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Toner Penetration into Paper at Fusing

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In the most popular toner fusing process, toner melts are fused onto the paper under the influence of heat and pressure exerted by the nip forming rollers. The degree of toner penetration influences the toners fuse level and depends on process conditions, toner rheological and paper properties. This article establishes a three-dimensional (3D) model for studying toner penetration onto the paper macroscopically. The model considers a toner melt layer sandwiched between a smooth fuser roll and a porous paper surfaces under heat and pressure. Toner penetration and their thickness variation within the nip are computed under various conditions of toner viscosity, paper permeability, applied pressure, dwell time, and toner layer thickness. Toner viscosity is a strong function of temperature. Under normal fusing conditions and using Xerox 4024 plain paper, toner images just fill the voids at the paper surface. Only the less viscous liquid, such as fuser oil, can easily penetrate into the paper. Modeling results compare favorably with the actual toner images. The model can be applied to other penetration studies also.

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Introduction

The physics of toner fusing includes melting, sintering, spreading, penetration, and vitrification of melted toner onto the receiving substrate. Fusing or fixing of toned images onto a substrate, such as paper, can be accomplished in many different ways, such as, vapor, cold pressure, radiant, hot plate, belt, or roll fuser. The most favored method is hot roll fusing under applied pressure. The fuser roll is typically heated internally and the outer layer is a low surface energy material that is also coated with release agent to ensure toner release. There is also a compliant rubber layer to form a fusing nip under pressure to improve contact and dwell time. The toner resin in the nip is softened or melted by the heat while pressure is applied to overcome the yield value of the toner. The toner particles are fused and penetrated to a cohesive mass that bonds to the paper after toner vitrification. The degree of toner penetration depends on process conditions (temperature, pressure, and dwell time), rheological properties of toners and paper properties. The penetrations of toner onto the paper during the fusing process is strongly related to the image fuse or fix level and can be solved microscopically or macroscopically. In microscopic study, the difficulty is that the surface topography of the paper is not uniform and not well defined. Poiseuille's law is typically applied to the penetration phenomenon, as in the case of Lucas–Washburn equation. It describes the depth of penetration as a function of pore radius, time, pressure and viscosity. For the paper, the radius of capillary pore is not well defined. It is, therefore, not suitable for its application microscopically. Macroscopically, paper properties are lumped into a single phenomenological parameter called permeability. The permeability takes into account the size, shape, spacing, and roughness of the paper as well as the physical properties of the fluid. Generally, the degree of image fuse level is judged macroscopically not microscopically.

This article studies toner penetration onto the paper surface macroscopically as a 3D case. A cross-sectional view of actual images on the paper is also included for qualitative comparison.

Analysis

As the toner images and the porous paper reach the fusing station under the influence of heat and pressure, the toner layer contacting the porous paper surface may flow into the paper substrate or stay on the surface.

The present study of toner penetration in fusing considers the toner melt sandwiched between two surfaces with one surface having a porous facing (i.e. paper) as shown in Fig. 1. The nomenclatures for Fig. 1 are a = width of the fusing nip, b = length of the fusing nip, h = thickness of the toner layer and H = paper thickness. It is assumed that the toner melt in the film region satisfies the modified Reynolds equation and the toner flow in the porous paper region satisfies the 3D Laplace equation. Together with the associated boundary conditions, the problem is solved analytically in series form.

