Optimizing Color-Matching Functions for Individual Observers Using a Variation Method

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The CIE system allows the specification of color matches for a standard observer using the color-matching functions (cmfs). However, the cmfs of an individual observer are different from those of the CIE within a finite range. This article describes the optimization of the cmfs of an individual observer based on metameric pairs using a variation method. This is a so much simplified method for estimating rough and ready cmfs of an individual observer in comparison with past experiments. The underlying assumption of the optimization is that the optimum cmfs will predict that the integrated cone responses of a metameric pair are equal. The feature of the proposed optimization method is that the color difference in a metamer pair can be optimized to 0 at a boundary condition in the variation method, and the smoothness of the modified cmfs results from the cost function of the least mean square of modified values in the variation method. The cost function of the variation method is generalized to consider the perception of color differences by the human visual system. Experiments using measured metamer spectral data demonstrate the validity of the proposed method. In the ***Supplemental Material (found on the IS&T website at www.imaging.org, for no less than 2 years from the date of publication),*** theorems and characteristics are presented and proved to demonstrate the rigid theoretical background supporting the experimental validity.

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Introduction

The Commission Internationale de l'Eclairage (CIE) system allows the specification of color matches for a standard observer using the color matching functions (cmfs). The cmfs of the standard observer are the fundamental basis of colorimetry. Studies performed by Guild[1] (1931) and Wright[2] (1928-29) are central to the work of the CIE, providing the foundation for the derivation of the CIE 1931 standard colorimetric observer. The CIE cmfs were defined simply as an average of the functions of observers with normal color vision, and the standard colorimetric observer was defined as a hypothetical one that has the average cmfs so defined. Therefore, the cmfs of real observers with normal color vision do not necessarily agree exactly with those of the CIE standard colorimetric observer's.

This article describes the optimization of the cmfs of an individual observer based on metameric pairs using a variation method.[3] This is a so much simplified method for estimating rough and ready cmfs of an individual observer in comparison with past experiments. The underlying assumption for the optimization is that the optimum cmfs will predict that the integrated cone responses of a metameric pair are equal. A feature of the optimization method is that the color difference in a metamer pair can be optimized to 0 at a boundary condition in the variation method, and the smoothness of the modified cmfs results from the cost function of the least mean square of modified values in the variation method. The cost function of the variation method is generalized by a Taylor expansion to consider the perception of color differences by the human visual system.

In the experiments, metamer data sources were obtained from four individuals. The visual experiment was designed to perform a color match between a hard copy card and an additive mixing device. The integrated cone responses of a metameric pair are not equal when these metamers are evaluated by the CIE cmfs because of the difference between the CIE cmfs and those of the individual observer. Using the proposed optimization method, the color difference in the metamer data has been decreased to $\Delta E = 0$ with the modified cmfs predicting the cmfs of the individual observer. The experimental results demonstrate the validity of the proposed method.

Deriving Optimal Color-Matching Functions

Basic Algorithm

The tristimulus values X, Y, Z of an object-color stimulus are given as follows:

$$X = \sum_{\lambda} \rho(\lambda) S(\lambda) \bar{x}(\lambda) \Delta\lambda,$$
$$Y = \sum_{\lambda} \rho(\lambda) S(\lambda) \bar{y}(\lambda) \Delta\lambda, \quad (1)$$
$$Z = \sum_{\lambda} \rho(\lambda) S(\lambda) \bar{z}(\lambda) \Delta\lambda.$$

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Supplemental Materials—Appendix can be found on the IS&T website (www.imaging.org) for a period of no less than 2 years from the date of publication.

where $\rho(\lambda)$ is the spectral reflectance of the object, $S(\lambda)$ is the spectral power distribution of the illuminant, and $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ are cmfs of the CIE standard colorimetric observer. The summation (Σ) is over the visible spectrum with a wavelength interval $\Delta\lambda$. The product $\rho(\lambda)S(\lambda)$ defines the object-color stimulus.

Two objects with different spectral reflectance functions, $\rho(\lambda)$ and $\rho'(\lambda)$, give rise to metamer stimuli when illuminated by $S(\lambda)$ if their corresponding tristimulus values, X,Y,Z and X',Y',Z', are equal as follows:

$$\begin{aligned}\sum_\lambda \rho(\lambda)S(\lambda)\bar{x}(\lambda)\Delta\lambda &= \sum_\lambda \rho'(\lambda)S(\lambda)\bar{x}(\lambda)\Delta\lambda,\\ \sum_\lambda \rho(\lambda)S(\lambda)\bar{y}(\lambda)\Delta\lambda &= \sum_\lambda \rho'(\lambda)S(\lambda)\bar{y}(\lambda)\Delta\lambda,\\ \sum_\lambda \rho(\lambda)S(\lambda)\bar{z}(\lambda)\Delta\lambda &= \sum_\lambda \rho'(\lambda)S(\lambda)\bar{z}(\lambda)\Delta\lambda.\end{aligned} \tag{2}$$

The proposed method for optimizing the cmfs of an individual observer using the variation method[4–10] is presented below. The underlying assumption for the optimization is that optimum cmfs will predict that the integrated cone responses to a metameric pair are equal. For convenience, symbols are defined as follows:

$$\begin{aligned}&X_1 = X,\ X_2 = Y,\ X_3 = Z,\\ &q_1(\lambda) = \bar{x}(\lambda),\ q_2(\lambda) = \bar{y}(\lambda),\ q_3(\lambda) = \bar{z}(\lambda),\\ &q(\lambda) = \left(q_1(\lambda),\ q_2(\lambda)\ q_3(\lambda)\right)^t,\\ &J(\lambda) = \rho(\lambda)S(\lambda).\end{aligned} \tag{3}$$

Let $q_i^*(\lambda),\ (i = 1,2,3)$ be the modified cmfs with a variation term $\Delta q_i(\lambda)$, $(i = 1,2,3)$

$$\begin{aligned}&q_i^*(\lambda) = q_i(\lambda) + \Delta q_i(\lambda),\ (i = 1,2,3),\\ &q^*(\lambda) = \left(q_1^*(\lambda),\ q_2^*(\lambda),\ q_3^*(\lambda)\right)^t,\\ &\Delta q(\lambda) = \left(\Delta q_1(\lambda),\ \Delta q_2(\lambda),\ \Delta q_3(\lambda)\right)^t.\end{aligned} \tag{4}$$

