Kubelka–Munk Theory and the Yule–Nielsen Effect on Halftones

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This report experimentally relates the Yule–Nielsen n parameter for optical dot gain to the scattering and absorption parameters, S and K, of Kubelka–Munk theory. The relationship between the parameters is made through another metric of light scatter, k_p, defined as the inverse of the frequency, ω in cy/mm, at which the MTF of the paper equals 1/2. The value of n is related exponentially to k_p, as shown in earlier work, and the current work indicates that $k_p = c/S$, where c is a constant equal approximately to 10. However, contrary to some intuition and to previous theoretical projections, the absorption coefficient of the paper, K, has no significant influence on k_p or the MTF of paper.

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Introduction

The overall reflectance, R, of a halftone image is described by the Murray–Davies Eq. 1 in which R_i and R_p are the reflectance values of the ink and the paper when the halftone is at an ink area fraction, F, between 0 and 1.[1]

$$R = F \bullet R_i + (1 - F) \bullet R_p \tag{1}$$

Equation 1 appears to indicate a linear relationship between R and F, but the reflectance values, R_i and R_p, are also functions of F.[2,3] This occurs because light scatters laterally inside the paper substrate, increasing the probability it will encounter ink and be absorbed. However, it is common practice to assume $R_i = R_b$ and $R_p = R_g$, where R_b and R_g are the reflectance values of the ink at $F = 1$ (black) and the reflectance of the paper at $F = 0$ (ground). When these approximations are used in Eq. 1, the calculated values of R are generally higher than the actual measured values. In order to compensate, an empirical n parameter can be used.[1,2,3]

$$R = \left[F \cdot R_b^{1/n} + (1-F) \cdot R_g^{1/n}\right]^n \tag{2}$$

Equation 2, the Yule–Nielsen equation, has been applied successfully to model tone and color reproduction in many halftone systems. If light does not scatter laterally in the paper to any significant extent, then $n = 1$, $R_i = R_b$, $R_p = R_g$, and Eq. 2 reduces to Eq. 1. If light scatters thoroughly in the paper and becomes completely scrambled over distances much larger than the dot size, then $n = 2$.[2–4] Thus, the n factor can be used as an index of the effect of light scattering on the halftone image. The purpose of this article is to examine how n is related to other indices of light scatter in printing substrates such as paper.

Which Dot Fraction? The effect of lateral light scatter on halftone images is called the Yule–Nielsen effect, but the term "optical dot gain" has also been used to characterize the phenomenon. If we assume $n = 1$ but increases the values of F in Eq. 2 to a larger value, called F', then the value of R we calculate can be made to fit the experimental value of R. In other words, it is assumed the ink dots behave as if they were larger than they really are, and an index called "optical dot gain" is defined as $G_o = F' - F$. Plots of G_o versus F are commonly reported in the literature. The G_o metric was developed by analogy with the "physical dot gain" metric, $G_p = F - F_c$, where F_c is the dot area fraction commanded by the printing process (screen, digital signal, etc.) The G_p metric actually measures the difference between the intended dot size and the size of the dot that is actually formed on the paper. By using F_c in Eq. 2, the empirical n parameter can still model tone and color reproduction in many halftone systems, but n values greater than 2 are often observed. The focus of the current work is on Yule–Nielsen effect of light scatter, and F values reported below are those measured by microdensitometry analysis of the printed images, as described perviously.[1,5]

Relating *n* to Other Scattering Metrics. Two theories commonly applied to describe the optical behavior of paper and other printing substrates are Kubelka–Munk theory[6] and Linear Systems theory. The former

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describes reflectance and transmittance properties of scattering materials in terms of a scattering coefficient, S, and an absorption coefficient, K. Linear Systems theory uses a point spread function, PSF(r), that describes the probability of light emerging from a paper at distance r from the point at which it entered the paper.[7,8] The PSF is often described in the Fourier domain as the Modulation Transfer Function, MTF(ω), where ω is spatial frequency in units of $1/r$. Intuition suggests that the Yule–Nielsen parameter, n, must be related both to the MTF of the substrate and also the S and K parameters of Kubelka–Munk theory.