Assumptions. The following assumptions are made in the analysis:

a) The toner melt in the film region is laminar
b) The toner melt is incompressible
c) The inertia effects for the toner melts are negligible
d) In the film region, the pressure is independent of the z-coordinate
e) The toner melt flow in the porous paper follows the modified Darcy's law

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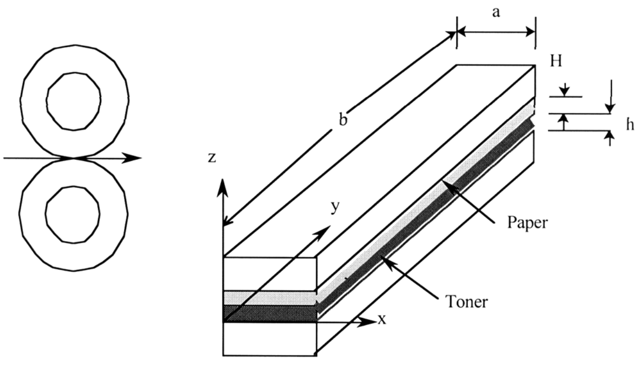

Figure 1. Sketch of a roll fuser and its fusing nip

f) The porous paper has uniform thickness and permeability.

Basic Equations. It can be shown that for the quasi-static case, the toner melt pressure in the film region satisfies the modified Reynolds equation. (see Appendix for derivation)

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{12\mu}{h^3}\left[\frac{dh}{dt} - \frac{\Phi}{\mu}\left(\frac{\partial \bar{p}}{\partial z}\right)_h\right] \tag{1}$$

where p = pressure, μ = viscosity, h = toner layer thickness, and Φ = permeability. Equation 1 can be obtained from integrating Navier–Stokes equations and satisfying the continuity equation. In the porous paper region, the pressure satisfies the Laplace equation (see Appendix for derivation).

$$\frac{\partial^2 \bar{p}}{\partial x^2} + \frac{\partial^2 \bar{p}}{\partial y^2} + \frac{\partial^2 \bar{p}}{\partial z^2} = 0 \tag{2}$$

At the edge or outside the roller, it is the ambient pressure. The ambient pressure is typically assumed to be equal to 0. For example, when two rollers are not engaged, there is no pressure between them except the ambient pressure. The boundary conditions are, therefore

$$p(0,y) = p(a,y) = p(x,0) = p(x,b) = 0 \tag{3}$$

$$\bar{p}(0,y,z) = \bar{p}(a,y,z) = \bar{p}(x,0,z) = \bar{p}(x,b,z) = 0 \tag{4}$$

$$\left(\frac{\partial \bar{p}}{\partial z}\right)_{h+H} = 0 \tag{5}$$

Because the pressure must be continuous at the paper—toner melt interface, we have an additional matching condition

$$p(x,y) = \bar{p}(x,y,h) \tag{6}$$

The problem is to solve Eqs. 1 and 2, and satisfying boundary conditions of Eqs. 3 to 6.

Solution. Equation 2 can be solved by separation of variables,[1] which gives

$$\bar{p}(x,y,z) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\sin(\alpha_m x)\sin(\beta_n y) \bullet e^{\gamma_{mn} z}\left[1 + e^{2\gamma_{mn}(h+H-z)}\right] \tag{7}$$

where $\alpha_m = m\pi/a$, $\beta_n = n\pi/b$, and $\gamma_{mn} = (m^2+k^2n^2)^{0.5}\pi/a$.

In solving Eq. 1, $p(x,y)$ is expressed by an infinite series[2] consisting of a complete set of orthogonal functions, each of which satisfies the boundary conditions given in Eq. 3,

$$p(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}\sin(\alpha_m x)\sin(\beta_n y) \tag{8}$$

The constant B_{mn} can be determined from the matching Eq. 6, and the result is

$$B_{mn} = A_{mn} e^{\gamma_{mn} h}(1 + e^{2\gamma_{mn} H}) \tag{9}$$

The pressure p is a sinusoidal function of y. It is symmetric to y but not independent of y. Substituting Eqs. 7 and 8 into Eq. 1, and rearranging it, one obtains

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} C_{mn} e^{\gamma_{mn} h}\sin(\alpha_m x)\sin(\beta_n y) = -\frac{12\mu}{h^3}\frac{dh}{dt} \tag{10}$$

with

$$C_{mn} = (\alpha_m{}^2 + \beta_n{}^2)(e^{2\gamma_{mn} H} + 1) + \frac{12\Phi}{h^3}\gamma_{mn}(e^{2\gamma_{mn} H} - 1) \tag{11}$$

