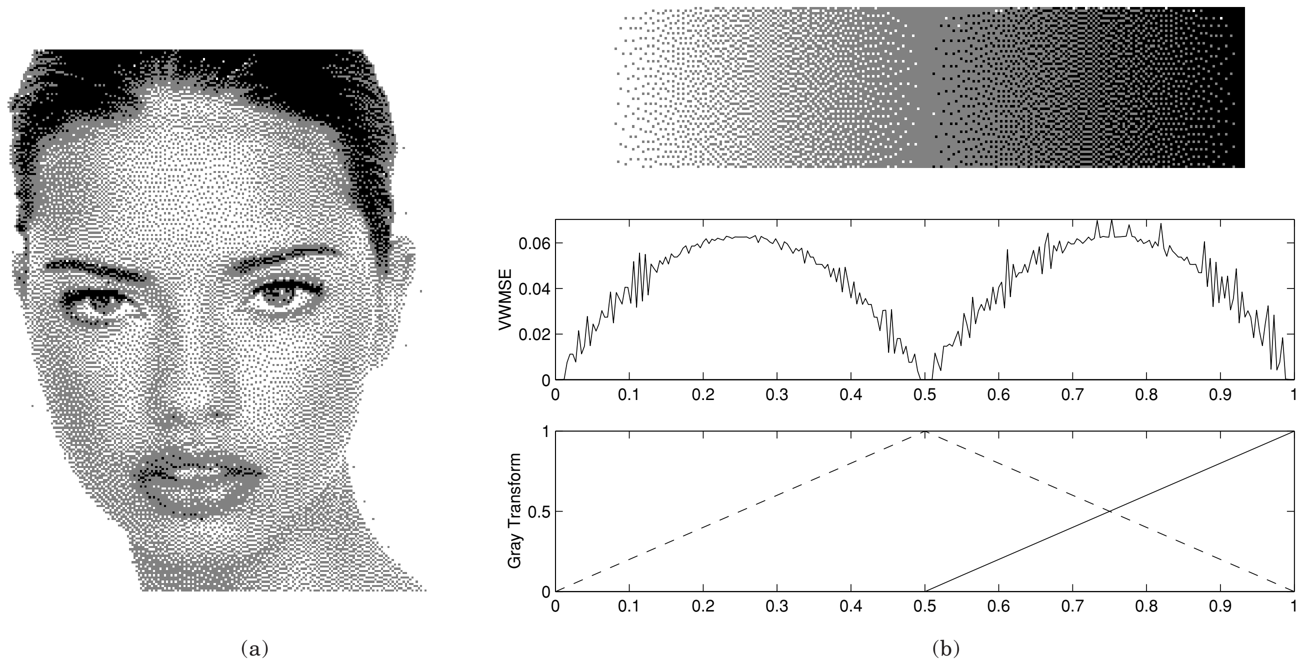


Figure 16. (a) *Adrian* halftoned using the conventional trilevel error diffusion algorithm and (b) *Gray scale ramp* halftoned using the same algorithm with the corresponding VWMSE and the gray level distribution plot.

with 50% noise added to account for the texture at gray levels 0.25 and 0.75 due to patterned arrangement of dots near these gray levels. Floyd–Steinberg weights, inherently, suffer from this patterning problem, at intermediate gray levels, and this texture should not be confused with the banding effect in multilevel halftoning—the problem, that we are primarily addressing, in this article.

To evaluate the multitoning results, a human visual model has been utilized to study the perceived mean square error between the original image and the multitoned version. This metric is referred to as the Visually Weighted Mean Square Error (VWMSE). Figure 15 shows the conceptual flow diagram of the process. If $h[m, n]$ is the point spread function of a human eye, then the error e is given by:

$$e = \sum\{x[m, n] * h[m, n] - y[m, n] * h[m, n]\}. \quad (15)$$

In spatial frequency domain, the equation is given by:

$$E = \sum\{X[i, j] - Y[i, j]\}.H[i, j]. \quad (16)$$

A model of low-contrast photopic modulation transfer function was used to characterize the human visual system.[13]

$$H[i,j] = \begin{cases} a(b + cf_{i,j})\exp[-cf_{i,j})^d] & f_{i,j} > f_{\max} \\ 1.0 & \text{otherwise} \end{cases} \quad (17)$$

where the constants a, b, c, d take on values 2.2, 0.192, 0.114 and 1.1 respectively. The unit of frequency is cycles/degree.