Let $\rho_r(\lambda)$, $\rho_m(\lambda)$ be a spectral reflectance of a reference data and that of a metamer data, respectively. The difference between the reference and the metamer object color stimuli is as follows:

$$\Delta J(\lambda) = \rho_r(\lambda)S(\lambda) - \rho_m(\lambda)S(\lambda). \tag{5}$$

The tristimulus values of $\Delta J(\lambda)$ related to $q_i^*(\lambda)$ are then given by

$$Q_i^* = \left(\Delta J(\lambda)\cdot q_i^*(\lambda)\right),\ (i = 1,2,3). \tag{6}$$

Constraints are imposed as follows on Eq. 6:

[Constraints on the Variation Method]

$$\Delta Q_i = \left(\Delta J(\lambda)\cdot q_i^*(\lambda)\right) = const_i\ (given),\ (i = 1,2,3). \tag{7}$$

[Cost Function of the Variation Method]

$$CF(\Delta q) = \sum_\lambda cf\left(\Delta q(\lambda)\right)\Delta\lambda, \tag{8}$$

where

$cf(\Delta q(\lambda))$ cost function for each λ. At $\Delta q = 0$ the minimum,

$CF(\Delta q)$: cost function for the entire wavelength range.

[Constraint with Unknown Parameters of the Variation Method]

$$\sum_\lambda\left(\sum_{i=1}^{3}\mu_i\Delta J(\lambda)\cdot q_i^*(\lambda)\right) = const. \tag{9}$$

where

$\mu_i (i = 1,2,3)$: Lagrangian unknown parameters,

The cf function is expanded using a Taylor expansion as

$$\begin{aligned}&cf_t\left(\Delta q(\lambda)\right) =\\ &cf\left(q(\lambda)\right) + \nabla cf\left(q(\lambda)\right)\Delta q(\lambda) + \frac{1}{2}\left(\Delta q(\lambda)\right)^t H\left(q(\lambda)\right)\Delta q(\lambda),\end{aligned} \tag{10}$$

where

$$\begin{aligned}\nabla cf\left(q(\lambda)\right) &= \left(\frac{\partial cf(q)}{\partial q_1}, \frac{\partial cf(q)}{\partial q_2}, \frac{\partial cf(q)}{\partial q_3}\right)^t\\ &= \left(cf_{q1}(q), cf_{q2}(q), cf_{q3}(q)\right)^t,\end{aligned}$$

$$H(q) = \begin{bmatrix}\frac{\partial^2 cf(q)}{\partial q_1\partial q_1} & \frac{\partial^2 cf(q)}{\partial q_1\partial q_2} & \frac{\partial^2 cf(q)}{\partial q_1\partial q_3}\\ \frac{\partial^2 cf(q)}{\partial q_2\partial q_1} & \frac{\partial^2 cf(q)}{\partial q_2\partial q_2} & \frac{\partial^2 cf(q)}{\partial q_2\partial q_3}\\ \frac{\partial^2 cf(q)}{\partial q_3\partial q_1} & \frac{\partial^2 cf(q)}{\partial q_3\partial q_2} & \frac{\partial^2 cf(q)}{\partial q_3\partial q_3}\end{bmatrix},\quad \textit{Hessian matrix},$$

and the following Lagrange (Eq. 11) function is derived.

[Lagrange Function of the Variation Method]

$$\begin{aligned}F = CF - 2const &= \sum_\lambda f\Delta\lambda = \sum_\lambda\left[cf_t\left(\Delta q(\lambda)\right) - 2\sum_{i=1}^{3}\mu_i\Delta J(\lambda)\cdot q_i^*(\lambda)\right]\Delta\lambda\\ &= \sum_\lambda\left[cf\left(q(\lambda)\right) + \sum_{i=1}^{3}\Delta q_i(\lambda)cf_{qi}\left(q(\lambda)\right) + \frac{1}{2}\left(\Delta q(\lambda)\right)^t H\left(q(\lambda)\right)\Delta q(\lambda) - 2\sum_{i=1}^{3}\mu_i\Delta J(\lambda)\cdot q_i^*(\lambda)\right]\Delta\lambda\\ &= \sum_\lambda\left[cf\left(q(\lambda)\right) + \frac{1}{2}\left(\Delta q_i(\lambda)\right)^t H\left(q(\lambda)\right)\Delta q(\lambda) - 2\sum_{i=1}^{3}\left(\mu_i\Delta J(\lambda) - \frac{1}{2}cf_{qi}\left(q(\lambda)\right)\right)\Delta q_i(\lambda) - 2\sum_{i=1}^{3}\mu_i\Delta J(\lambda)\cdot q_i(\lambda)\right]\Delta\lambda\\ &= \sum_\lambda\left[cf\left(q(\lambda)\right) + \frac{1}{2}\left(\Delta q(\lambda)\right)^t H\left(q(\lambda)\right)\Delta q(\lambda) - 2V^{(1)}\Delta q(\lambda) - 2V^{(2)}q(\lambda)\right]\Delta\lambda,\end{aligned} \tag{11}$$

f: Lagrange function of the variation method for each λ,

F: Lagrange function of the variation method for all wavelengths, where

$$V^{(1)}(\lambda) = \left(V_1^{(1)}(\lambda), V_2^{(1)}(\lambda), V_3^{(1)}(\lambda)\right)^t$$

$$= \left(\mu_1 \Delta J(\lambda) - \frac{1}{2} cf_{q1}(q(\lambda)),\ \ \mu_2 \Delta J(\lambda) - \frac{1}{2} cf_{q2}(q(\lambda)),\right.$$

$$\left.\mu_3 \Delta J(\lambda) - \frac{1}{2} cf_{q3}(q(\lambda))\right)^t,$$

$$V^{(2)}(\lambda) = \left(V_1^{(2)}(\lambda), V_2^{(2)}(\lambda), V_3^{(2)}(\lambda)\right)^t$$

$$= \left(\mu_1 \Delta J(\lambda),\ \mu_2 \Delta J(\lambda),\ \mu_3 \Delta J(\lambda)\right).$$

A modified value Δq is independent of q and, based on Eq. 11, the following equation is derived:

$$\frac{\Delta f}{\Delta q} = H(q(\lambda))\Delta q(\lambda) - 2V^{(1)}(\lambda) = 0. \tag{12}$$

Equation 7 is converted as follows:

$$\left(\Delta J\ \ (\lambda) \cdot \Delta q_i(\lambda)\right) = const_i', \tag{13}$$

where

$$const_i' = const_i - \left(\Delta J(\lambda) \cdot q_i(\lambda)\right).$$

By eliminating $\Delta q_i(\lambda)$, (i = 1,2,3) in Eqs. 12 and 13, a linear equation of the parameters μ_i, (i = 1,2,3) are derived. By solving the equation for the parameters μ_i, (i = 1,2,3) the Δq value and the modified cmfs are derived as shown in Eqs. 14 and 15.

$$\Delta q(\lambda) = 2H^{-1}(q(\lambda))V^{(1)}(\lambda), \tag{14}$$

$$q_i^*(\lambda) = q_i(\lambda) + \Delta q_i(\lambda),\ (i = 1,2,3) \tag{15}$$

The method derives the optimum solution over the entire wavelength range (see ***Supplemental Material***).