The constant A_{mn} can then be determined by using the orthogonality of the eigenfunctions $\sin(\alpha_m x)$ and $\sin(\beta_n y)$. Multiplying Eq. 10 with $\sin(\alpha_m x)\sin(\beta_n y)$ and integrating from 0 to a for x and 0 to b for y, respectively, one obtains

$$\begin{aligned} A_{mn} &= -\frac{192\mu dh/dt}{C_{mn} mn\pi^2 h^3 e^{\gamma_{mn} h}} && \text{m, n odd} \\ A_{mn} &= 0 && \text{m, n even} \end{aligned} \tag{12}$$

Substituting Eq. 12 into Eq. 7, one has the 3D pressure distribution in the porous paper region. Substituting Eqs. 9 and 12 into Eq. 8, we have the pressure distribution in the toner melt region.

The loading at the nip is the integration of the pressure over the nip area

$$\begin{aligned} W &= \int\int p(x,y)dxdy = p_{avg}ab \\ &= -\frac{768\mu ab(dh/dt)}{\pi^4 h^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1 + e^{2\gamma_{mn} H}}{m^2 n^2 C_{mn}} \end{aligned} \tag{13}$$

Equation 13 can be used to estimate toner layer thickness change rate dh/dt, and relationship between toner layer thickness h and time t. The toner layer thickness and time relation as it passes through the nip can be obtained by integrating Eq. 13. If the initial toner layer

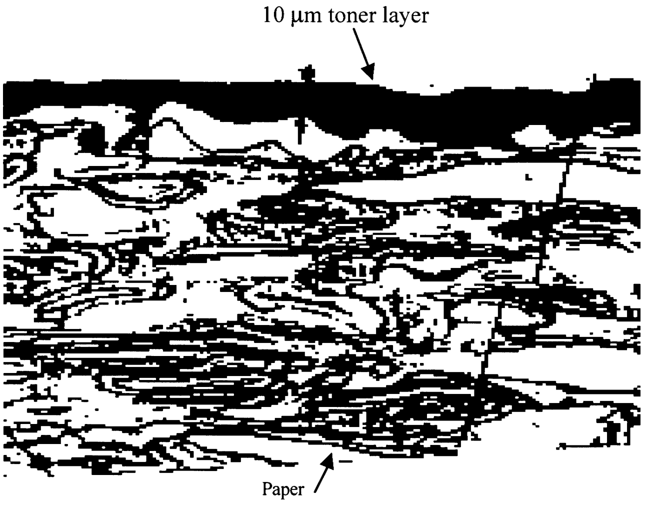

Figure 2. Cross-sectional view of MICR toner images fused on 4024 paper, 500X.

thickness entering the nip is h_o at $t = 0$, the time required for it to reach a new value h is

$$t=-\frac{768\mu a^3 b}{W\pi^6}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{m^2n^2(m^2+k^2n^2)D_{mn}^{2}}\left[\frac{1}{6}\ln\frac{(D_{mn}+h)^2}{D_{mn}^{2}-D_{mn}h+h^2}+\frac{1}{\sqrt{3}}\tan^{-1}\frac{2h-D_{mn}}{\sqrt{3}D_{mn}}\right]_{ho}^{h} \qquad (14)$$

where $D_{mn}^3 = 12\Phi a(e^{2\gamma_{mnH}}-1)/\pi(m^2+k^2n^2)^{0.5}(e^{2\gamma_{mnH}}+1)$.