Figures 16(a,b) show the results for halftoning, using conventional trilevel halftoning algorithm. A band at gray level 0.5 is evident in the *gray scale ramp* and in the image *Adrian* at the hairline, lips and the eye-line. Corresponding MSE curve has a zero drop at the gray level 0.5, which, infact, is the cause of banding. The printable gray level distribution shows that the g curve corresponding to the gray level 0.5 peaks up to the maximum value and then drops down at the same rate. The *b* curve corresponding to the black level starts growing after the peak of the gray level 0.5 has occurred.

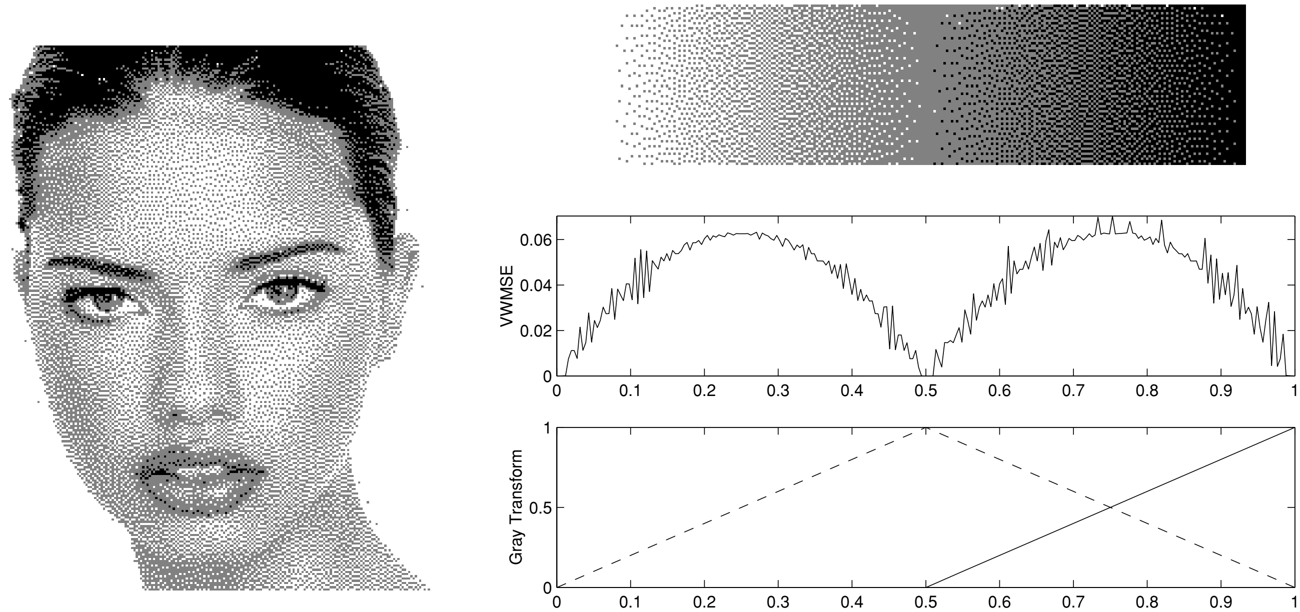
Figures 17(a,b) show the results for halftoning using the proposed method. The peak of the *g* curve is flattened in this case, thus demanding the introduction of black pixels in this flat region to maintain the gray level balance. This result in the new distribution of the b curve, as shown. This redistribution results in the introduction of black and white pixels in the banding region. In the *gray scale ramp* the black pixels extend to the right of the gray level 0.5 and vice versa for the white pixels, thus eliminating the band. However the MSE curve shows that at the expense of elimination of the band at gray level 0.5, quantization noise is introduced in the flat region resulting in increased graininess in the image. From the image *Adrian* it is clearly evident that band present at the hairline, lips and eye-line is gone at the expense of the introduction of noise in these regions.