The MTF is often modeled empirically as follows.

$$\mathrm{MTF} = \frac{1}{1+(k_p \cdot \omega)^m} \tag{3}$$

In this function, ω is spatial frequency in cy/mm and k_p is a constant proportional to the mean distance light scatters in paper between entering and exiting as reflected light. The m parameter is an empirical parameter used to adjust the shape of the function to fit experimental data. The value of k_p is equal to $1/\omega_o$, where ω_o is the value of ω where MTF = 1/2. Thus, regardless of the value of m, k_p is a definable, experimental metric for lateral scattering in the paper.

The k_p metric and the n metric are clearly related, but attempts to derive n quantitatively from application of linear systems theory leads only to approximations because n is really an empirical parameter used to fit an empirical model to experimental data. However, within typical limits of experimental error, it has been shown experimentally that n can be related to k_p through the following expression.[10]

$$n \cong 2 - \exp\left\{-(A \cdot k_p \cdot \upsilon)\right\} \tag{4}$$

The value of the constant A is dependent on the particular geometric pattern of the halftone.[10] For an AM clustered dot halftone, ν is the frequency of occurrence of halftone dots. For an FM halftone, ν is the inverse of the distance, L, between the centers of adjacent dots in the halftone. Note that ν is in mm^{-1} for both AM and FM systems, and k_p is in mm.

Equation 5 has been derived by Engeldrum and Pridham[11] from Kubelka–Munk theory to relate the MTF of a material to the S and K constants.

$$MTF(\omega) = \frac{1}{-\ln(1-R_\infty^2)} \cdot \sum_{j=1}^{\infty}\left[\frac{R_\infty^{2j}}{j} \cdot \left[1+\left(\frac{2\pi\omega}{2jbS}\right)^2\right]^{-3/2}\right] \tag{5}$$

In this function $a = 1 + K/S$, $b = (a^2 - 1)^{1/2}$, and $R_\infty = a - b$. If Eq. 5 is correct, then one need only measure K and S of the printed substrate to calculate MTF versus ω. Then the value of $\omega = \omega_o$ at which MTF = 1/2 should provide the value $k_p = 1/\omega_o$. The value of k_p can then be applied to Eq. 4 to calculate the Yule–Nielsen n factor.

Equation 5 indicates that the absorption coefficient K should have a significant impact on k_p, and in turn, on the Yule–Nielsen n factor. In addition, because K is strongly dependent on the wavelength of light, λ, then Eq. 5 also predicts a strong influence of k_p and n on λ. However, as will be shown, the impact of K on k_p is not significant, and n is not strongly dependent on λ.

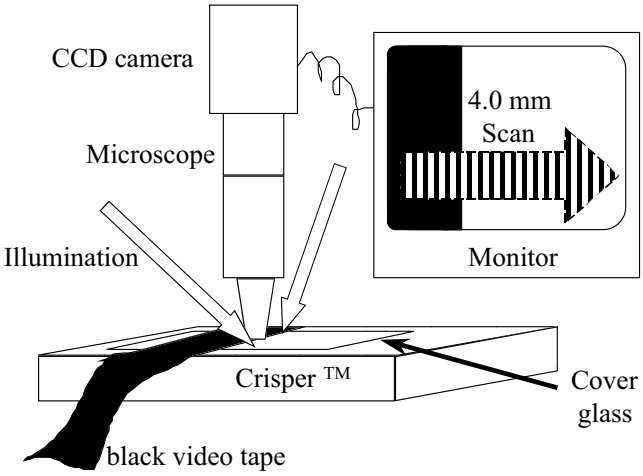

Figure 1. Microdensitometer.

Measuring MTF

A sample of a commercial plastic substrate called Crisper™ (Toyobo Co., Ltd.) was placed under a reflection microscope with a CCD video camera as shown in Fig. 1. The camera was attached to a frame grabber, and digital pixel values in this system were linearly related to sample luminance. This system was used as a reflection microdensitometer, and the MTF of the instrument was measured as 0.5 at 40 cy/mm. A piece of black video tape, 20 μm thick, was placed on top of the Crisper™ sample with the edge of the tape positioned to run through the center of the field of view of the microdensitometer. The tape was clamped in place with a microscope cover glass, and the tape edge was illuminated with two fiber optic bundles 45 degrees from the vertical on each side, and in directions collinear with the tape edge. The image of the edge was captured, as illustrated in Fig. 1, and software was used to scan a section of the edge. With the edge in the vertical direction as shown, the scan was done in the horizontal direction across 4 mm of the sample. The scan was achieved by averaging pixel values in a vertical column of 100 pixels (0.8 mm) to produce an average pixel value at each horizontal pixel position in the scan. The scanned edge of the plastic substrate is shown in Fig. 2 as pixel value, P, versus position, x, in millimeters.