Description of the Cost Function

The cost function of the variation method is derived considering the CIE L*a*b* color space of the perception of color differences by the human visual system.

$$\begin{aligned} L &= 116 g\left(\frac{Y}{Y_n}\right) - 16, \\ a &= 500\left\{g\left(\frac{X}{X_n}\right) - g\left(\frac{Y}{Y_n}\right)\right\}, \\ b &= 200\left\{g\left(\frac{Y}{Y_n}\right) - g\left(\frac{Z}{Z_n}\right)\right\}. \end{aligned} \tag{16}$$

$$g(r) = \begin{cases} r^{\frac{1}{3}}, & > 0.008856, \\ 7.787r + \dfrac{16}{116}, & r \le 0.008856. \end{cases}$$

The cost function measures the CIE L*a*b* sensitivity depending on variations in the CIE XYZ cmfs as shown in Eq. 17 where w $(0< w)$ is a coefficient in the cost function.

The first term $cf_1(\Delta q(\lambda))$ in Eq. 17 is for the L*a*b* sensitivity and the second term $cf_2(\Delta q(\lambda))$ in Eq. 17 is for the smoothness of the modified cmfs. Equation 17 is expanded using the Taylor expansion as shown in Eq. 18.

$$\begin{aligned} cf(\Delta q(\lambda)) &= cf_1(\Delta q(\lambda)) + cf_2(\Delta q(\lambda)) = \left\{(\Delta L)^2 + (\Delta a)^2 + (\Delta b)^2\right\} + w\left\{(\Delta q_1(\lambda))^2 + (\Delta q_2(\lambda))^2 + (\Delta q_3(\lambda))^2\right\} \\ &= \left[\left\{116 g\left(\frac{q_2(\lambda) + \Delta q_2(\lambda)}{Y_n}\right) - 16\right\} - \left\{116 g\left(\frac{q_2(\lambda)}{Y_n}\right) - 16\right\}\right]^2 \\ &+ \left[500\left\{g\left(\frac{q_1(\lambda) + \Delta q_1(\lambda)}{X_n}\right) - g\left(\frac{q_2(\lambda) + \Delta q_2(\lambda)}{Y_n}\right)\right\} - 500\left\{g\left(\frac{q_1(\lambda)}{X_n}\right) - g\left(\frac{q_2(\lambda)}{Y_n}\right)\right\}\right]^2 \\ &+ \left[200\left\{g\left(\frac{q_2(\lambda) + \Delta q_2(\lambda)}{Y_n}\right) - g\left(\frac{q_3(\lambda) + \Delta q_3(\lambda)}{Z_n}\right)\right\} - 200\left\{g\left(\frac{q_2(\lambda)}{Y_n}\right) - g\left(\frac{q_3(\lambda)}{Z_n}\right)\right\}\right]^2 \\ &+ w\left\{(\Delta q_1(\lambda))^2 + (\Delta q_2(\lambda))^2 + (\Delta q_3(\lambda))^2\right\}, \end{aligned} \tag{17}$$

$$\begin{aligned} cf_t(\Delta q(\lambda)) = 116^2 &\left[\left.\frac{dg(r)}{dr}\right|_{r=q_2(\lambda)/Y_n} \cdot \frac{\Delta q_2(\lambda)}{Y_n}\right]^2 \\ +500^2 &\left[\left.\frac{dg(r)}{dr}\right|_{r=q_1(\lambda)/X_n} \cdot \frac{\Delta q_1(\lambda)}{X_n} - \left.\frac{dg(r)}{dr}\right|_{r=q_2(\lambda)/Y_n} \cdot \frac{\Delta q_2(\lambda)}{Y_n}\right]^2 \\ +200^2 &\left[\left.\frac{dg(r)}{dr}\right|_{r=q2(\lambda)/Y_n} \cdot \frac{\Delta q_2(\lambda)}{Y_n} - \left.\frac{dg(r)}{dr}\right|_{r=q_3(\lambda)/Z_n} \cdot \frac{\Delta q_3(\lambda)}{Z_n}\right]^2 \\ +w &\left[(\Delta q_1(\lambda))^2 + (\Delta q_2(\lambda))^2 + (\Delta q_3(\lambda))^2\right]. \end{aligned} \tag{18}$$

$$H(q(\lambda)) = \begin{pmatrix} \left(\frac{500}{X_n}\right)^2\left(\frac{dg}{dr}\left(\frac{q_1(\lambda)}{X_n}\right)\right)^2 + w & -\frac{500^2}{X_nY_n}\frac{dg}{dr}\left(\frac{q_1(\lambda)}{X_n}\right)\frac{dg}{dr}\left(\frac{q_2(\lambda)}{Y_n}\right) & 0 \\ -\frac{500^2}{X_nY_n}\frac{dg}{dr}\left(\frac{q_1(\lambda)}{X_n}\right)\frac{dg}{dr}\left(\frac{q_2(\lambda)}{Y_n}\right) & \frac{116^2+500^2+200^2}{Y_n^2}\left(\frac{dg}{dr}\left(\frac{q_2(\lambda)}{Y_n}\right)\right)^2 + w & -\frac{200^2}{Y_nZ_n}\frac{dg}{dr}\left(\frac{q_2(\lambda)}{Y_n}\right)\frac{dg}{dr}\left(\frac{q_3(\lambda)}{Z_n}\right) \\ 0 & -\frac{200^2}{Y_nZ_n}\frac{dg}{dr}\left(\frac{q_2(\lambda)}{Y_n}\right)\frac{dg}{dr}\left(\frac{q_3(\lambda)}{Z_n}\right) & \left(\frac{200}{Z_n}\right)^2\left(\frac{dg}{dr}\left(\frac{q_3(\lambda)}{Z_n}\right)\right)^2 + w \end{pmatrix}. \quad (19)$$

The Hessian matrix of the cost function is calculated as shown in Eq. 19.