Figures 18(a,b) show the results of halftoning using the uniform gray level transform, with increasing degree of flattening of the *g* curve, resulting in further elimination of the banding artifacts at the cost of increasing graininess at the midtones.

Figures 19 and 20 show the results of halftoning using the non-uniform gray level transform. The introduction of undesirable black pixels in the regions of lower gray level can be clearly observed in Fig. 19(a). Similarly in Fig. 20(a), the appearance of undesirable black pixels, in the neck and the cheek regions of the image *Adrian*, can clearly be observed with the increasing degree of skewness at higher gray levels.

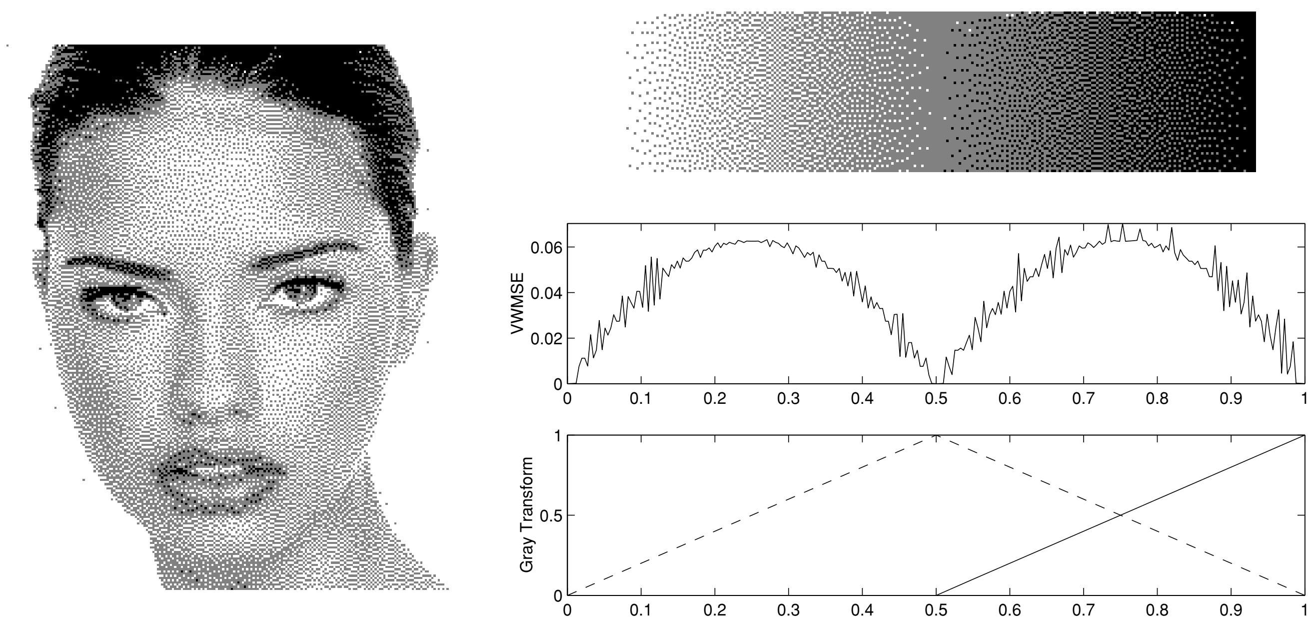
Optimization of the Gray Level Separation

From the above discussion, we observe that in case of non-uniform or skewed intermediate gray level distributions, the mean square error (MSE) peaks up in the di-



(a) (b)

Figure 17. (a) *Adrian* halftoned using the proposed trilevel error diffusion algorithm and (b) *Gray scale ramp* halftoned using the same algorithm with the corresponding VWMSE and the gray level distribution plot (10% flattening).