The MTF was calculated from the edge scan by Fourier analysis. The P versus x data from Fig. 2 was differences point-to-point, $dP = (P_{i+1} - P_i)$, to calculate the derivative of the edge as shown in Fig. 3. The pixels under the tape edge could not be measured experimentally and were assumed to have the same distribution as the pixel values on the measured side of the edge. Thus, the derivative at $x < 0$ (under the edge) was assumed to be the same as $x > 0$ where the data was taken. A Fourier transform algorithm was applied to the data, and the MTF of the reference material was calculated as the modulus of the Fourier transform, normalized to 1.00 at zero frequency, as shown by the data points in Fig. 4. The data was fit by Eq. 3 with m = 1.2 and k_p = 0.14 millimeters.

Measuring the Yule–Nielsen Effect

The k_p of other printing substrates might be found through a similar line scan analysis. However, many printing substrates have rough surfaces and are not easily measured by this technique. The rough surface

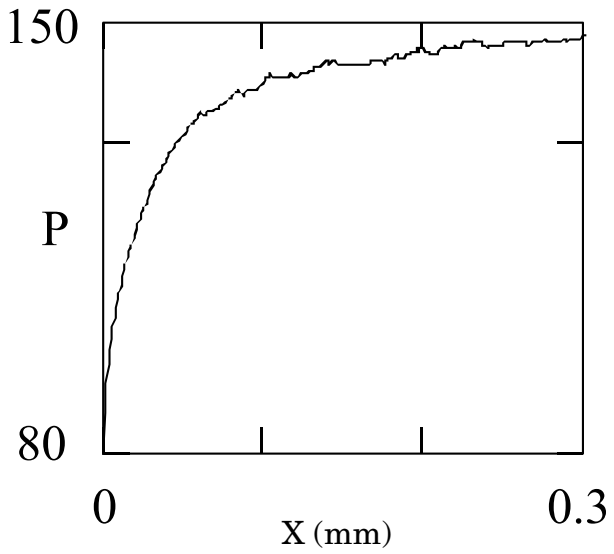

Figure 2. Edge scan of Crisper™ as pixel value, P, versus position in millimeters. X = 0 at the edge.

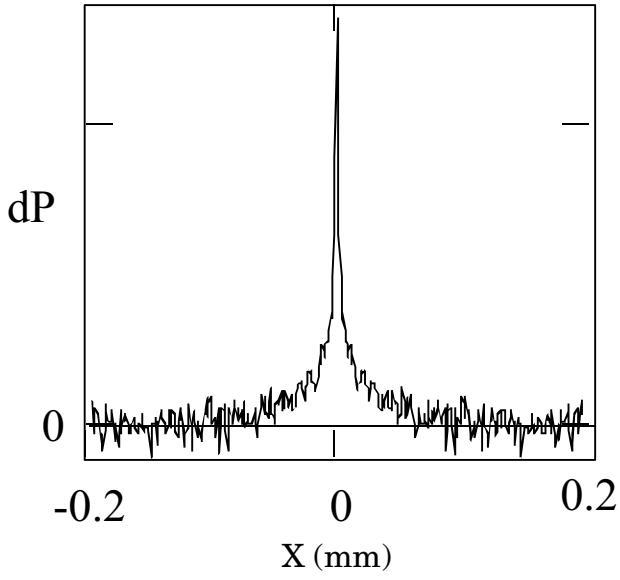

Figure 3. Derivative of edge scan in Fig. 2.