Toner Penetration into the Paper. The velocity component in the z-direction is

$$\frac{dz}{dt}=\bar{w}=\frac{dh}{dt}-\frac{\Phi}{\mu}\frac{\partial\bar{p}}{\partial z} \qquad (15)$$

where $dt = dx/U$ and U is the process speed. The pressure gradient can be obtained from differentiating Eq. 7. The depth of toner penetration into the paper can be obtained from the integration of Eq. 15. This is different to the conventional penetration calculation by simply employing Darcy's law for flow, i.e.

$$\frac{dz}{dt}=-\frac{\Phi}{\mu}\frac{\partial\bar{p}}{\partial z} \qquad (16)$$

Results of Toner Penetration. Figure 2 (courtesy of John Berkes[3]) shows a cross-sectional view of fused toner image on a 24# Xerox 4024 DP paper taken from a Xerox 4197 MICR printer. The micrograph has a magnification of 500X. It shows that the toner has practically no penetration into the paper but does fill the voids on the paper surface.

We will estimate the penetration of this toner layer into the paper at the roll-fusing nip. The permeability of the 24# 4024 DP paper is estimated to be 1.09E-8 mm^2 (or 1.69E-11 in^2). The other input data are: pressure = 1.02E6 pascal (or 148 psi), a = 3.048 mm, b = 370.84 mm, U = 50 mm/s, viscosity = 2.5 × 10^3 poises (at a temperature of 130°C), and initial toner layer thickness of 10 μm. The dwell time in the fusing nip is 0.06 s. Figure 3 shows modeling results of toner penetration as a function of dwell time with these input data. The toner penetration is in an order of 10^{-2} μm only. The penetration predicted by Darcy's law, Eq. 16, is much lower than the results estimated by this new theory, i.e. Eq. 15. The fuser roll surface temperature is typically about 200°C, but the toner temperature inside the nip is between 120°C and 140°C only. The toner viscosity inside the fusing nip, paper permeability and applied pressure are the key parameters for toner penetration.

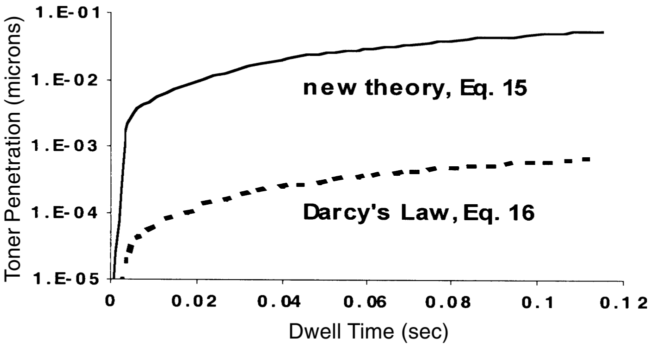

Figure 3. Toner penetration into 4024 plain paper μ = 2.5E3 poises, p = 1.02E6 pascal, Φ = 1.09E-8 mm^2, h_o = 10 μm

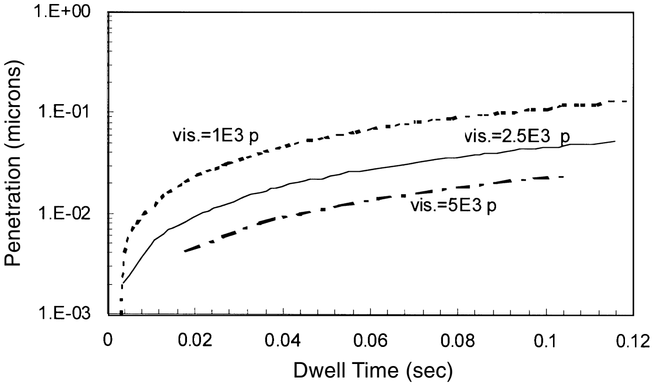

Figure 4. Toner penetration into 4024 plain paper with various toner viscosities, p = 1.02E6 pascal, Φ = 1.09E-8 mm^2, h_o = 10 μm.

The toner viscosity is a strong function of temperature. It decreases exponentially as the temperature increases. Figure 4 shows examples of toner penetration as a function of dwell time with three toner viscosities of 1 × 10^3, 2.5 × 10^3, and 5 × 10^3 poises, at temperatures of about 140, 130 and 120°C, respectively. Toner with lower viscosity, i.e. at higher temperature, has deeper penetration.