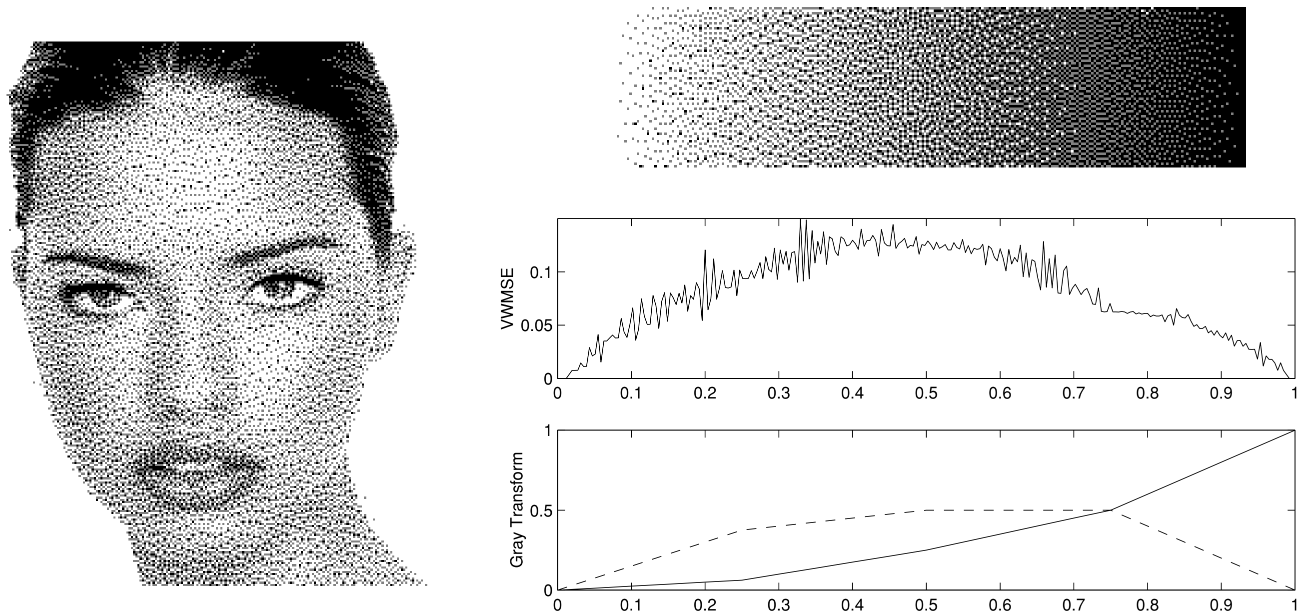


(a) (b)

Figure 18. (a) *Adrian* halftoned using the proposed trilevel error diffusion algorithm and (b) *Gray Scale Ramp* halftoned using the same algorithm with the corresponding VWMSE and the gray level distribution plot (20% flattening).