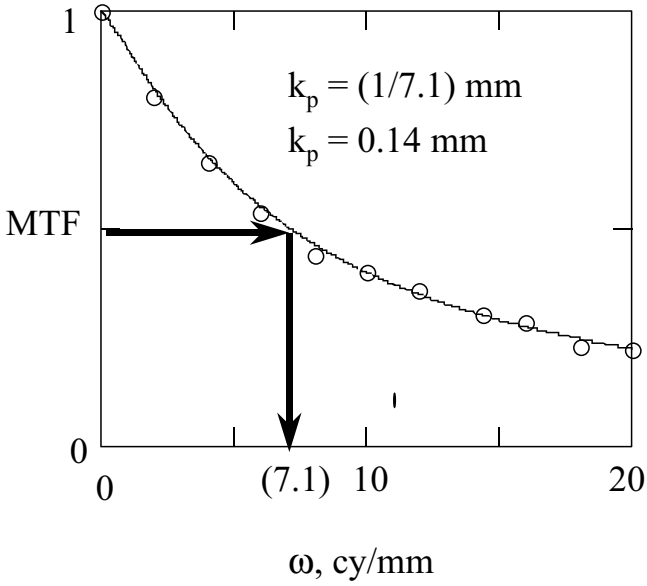

Figure 4. Crisper™ standard. The dots (O) were calculated from the edge scan, and the line is Eq. 3 with k_p = 0.14 mm and m = 1.2

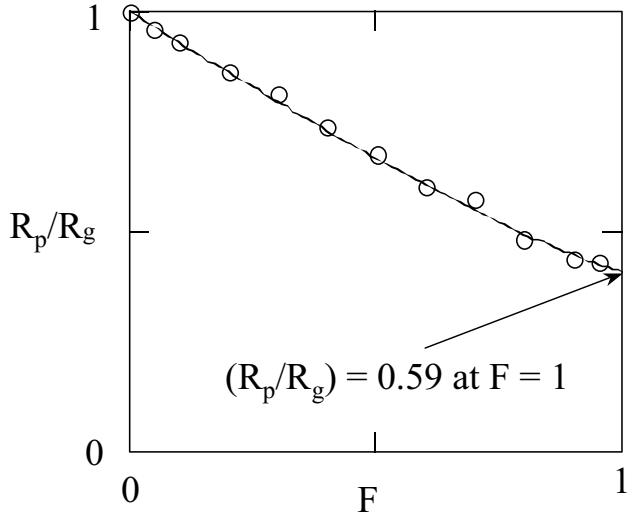

Figure 5. Crisper™ R_p versus F.

adds significant noise to the edge scan, and many scans must be averaged to estimate the average k_p of the sample. An alternative approach is to place a halftone image on the paper and measure the average Yule–Nielsen effect of the substrate and relate the effect to the value of k_p. In the current work, the Yule–Nielsen effect was measured by placing a graphic arts "lith" film with a stochastic halftone wedge (F = 0 to F = 1) in vacuum contact with the substrate, emulsion to paper. The stochastic halftone was generated by a proprietary error diffusion process and printed on a commercial image-setter. The halftone dot pattern was generated as a grid of square dots and spaces of L = 0.042 mm on each side. Figure 5 shows R_p versus F measured from a histogram analysis of this halftone using microdensitometric analysis techniques described perviously.[1,5]

The R_p versus F function for a stochastic halftone has been shown to be well modeled with Eq. 6 when R_i is zero,[10] and the value of R_i was zero within experimental error in the current work.

$$R_p / R_g = 1 - w \cdot \left[1 - (1-F)^B\right] \tag{6}$$

A power factor of B = 1.2 was found to fit measurements made with the halftone film in contact with all paper and synthetic printing substrates used in this study. The value of w is the value of $(1 - R_p/R_g)$ extrapolated to F = 1.

The value of w has been shown experimentally to be related to k_p by Eq. 7.[10,12]