The toner penetration also changes with paper permeability. Figure 5 shows examples of toner penetration as a function of dwell time at three different paper permeabilities, i.e. Φ = 1.09E-9, 1.09E-8, and 1.09E-7 mm^2 (or 1.69E-12, 1.69E-11 and 1.69E-10 in^2 respectively). The toner penetration increases with dwell time and permeability. When the paper permeability is less than 1.09E-8 mm^2 (or 1.69E-11 in^2), the toner penetration does not change further for this case. In fact, the curves for the paper permeability at 1.69E-11 in^2 and 1.69E-12 in^2 are identical.

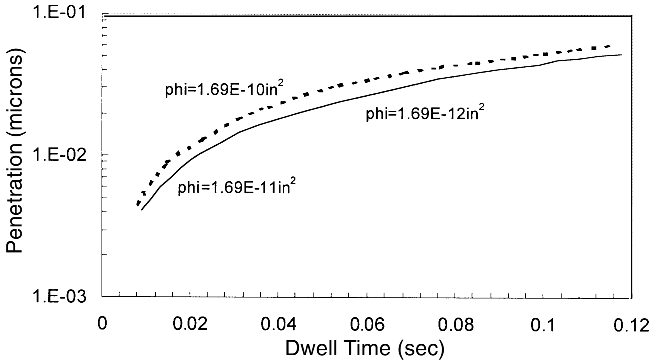

Figure 5. Toner penetration into paper with various paper permeability, μ = 2.5E3 poises, p = 1.02E6 pascal, h_o = 10 μm.

The toner penetration also increases with applied pressure. Figure 6 shows examples of toner penetration with applied pressure, p, at 0.51E6, 1.02E6 and 2.04E6 pascal (or 74, 148 and 296 psi, respectively).

As the original toner layer thickness changes, the toner penetration is also expected to be different. Figure 7 shows toner penetration curves with three different toner layer thickness.

Summary

A model using paper permeability as the macroscopic parameter is introduced to compute the toner penetration into the paper and their thickness variation in the fusing nip. The viscosity of toner, applied pressure, and toner layer thickness dominate the toner penetration into the paper. The toner melt viscosity varies inversely with the exponent of temperature.

The penetration of the MICR toner into the Xerox 4024 DP paper is in an order of 10^{-2}μm. There is practically no toner penetration into the paper. The toner images just fill the voids at the top surface of the paper. The thickness of the fuser oil in the dry powder fusing is about 3×10^{-2} μm. It doesn't have significant amount of oil for deep penetration. The filling of the voids at the paper surface can be studied microscopically also if we have the details of the paper surface topography. The surface and structure of the paper is very non-uniform which makes the microscopic study un-realistic.

The penetration of the toner into the paper increases with lower toner viscosity, higher paper permeability, higher applied pressure, longer dwell time and thicker toner layer. Modeling results compare favorably with cross-sectional view of actual images at least qualitatively.

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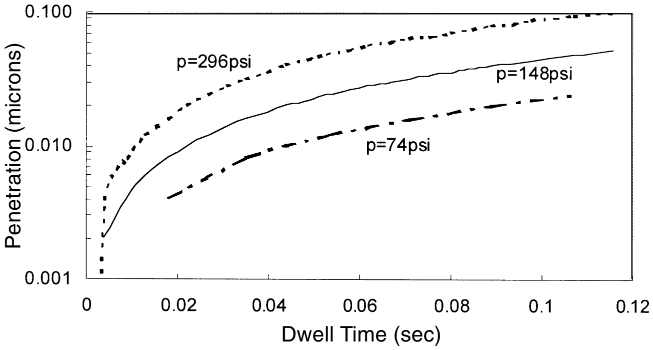

Figure 6. Toner penetration into 4024 plain paper at various pressure, viscosity=2.5E3 poises, Φ = 1.09E-8 mm^2, h_o= 10 μm.