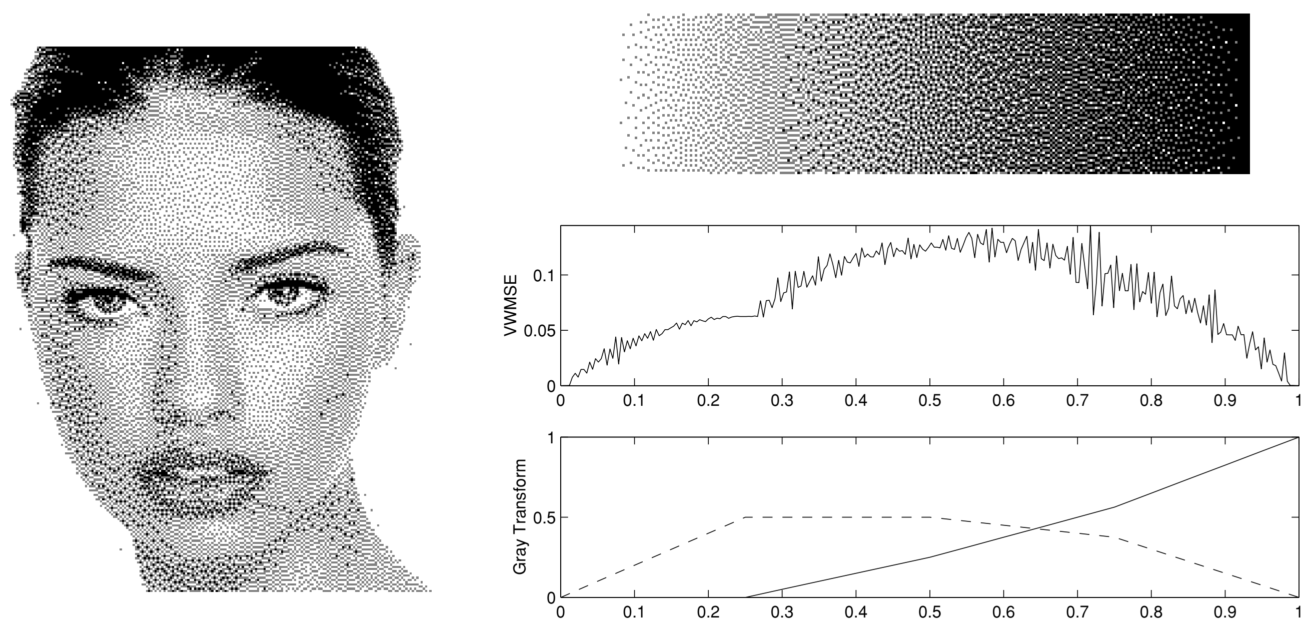
rection of the skew because of an early introduction of the black and white pixels. However, in case of uniform distributions, we see an increase in the MSE only in the neighborhood of the printable gray levels, thus, getting rid of the banding artifacts with an increase in graininess in the banding regions but not in the farther neighborhood where the MSE distribution is already at the peak. The occurrence of the band at the intermediate printable gray levels can be explained by the MSE distribution for the *gray scale ramp* using 3 level halftoning. We see that the banding artifacts are the most prominent in the regions where there is a maximum MSE differential between the neighboring gray levels. For trilevel halftoning, the depression for the MSE curve occurrs at gray level 0.5, where a very visible band can be observed. Thus, to remove banding, the MSE distribution has to be equalized. Hence the optimality criterion is to equalize the MSE distribution over the intermediate gray levels.

For the gray level distributions where the gray curve flattening is sufficient enough to get rid of the band completely, we can also observe that the MSE value at the intermediate gray level is almost equal to the neighboring values. So a uniform gray level distribution with a specific degree of flattening gives the near optimal MSE



(a) (b)

Figure 19. (a) *Adrian* halftoned using the proposed trilevel error diffusion algorithm and (b) *Gray scale ramp* halftoned using the same algorithm with the corresponding VWMSE and the gray level distribution plot (skewed distribution at lower gray levels).



(a) (b)

Figure 20. (a) *Adrian* halftoned using the proposed trilevel error diffusion algorithm and (b) *Gray scale ramp* halftoned using the same algorithm with the corresponding VWMSE and the gray level distribution plot (skewed distribution at higher gray levels).

distribution and hence the near optimal solution for multitoning. However, there is a slight drawback associated, even with the uniform distributions. With the flattening of the gray level distribution, two new bands are introduced at the neighboring gray levels depending upon the extent of flattening. But the magnitude of these two new bands, in terms of mean square error differential, is very low as compared to the highly visible band at 0.5.

A possible solution to the optimal (equalized) MSE distribution is to slightly tilt the edges of the gray level distributions, as shown in Fig. 22. Using these gray level curves, we can get rid off the smaller bands, as well, by equalizing the MSE distribution and, hence, achieving optimality. Figure 21 shows the *gray scale ramps* halftoned using the modified gray level distributions. The corresponding MSE distributions are shown in Fig. 23.