The first derivative of the cost function is as follows:

$$\nabla cf(q(\lambda)) = \left(\frac{\partial cf(q)}{\partial q_1}, \frac{\partial cf(q)}{\partial q_2}, \frac{\partial cf(q)}{\partial q_3}\right)^t = \left(cf_{q1}(q), cf_{q2}(q), cf_{q3}(q)\right)^t = \left(0,\ 0,\ 0\right). \quad (20)$$

Equations 19 and 20 are applied to the calculation of Eq. 14.

The Hessian matrix (Eq. 19) is semipositive for any $q(\lambda)$, as shown in Characteristic 1 in the ***Supplemental Material***, so the truncated cost function (Eq. 10) is convex based on Theorems 1 and 2 in the ***Supplemental Material***.

Experiments

Experimental Data Source

This section discusses the experimental setup and data collection procedures.

Experiments—Shaw and Fairchild[11,12]

Shaw and Fairchild (1999) designed a visual experiment to allow observers to perform visual color matching between a neutral gray card of $L^* = 50$ created with a Fujix Pictrography 3000 color printer, and an ACS VCS 10 additive mixing device. The ACS VCS 10 consists of seven colored discs, all rotating at high speed to stimulate an additive integral visual response. The proportions of each colored disc are adjusted by an observer using the controls to simulate the stimuli. The viewing booth had both fluorescent daylight and incandescent illumination to view the colors. The seven discs in the ACS VCS 10 were white, red, green, blue, yellow, purple and black. Independent control was allowed on any three primaries at any one time using the control panel. The goal was to generate a metameric match between the stimuli and the three primaries. Of the five colored primary discs, three primaries were chosen: red, green and blue (RGB).

The total number of observers was four. All observers claimed to have normal color vision, but this was not tested. The matching field was 8 cm × 9 cm, subtending a visual angle of 7°. Observers were seated 75 cm from the stimuli and asked to make an exact match to the gray card using only the three primaries specified. When a color match was achieved, a PhotoResearch PR650 was used to measure the spectral radiance of the metamer from the observer's angle of view. This was considered very important due to the angular properties of the colored discs, whereby a color match was perceived as a different color when viewed from a different angle. Each observer was asked to repeat the experiment 10 times.

The precision and accuracy of the PhotoResearch was not tested. A previous evaluation performed by Alfvin and Fairchild[13,14] indicated that the systematic and random errors associated with the instrument were minimal and acceptable for the purpose of this research.

Optimization of cmfs

In optimizing cmfs, the metamer data, above were employed. The experimental data were within a common wavelength range of 400 nm-700 nm (in 5 nm steps). The spectral data for each observer were averaged to reduce experimental noise. In the experiments, the cmfs of the CIE 1931 standard colorimetric observer were used as the standard reference to derive the modified cmfs optimized to an individual observer. The cost function described above was employed. The weighting coefficient was $\omega = 10^2$.

Figures 1 through 4 show the modified cmfs for observers 1 through 4, respectively. In the optimization of Figs. 1 through 4, the constraint of Eq. 7 was imposed on the variation method and $const_i = 0$, $(i = 1,2,3)$ for $\Delta E = 0$. Figure 1 depicts almost the same cmfs as the CIE 1931 cmfs. In Fig. 2, except for the change in the wavelength range from 550 nm – 700 nm in the modified $\bar{z}(\lambda)$ function, the modified cmfs are smooth and realistic. In Fig. 3, although there is a non-negligible difference in the wavelength range from 640 nm – 700 nm in the modified $\bar{y}(\lambda)$ function, the modified cmfs are realistic. In Fig. 4, the modified $\bar{z}(\lambda)$ function is the same as the CIE 1931 $\bar{z}(\lambda)$, except for the slight difference in the wavelength range from 660 nm – 700 nm. As for the smoothness of the modified cmfs, the least mean square does not necessarily ensure the continuity of the first-order derivative (see ***Supplemental Material***), although it is a general constraint for smoothness.

Discussion

In general, the problem studied here has very large dimensionality (61) in the wavelength range from 400 nm – 700 nm (in 5 nm steps). It is not easy to determine the optimal solution. In the proposed optimiza-