$$w = 1 - \exp\left\{\frac{-A \cdot k_p}{L}\right\} \tag{7a}$$

or

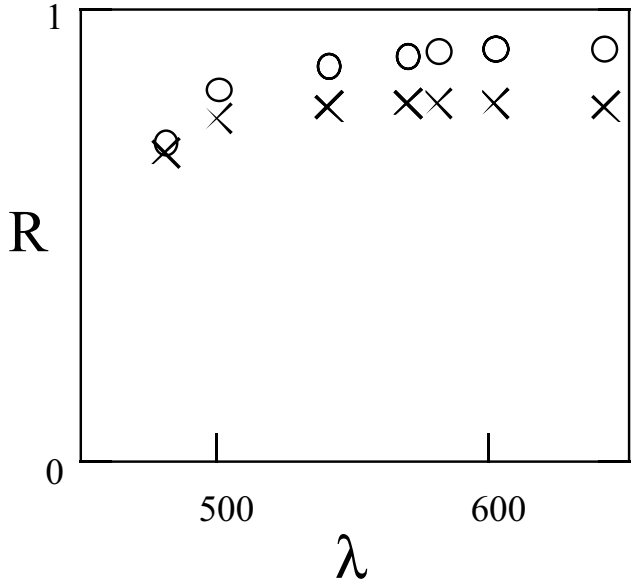

Figure 6. Reflection spectrum of yellow plain paper at infinite thickness, $R\infty$ (O), and over black, R_b (X). Paper is 0.072 mm thick.

$$k_p = \frac{-L \cdot \ln(1-w)}{A} \tag{7b}$$

The value of L = 0.042 mm for this stochastic halftone, k_p = 0.14 mm and w = 0.59 for the Crisper™ reference material. Thus, the value of A = 0.267 can be calculated.

The physical justification for Eq. 7 is based on the probability model of halftone optics discussed previously.[8,13] For R_i = 0, $P_a = (1 - w)$ where P_a is defined as the fraction of light which emerges from a single square area of the paper of dimension L after entering that same area and scattering within the paper. The value of P_a must be unity for a paper of zero scattering length, k_p = 0, with w = 0. But as the scattering length, k_p, increases, P_a decreases and w increases. As k_p approaches infinity, w must approach unity and P_a must approach zero asymptotically. Equation 7 has the required limit of w = 0 at k_p = 0 and w = 1 as $k_p \rightarrow \infty$ and thus is a reasonable empirical model for w versus k_p.

The values of L = 0.042 mm and A = 0.267 were used with this halftone film to measure R_p versus F for all substrates in this project. Equation 7 was applied to calculate the corresponding value of the MTF constant k_p. By using 20 nm band pass filters, k_p values versus λ were measured.

***K* and *S* for Yellow Plain Paper**. K and S values for several papers and synthetic substrates were determined by measuring the reflectance, R_∞, of a thick stack of the substrate and then measuring R_b, the reflectance of a single sheet of the substrate with a black background. The thickness of the substrate was also measured, and the values of K and S were calculated by a numerical solution of the Kubelka–Munk equations for $R\infty$ and R_b.[6] As a check of the experimental method, an integrating sphere spectrophotometer was used to measure the total transmitted light through the samples, T, and values of K and S were calculated from the Kubelia–Munk equations for R_∞ and T. The two methods gave the same K and S values within experimental error. T data lead to lower experimental error for highly absorbing samples.

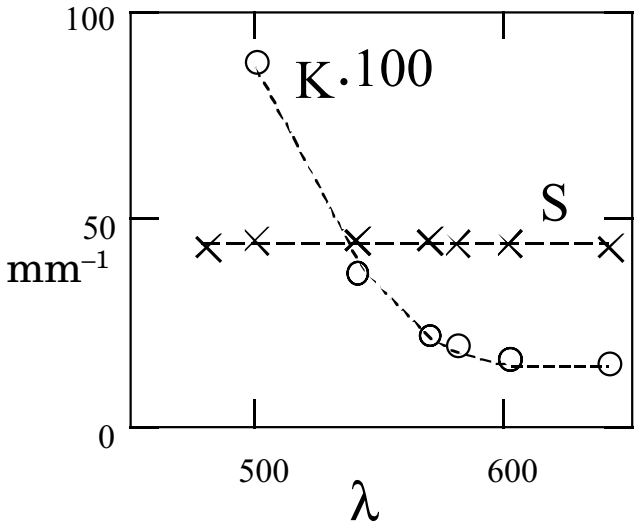

Figure 7. Values of S in mm^{-1} and $100{\cdot}K$ in mm^{-1} from data in Fig. 6.