Conclusion

In this article we introduce a novel multilevel halftoning technique, based on the idea of gray level separation. The proposed method, in addition to eliminating the unwanted banding artifacts, near the intermediate gray levels, in the halftoned image, also allows the growth of

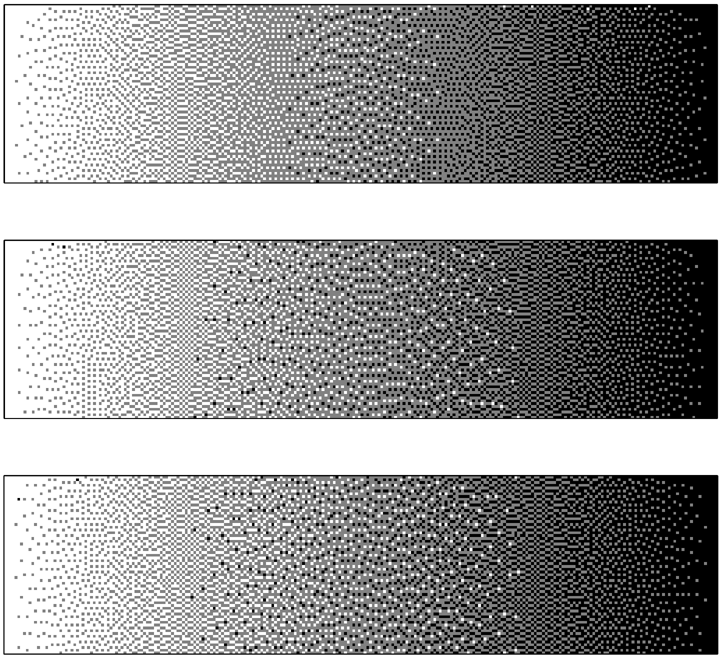


Figure 21. *Gray scale ramps*, halftoned using the modified gray level distributions, as shown in Fig. 22.

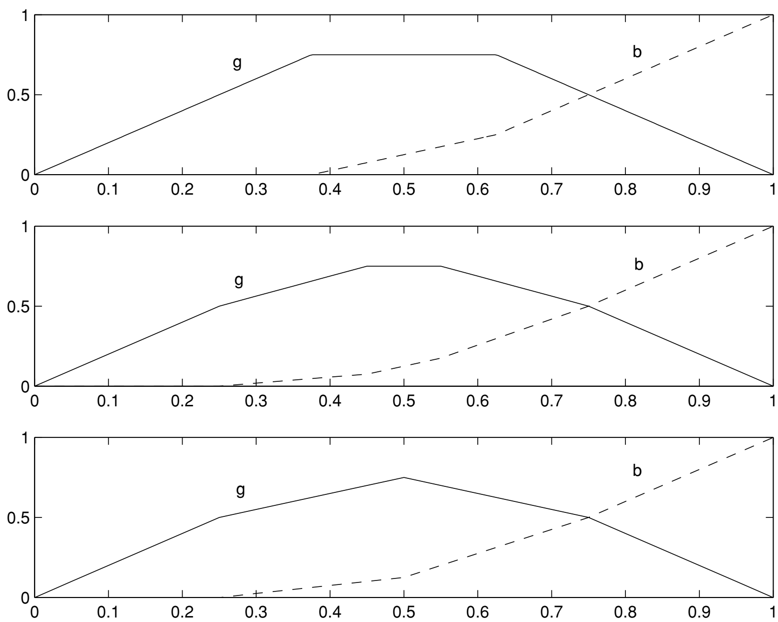


Figure 22. The modified gray level distributions to achieve optimal solution.

any conceivable dot pattern, rendering a wide variety of characteristics to the resulting halftones. To achieve the gray level separation, we have introduced a generalized gray level transform to decompose the image into the constituent gray level images, according to the dot growth pattern defined by the transform. Experiments are performed on the *gray scale ramp* and *Adrian* and a comparative discussion has been presented with the conventional multilevel error diffusion and it's variants, proposed earlier.

The new method is simple to implement and mean preserving and much less complex as compared to the other approaches,[6,7,10–12] that mix gray level pixels to eliminate banding. The results clearly indicate the elimination of the unwanted texture, in the halftoned gray scale ramp, at the cost of increasing graininess at the midtones. Further, what our new approach gives us is a simple and effective way to extend any and all bilevel algorithms to N-level. This is especially advantageous given the amount of prior research that has been done in the area of bilevel halftoning using error diffusion.