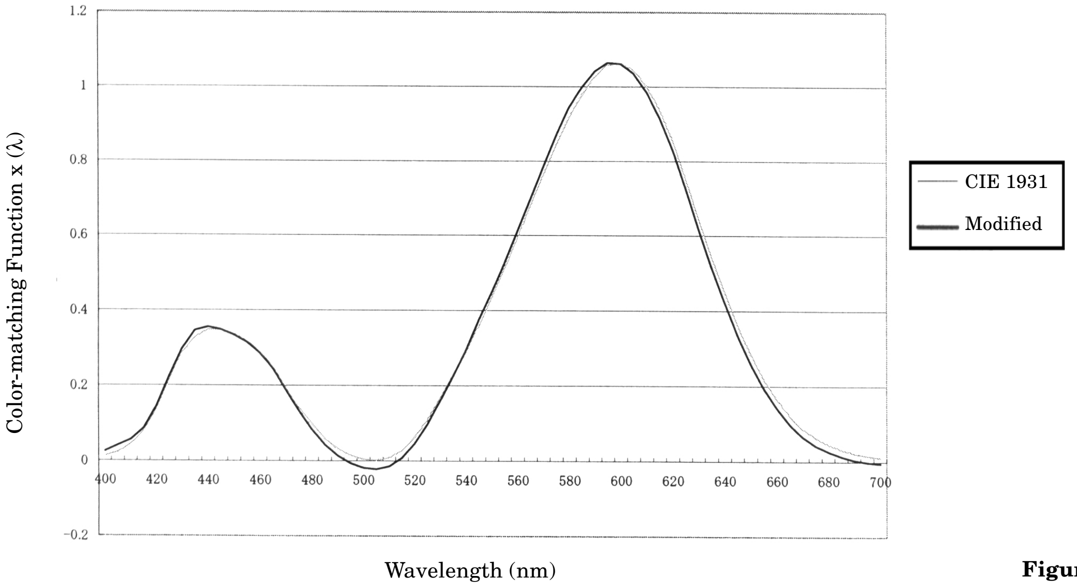

Figure 1a. $\bar{x}(\lambda)$

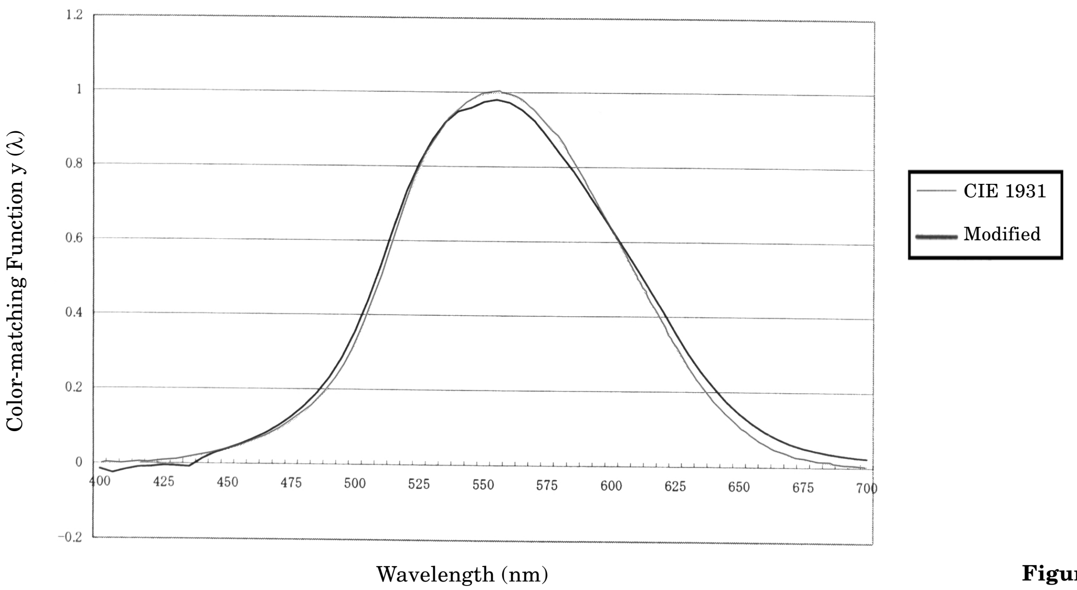

Figure 1b. $\bar{y}(\lambda)$

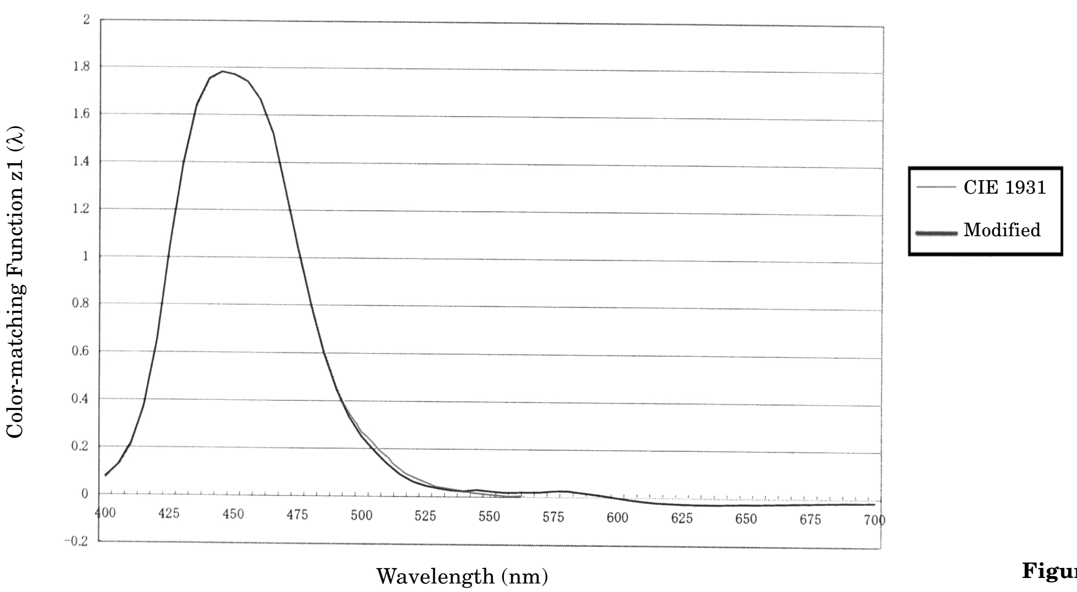

Figure 1c. $\bar{z}(\lambda)$

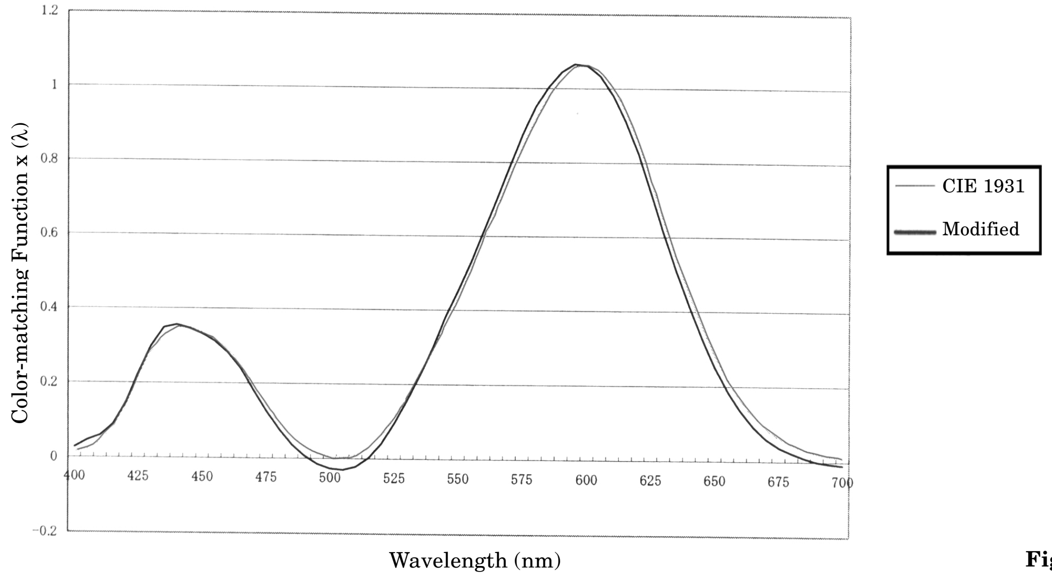

Figure 2a. $\bar{x}(\lambda)$

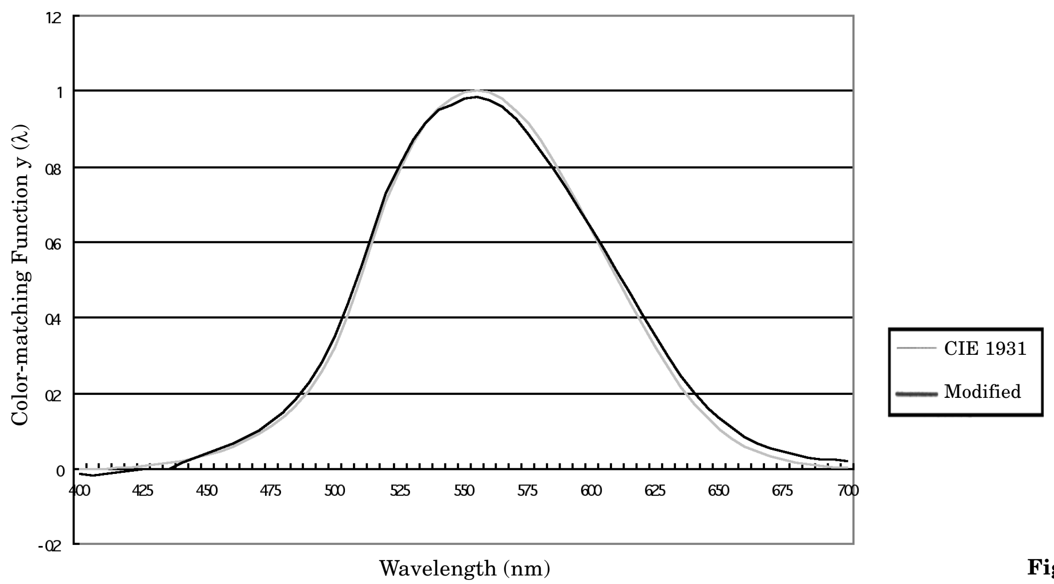

Figure 2b. $\bar{y}(\lambda)$

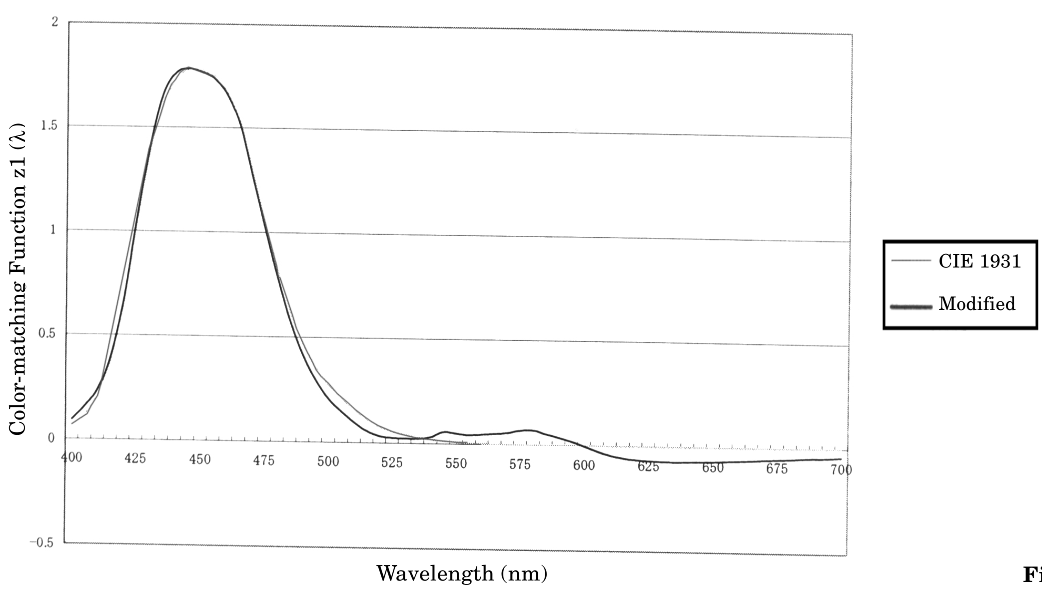

Figure 2c. $\bar{z}(\lambda)$

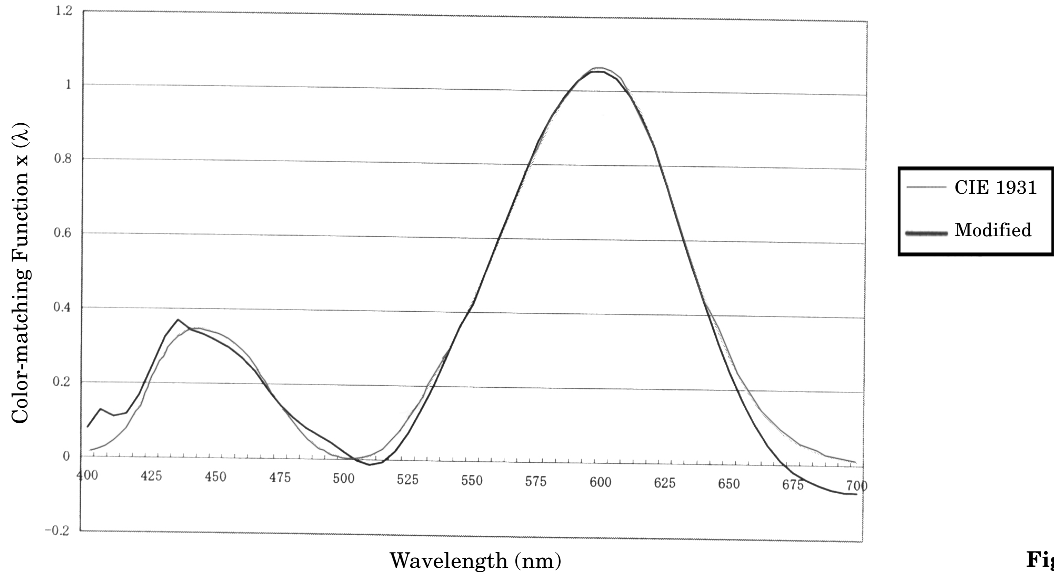

Figure 3a. $\bar{x}(\lambda)$

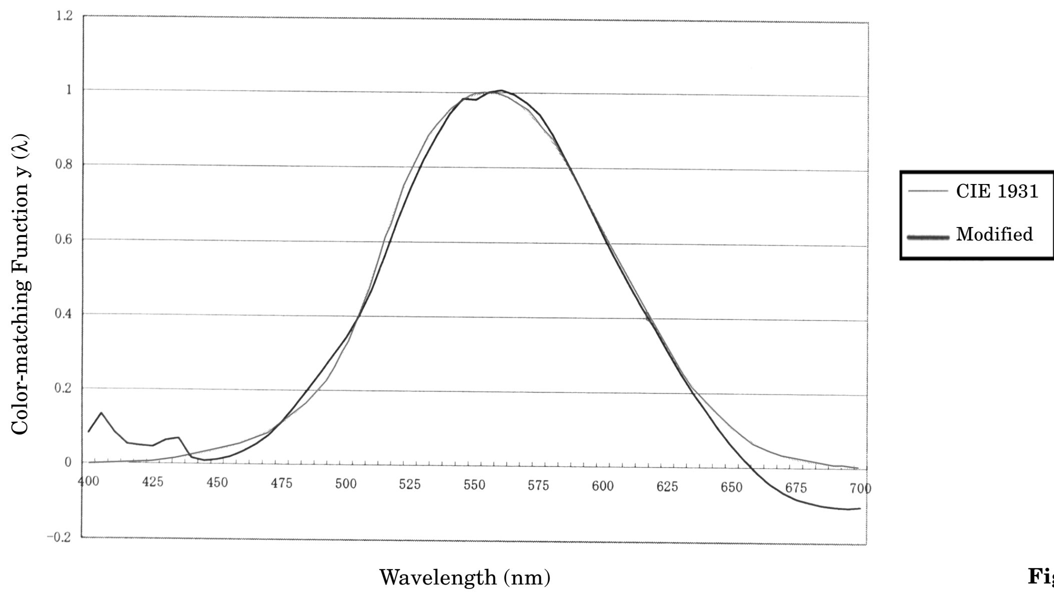

Figure 3b. $\bar{y}(\lambda)$

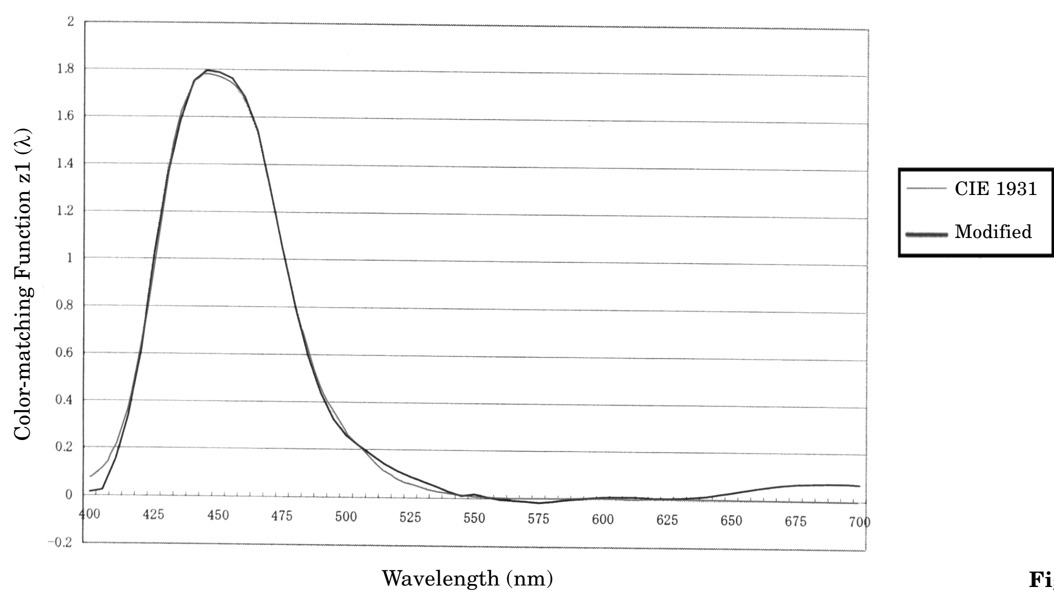

Figure 3c. $\bar{z}(\lambda)$

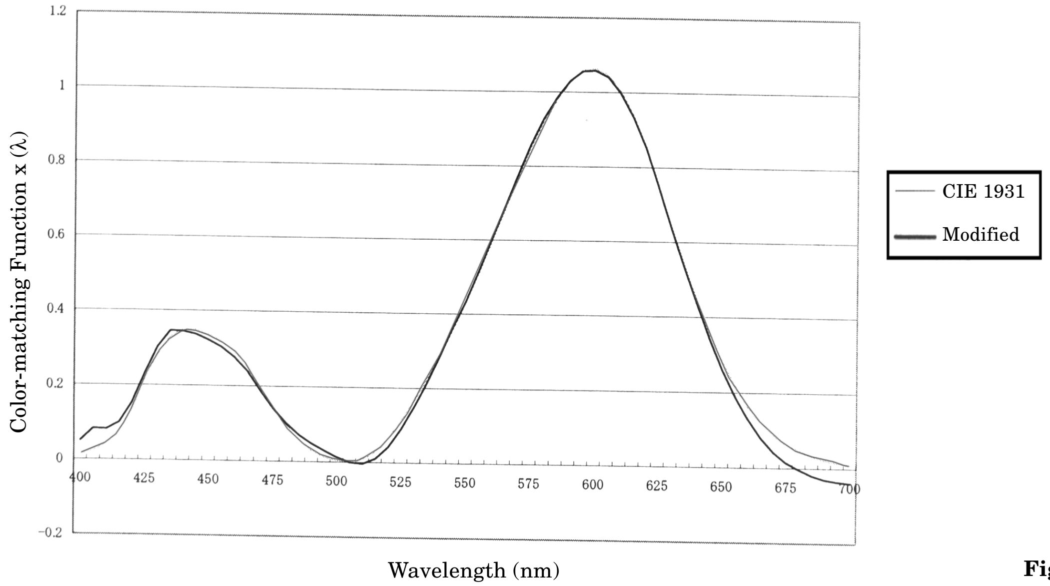

Figure 4a. $\bar{x}(\lambda)$

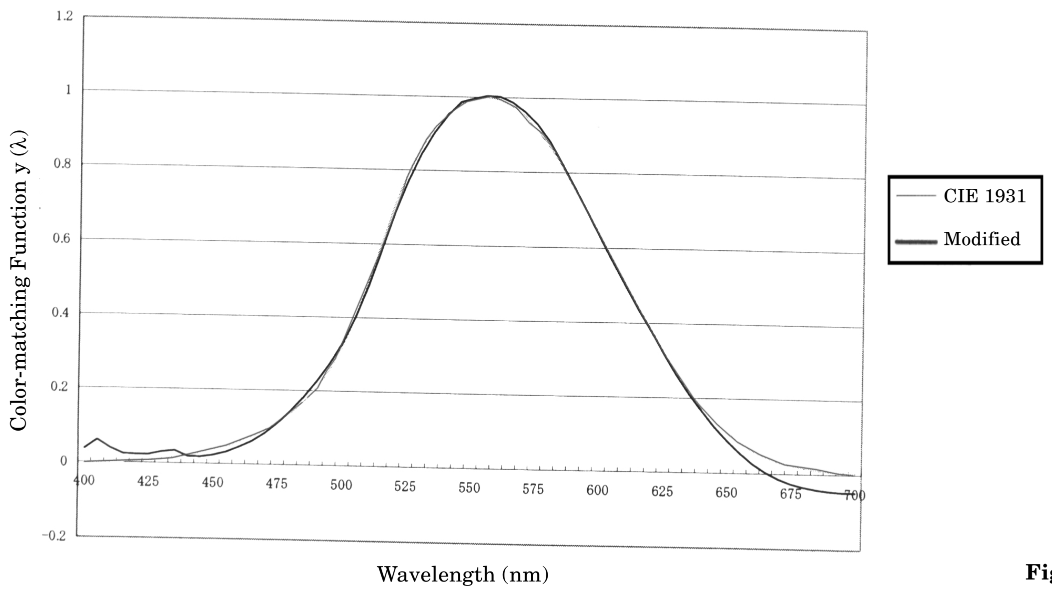

Figure 4b. $\bar{y}(\lambda)$

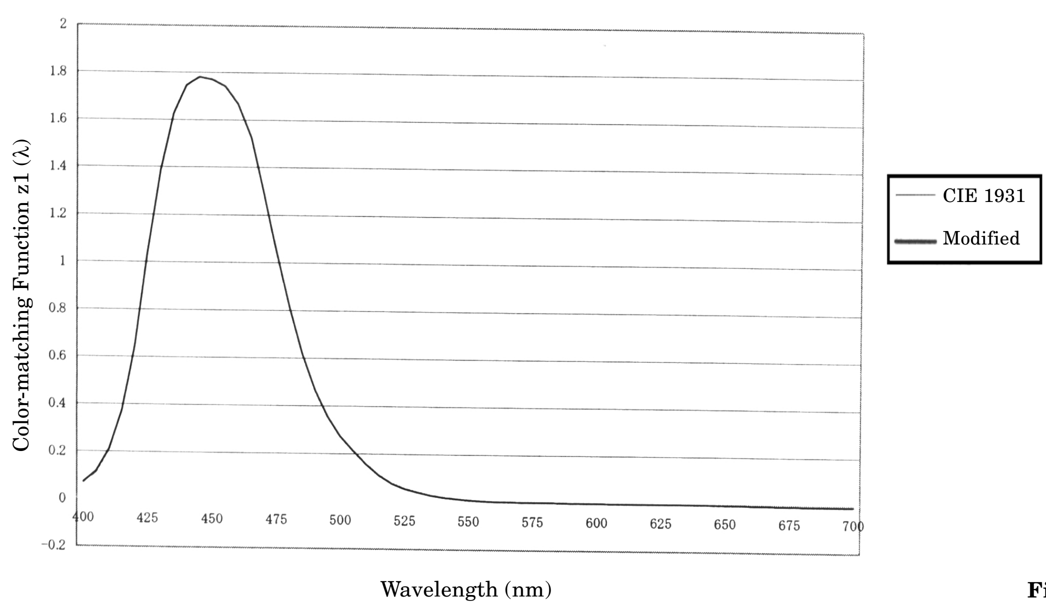

Figure 4c. $\bar{z}(\lambda)$