TABLE I. Spectral Data on Yellow Plain Paper

λ, nm	S in mm^{-1}	K in mm^{-1}	k_p, in mm, measured from R_p versus F	$1/\omega_o$, in mm, from Eq. 5
480	43	2.66	0.23	0.22
500	45	0.89	0.24	0.32
540	45	0.38	0.23	0.44
568	45	0.23	0.28	0.53
580	45	0.20	0.24	0.56
600	44	0.17	0.26	0.60
640	43	0.26	0.29	0.61

Figure 6 shows R_∞ and R_b for a commercially manufactured sample of yellow plain paper. The corresponding values calculated for of K and S are shown in Fig. 7. Note the value of S does not change significantly, but the value of K does. Equation 5 was applied to each pair of K and S values to calculate the MTF versus ω function. The value of $\omega = \omega_o$ at which MTF = 0.5 was noted. Table I summarizes the values of K, S, k_p, and $1/\omega_o$. If Eq. 5 is correct, then $1/\omega_o$ should be a good correlate of k_p. However, as shown in Fig. 8, the change in K over the range of this experiment does not result in the change in k_p predicted by $1/\omega_o$ from Eq. 5.

***K* and *S* for Coated Paper and Crisper™**. An experimental correlation between S and k_p has previously been shown.[14] In the current work five substrates, shown in Table II, were examined at two wavelengths, 500 nm and 580 nm, using 20 nm band pass filters. Values of S, K, and k_p were measured as described above, and the results are summarized in Table II. These samples show only small differences in K but significant differences in S. Figure 9 shows the relationship between k_p and $1/S$. For a negligible absorption coefficient, then, this data indicates Eq. 8 is an adequate model of k_p versus S within experimental error, with a unitless constant of c = 10.

$$k_p = c/S \tag{8}$$

In order to determine the impact of higher values of K on k_p, samples of the Crisper™ standard were dyed by boiling in a proprietary cyan dye solution. The dye was meant for use with this type of sample and produced darker cyan color as a function of boiling time. Samples

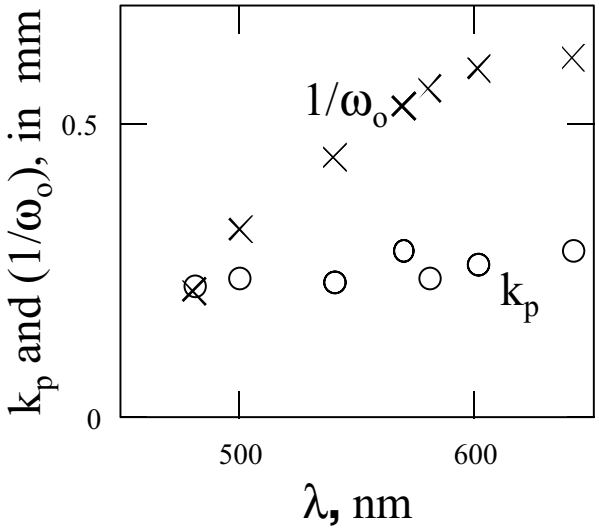

Figure 8. k_p and $1/\omega_o$ versus wavelength.

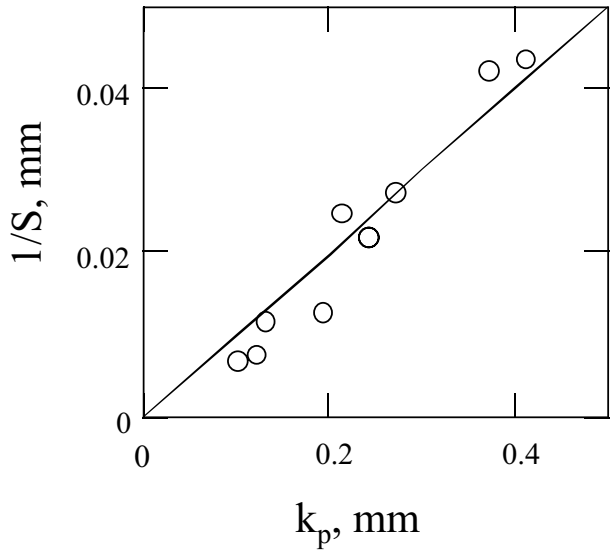

Figure 9. $1/S$ versus k_p from Table II.

of the coated paper were also dyed by soaking in an aqueous solution of a proprietary cyan ink jet dye. Again, dying time governed the darkness of the color. Figure 10 illustrates the reflection spectra for the coated paper samples.