Future Work

As can be observed from the halftoning results of the image *Adrian*, produced by the suggested method, the redistribution of the pixels trades off the introduction of high frequency graininess with the elimination of the banding texture. The graininess is more visible in the dc regions as compared to the high frequency regions. This suggests the use of the proposed algorithm in an image dependent way, where different dot growth patterns will

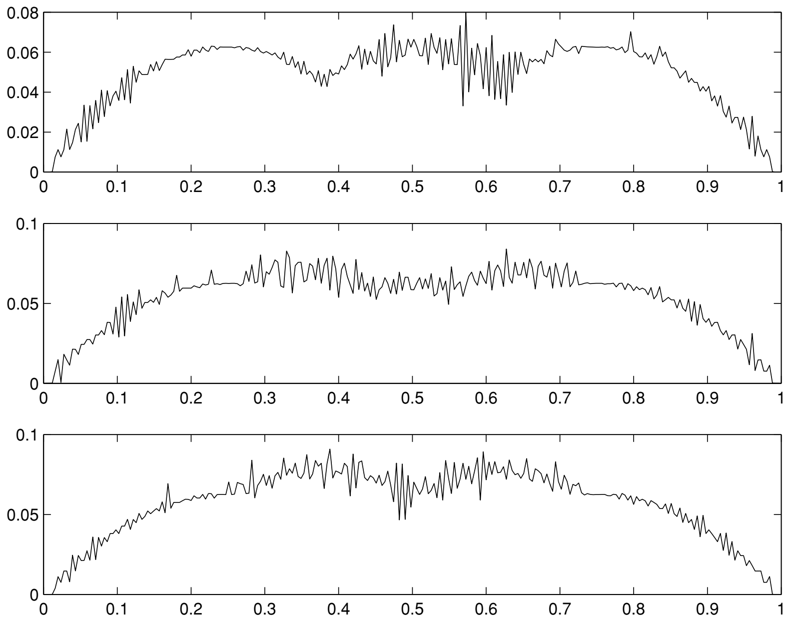


Figure 23. The optimal MSE distributions for the *gray scale ramps*, halftoned using the modified gray level distributions, as shown in Fig. 21.

be used for the low and the high frequency regions. In the future we have plans to investigate the results of image dependant multitoning using our algorithm.

The algorithm presented in this article can easily be extended to multilevel color halftoning. We plan to investigate the application of the proposed algorithm to ink jet printers with six ink options (light cyan and light magenta in addition to the conventional CMYK colors). We suggest the application of the proposed transform to the cyan and magenta channels separately and use a correlation metric to correlate the individual quantizer decisions, in the generalized error diffusion algorithm, to constrain the printing of the dots of individual colors as well as the midtones of cyan and magenta, in the final output. Unlike the case of multitoning in monochrome ink jet printers, where the purpose of redistribution of the gray levels is the elimination of unwanted texture, in six color halftoning the proposed algorithm, along with appropriate correlation of the individual channels, can be used to suggest a method to achieve true color perception.

The suggested halftoning method is appropriate for devices capable of reproducing more than one gray levels, like the ink jet printers with multiple ink option. However there are some printing devices that inspite of lacking the capability to reproduce more than one shade of gray physically, can achieve closer performance by using high addressibility, thanks to the capability of the human visual system to perform averaging over a unit area to perceive different gray levels. For example, there are a number of electrophotographic printers in which the printable dot size remains the same, but the printing head resolution is much higher,[17] allowing the individual dots to be printed at sub-pixel locations. There are ink jet printers with variable sized dot options to produce the illusion of multiple gray levels when averaged over a unit area. The proposed halftoning technique can be extended to these high addressibility devices, but with added constraints imposed by the appropriate printer model for the printer in question. Our future work will be directed towards applying the proposed algorithm to such devices, incorporating the suggested method in model based halftoning algorithms.

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