tion method, the problem is constrained by Eq. 7. The linkage of all dimensions is described by linear equations of Eq. 7, so the optimization problem is replaced by a linearly combined multidimensional problem that has only partial differentials operating on the cost function about one parameter included in the solution framework. The elimination of cross terms in partial differentials makes the proposed method simple to solve and successful.

In optimization problems, there is often difficulty in distinguishing between experimental error and the optimization error. In the proposed optimization method, $\Delta E = 0$ is certain and thus, the optimization error can be neglected, and only experimental error is included in the results. This technique can be repeated across a number of metameric matches to obtain a good statistical estimate of an individual observer's cmfs. The solution of the method has an unbiased property against the cmfs of an individual observer based on expectations (see ***Supplemental Material***).

Conclusions

The optimization of the cmfs of individuals using a variation method has been described. In the variation method, metamer data was employed as the source data. The underlying assumption for the optimization is that the optimum cmfs will predict that the integrated cone responses to a metameric pair are equal.

A feature of the optimization method is that the color difference in a metamer pair can be optimized to 0 at a boundary condition in the variation method, and the smoothness of the modified cmfs results from the cost function of the least mean square of modified cmfs values in the variation method. The cost function of the variation method has been generalized using a Taylor expansion to consider the perception of color differences by the human visual system.

This work has utilized experimental data from previous visual experimental data by Shaw and Fairchild (1999). The modified cmfs for observer 1 were almost the same as the CIE 1931 cmfs. For observer 2, except for the fluctuation in the wavelength range from 550 nm – 700 nm in the modified $\bar{z}(\lambda)$ function, the modified cmfs were smooth and realistic. For observer 3, although there was a non-negligible difference in the wavelength range of 640 nm – 700 nm in the modified $\bar{y}(\lambda)$ function, the modified cmfs were realistic. For observer 4, the modified $\bar{z}(\lambda)$ function was the same as the CIE 1931 $\bar{z}(\lambda)$, except for a slight difference in the wavelength range from 660 nm – 700 nm. As for the smoothness of the modified cmfs, the least mean square does not necessarily ensure the continuity of the first-order derivative (see ***Supplemental Material***), although it is a general constraint for smoothness.

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