Both the dyed and not dyed samples of both substrates were analyzed for K and S values as before. S was found not to change, but K increased with the dying time, as expected. Measurements of k_p were made at 500 nm where R_∞ and K changed only slightly, and at 580 nm where a significant changes in R_∞ and K were observed. The results are summarized in Table III. Figure 11 shows the K as a function of k_p.

Discussion

Figure 11 does not indicate any significant impact of the Kubelka–Munk absorption coefficient K on the Yule–Nielsen effect, represented by the k_p metric. Intuition might suggest that a high value of K would decrease the average distance that light scatters before emerg-

TABLE II. Scattering Properties of Different Substrates

Material	λ, nm	S, mm^{-1}	K, mm^{-1}	kp, mm
coated paper	500	39.8	0.51	0.21
coated paper	580	36.5	0.70	0.27
Crisper™	500	85.7	0.26	0.13
Crisper™	580	77.8	0.17	0.19
HS plastic	500	143.0	0.23	0.10
HS plastic	580	130.0	0.20	0.12
LS plastic	500	23.6	0.05	0.37
LS plastic	580	22.8	0.05	0.41
plain paper	500	45	0.89	0.24
plain paper	580	45	0.20	0.24

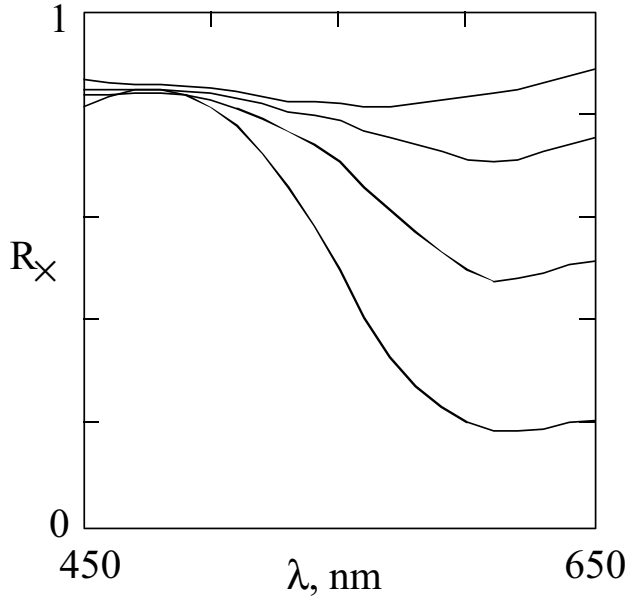

Figure 10. Spectra of coated paper at three dye levels.

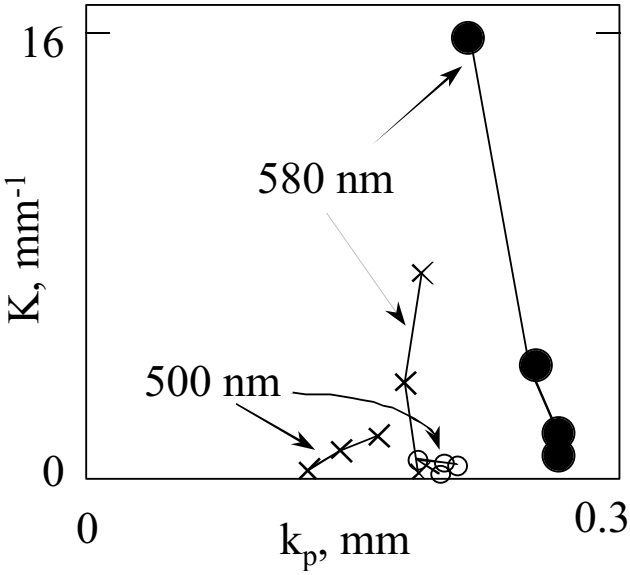

Figure 11. Dye concentration series, K versus k_p for: Coated paper at 500 nm (O); Coated paper at 580 nm (●); and Crisper™ at both at 500 and 580 nm, (X).