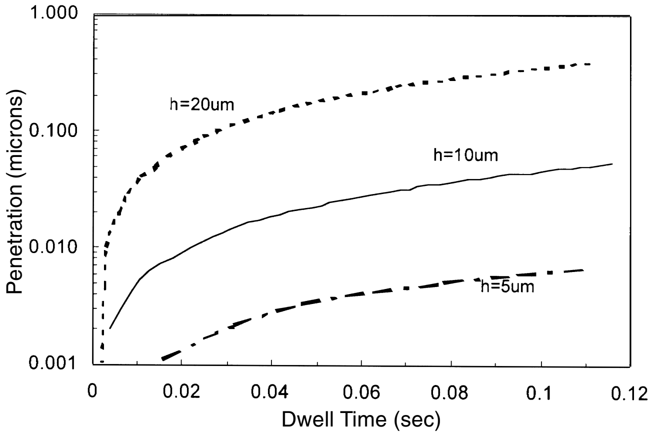

Figure 7. Toner penetration into 4024 plain paper with various toner layer thickness, μ = 2.5E3 poises, p = 1.02E6 pascal, Φ = 1.09E-8 mm^2

Appendix: Derivation of Eqs. 1 and 2

On the basis of the preceding assumptions, for the quasistatic case, the Navier–Stikes equations become

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu}\frac{\partial p}{\partial x} \tag{A1}$$

$$\frac{\partial^2 v}{\partial z^2} = \frac{1}{\mu}\frac{\partial p}{\partial y} \tag{A2}$$

where u = velocity in the x-direction, v = velocity in the y-direction, p = pressure and μ = viscosity. Integrating Eqs. (A1) & (A2) twice with respect to z and applying the non-slip boundary condition on both surfaces yields

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}(z^2 - hz) + U \tag{A3}$$

$$v = \frac{1}{2\mu}\frac{\partial p}{\partial y}(z^2 - hz) \tag{A4}$$

where U = paper surface velocity or process speed. The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \text{(A5)}$$

where w = velocity in the z-direction. Substituting Eqs. (A3) and (A4) into (A5) and integrating across the toner layer thickness h, one obtains

$$w)_h = \frac{h^3}{12\mu}\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right) \qquad \text{(A6)}$$

at $z = h$.

Because it is assumed that the flow in the porous media follows Darcy's law,[4] one has

$$\bar{u} = -\frac{\Phi}{\mu}\frac{\partial \bar{p}}{\partial x} \qquad \text{(A7)}$$

$$\bar{v} = -\frac{\Phi}{\mu}\frac{\partial \bar{p}}{\partial y} \qquad \text{(A8)}$$

$$\bar{w} - \frac{dh}{dt} = -\frac{\Phi}{\mu}\frac{\partial \bar{p}}{\partial z} \qquad \text{(A9)}$$

Similar to Eq. (A5), the continuity equation in the porous region is

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 \qquad \text{(A10)}$$

Substituting Eqs. (A7) through (A9) into Eq. (A10), one obtains the Laplace equation for pressure in the porous region

$$\frac{\partial^2 \bar{p}}{\partial x^2} + \frac{\partial^2 \bar{p}}{\partial y^2} + \frac{\partial^2 \bar{p}}{\partial z^2} = 0 \qquad \text{(A11)}$$

Because the velocity component in the z-direction must be continuous at the paper-toner interface, we have

$$w)_h = \bar{w})_h = \frac{dh}{dt} - \frac{\Phi}{\mu}\frac{\partial \bar{p}}{\partial z})_h \qquad \text{(A12)}$$

Substituting Eq. (A12) into Eq. (A6), one obtains the modified Reynolds equation for this type of squeeze film

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{12\mu}{h^3}\left[\frac{dh}{dt} - \frac{\Phi}{\mu}\left(\frac{\partial \bar{p}}{\partial z}\right)_h\right] \qquad \text{(A13)}$$