TABLE III. The Effect of Cyan Dye on the Properties of Substrates

Paper/dye	K in mm at 500 nm	k_p in mm at 500 nm	R_∞ at 500 nm	K in mm at 580 nm	k_p in mm at 580 nm	$R\infty$ at 580 nm
P.Paper_0	0.51	0.21	0.830	0.70	0.27	0.823
P.Paper_1	0.56	0.20	0.853	1.41	0.27	0.744
P.Paper_2	0.65	0.19	0.842	3.80	0.25	0.574
P.Paper_3	0.16	0.20	0.736	15.10	0.21	0.278
Crisper_0	0.26	0.13	0.925	0.17	0.19	0.935
Crisper_1	0.95	0.14	0.854	3.25	0.18	0.704
Crisper_2	1.39	0.16	0.834	6.93	0.19	0.600

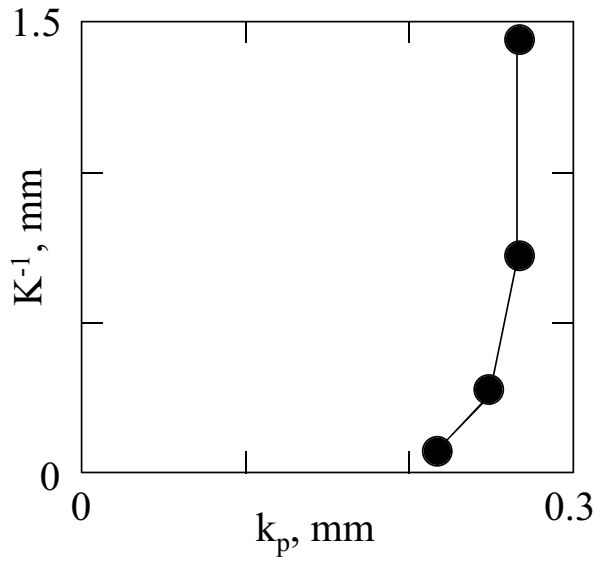

Figure 12. Coated Paper at 580 nm.

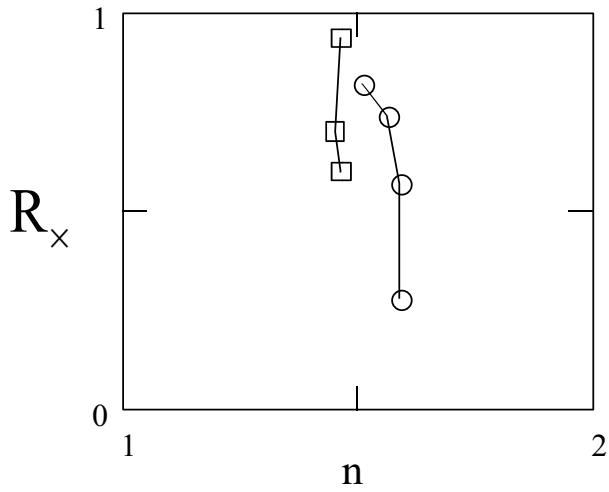

Figure 13. Coated paper and Crisper™ at both at 580 nm. Calculated from the data shown in Fig. 11 with A = 0.267 and a dot size of 80 μm.

ing from the paper substrate. Thus, one might expect a decrease in k_p as K increases. The only data that might suggest such behavior is the dyed coated paper at 580 nm in Fig. 11. This data was re-plotted as K^{-1} versus k_p as shown in Fig. 12, and there is no convincing evidence for the kind of linear trend observed between k_p and S^{-1}.

It is not possible to reach the conclusion that K has no impact on the Yule–Nielsen effect. However, it is possible to reach practical conclusions. Figure 13 shows the values of R_∞ versus n for the dyed samples at 580 nm, where n is calculated from the k_p values in Table III by using Eq. 4. Over this range of R_∞ there is no evidence of a correlation of any kind between K and k_p or n. Therefore, if K has an effect on k_p it is not of sufficient magnitude to be of any practical significance. Thus, Eq. 7 rather than Eq. 5, coupled with Eq. 4, provides a useful connection between Kubelka–Munk theory and the Yule–Nielsen effect.

A final caution about experimental accuracy in this work is in order. The measured values of k_p are internally consistent and are based on the same experimental technique. The experimental precision is indicated by the scatter in the data. However, the experimental accuracy is based on a single Fourier analysis of an edge scan of a reference material. Thus, while the trends and functional relationships are well represented, the calibration constants A and c may or may not be accurate. Moreover, the particular value of A may depend on the particular geometry of the halftone one uses. While it is easy to compare relative values of k_p, absolute values are much more difficult to measure with confidence.

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