Introducing Correction for Optical Dot Gain into Pollak's Equation: Application to Reflectance through R, G, and B Filters

Michitaka Nonaka*† and Masatsugu Isoda

Department of Image Information Engineering, Faculty of Engineering, Tokyo Institute of Polytechnics, Atsugi, Kanagawa, Japan

The Neugebauer equation is generally used to predict full color reproduction for printing. Pollak resolved the Neugebauer equation into factors, assuming the additivity of superimposed solid optical densities. Because of differences between predicted and actual results, various modifications of the formula have been suggested. Yule and Nielsen introduced the parameter n. We have now developed a quadratic equation with a factor corresponding the Yule-Nielsen n. By introducing our correction term to Pollak's equation, it is possible to correct approximately for optical dot gain in the case of halftone dots superimposed on paper.

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Introduction

Color images are reproduced using halftone dots in the printing process. Because various papers and inks are used, it is important to be able to accurately predict the resulting colors. High quality color images use a four-color process, with separate plates for cyan, magenta, yellow, and black halftone dots. A color printing press normally prints these colors in sequence on white paper.

To predict the color of the halftone reproduction, Neugebauer's equation[1] is generally used. This equation uses variables to describe the fractional dot areas of the three colors. We expressed Neugebauer's equation for four-color prints and included the reflectance of paper in it. The fractional dot area of cyan, magenta, yellow, black are described with the variables c, m, y, bk, respectively, and the reflectance of the paper is R_p. The variable R_{SC} denotes the reflectance measured with red light for the cyan solid print. R_{SCM} is the reflectance measured with red light for the magenta ink printed over cyan, and so on. The reflectance of the four-color combination print through the red filter is as follows.

$$\begin{aligned} R = R_p\, [&(1-c)(1-m)(1-y)(1-bk) + c(1-m)(1-y)(1-bk)R_{SC} + (1-c)m(1-y)(1-bk)R_{SM} \\ &+ (1-c)(1-m)y(1-bk)R_{SY} + (1-c)(1-m)(1-y)bkR_{SBK} + cm(1-y)(1-\mathrm{bk})R_{SCM} \\ &+ c(1-m)y(1-bk)R_{SCY} + c(1-m)(1-y)bkR_{SCBK} + (1-c)my(1-bk)R_{SMY} \\ &+ (1-c)m(1-y)bkR_{SMBK} + (1-c)(1-m)ybkR_{SYBK} + cmy(1-k)R_{SCMY} \\ &+ cm(1-y)bkR_{SCMBK} + c(1-m)ybkR_{SCYBK} + (1-c)mybkR_{SMYBK} + cmybkR_{SCMYBK}]. \end{aligned} \qquad (1)$$

There are two similar equations for the green and blue reflectance, where G and B, respectively, are substituted for R.

Pollak derived a modified form of Neugebauer's equation as the product of four terms, assuming that the solid ink densities were additive,[2,3] and that the reflectance of an ink combination was equal to the product of the reflectance of its components. Thus:

$$R = R_p(1-c+cR_{SC})(1-m+mR_{SM})(1-y+yR_{SY}) \cdot (1-bk+bkR_{SBK}). \qquad (2)$$

There are also two similar equations for the blue and green reflectance, where B and G are substituted for R.

Compared to Neugebauer's equation, we believe that Pollak's equation can more directly predict the color of halftone reproductions because of its simple form and its use of terms describing the paper and each ink film; each term is constructed in the form of the Murray-Davies[4] equation. However, the reflectance represented by each of the terms does not agree with the measured reflectance of a single color halftone print. Therefore, another correction is required. That is the correction for optical dot gain.

The original Murray-Davies equation represents the relation between a single color image and the ratio of the area occupied by the halftone dots. The density that is calculated with this equation does not perfectly predict the actual printing density. Therefore, it appears that the density difference between the value calculated from the Murray-Davies equation and the measured value is due to the behavior of the light absorbed in the paper by the halftone dots. The difference is called 'optical dot gain.' The optical dot gain reaches its maximum in areas with halftone and decreases with decreasing or increasing ratios of the area covered by printed dots versus white paper. Therefore, Yule and Nielsen introduced the coefficient n to improve the calculated predictions. Generally a quadratic equation can be fitted to the shape of the curve of the optical dot gain versus the fractional dot area. The coefficient k used in the quadratic equation can be determined from the height of the peak of the curve in the center of gradation.

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† IS&T Member

Modified Murray-Davies Equation Using the Quadratic Equation. The symbol a represents the ratio of the halftone dot area per unit area. This equation can be rewritten in terms of the reflected light in the form:

$$R = 1 - a + aR_S \quad (3)$$

Here we must add the corrective factor. The term $ka(1 - a)$. $f(a) = ka(1 - a)$ is a quadratic equation, and when $a = 0, 1$, then $f(a) = 0$. When $a = 0.5$, the value of the function reaches its maximum. The correction coefficient k is determined by the difference between the measured values and the calculated values of the Murray-Davies equation for the case where $a = 0.5$. The final approximation equation is as follows:

$$R = 1 - a + aR_S - ka(1 - a) \quad (4)$$

Yule and Nielsen introduced the coefficient n to improve the prediction of the Murray-Davies equation. Ruckdeschel and Hauser[5] analyzed the physical properties underlying Yule and Nielsen's n. They convoluted the point spread function for the substrate with the transmittance distribution of the halftone dots. That result was compared with the actual results using the Yule-Nielsen equation. Then n was calculated to equalize the results of the two formulas. They found that the range of values for n was $n = 1$ if the surface of the substrate was a perfect reflector (a mirror), and $n = 2$ if the surface of the substrate was a perfect light diffuser. They also reported that the n value is unique and independent where the saturation is less than 50%, but the value is not unique for saturation over 50%. They applied their theory to photography, so their approach is not directly applicable to normal printing because of the delay in the ink transfer from the plate to the substrate, and because of instability in the location of the plane of the printing process.

Nagayama, Yuasa and Mishina[6] reported on the influence of the shape of halftone dots made by a laser printer. Their work was based on a simplified version of the method of Ruckdeshel and Hauser.[5] In their paper it was reported that the correlation between the dot gain and the perimeter was very strong, but as the point spread became smaller the correlation became weaker.

Teraoka and Taguchi[7] reported that they derived the line spread function (LSF) of the paper from the MTF of a rectangle pattern of preproof and there was a close correlation between optical dot gain and the relative intensity of LSF at 50 µm distance from the LSF center. Inoue, Tsumura and Miyake[8] reported that the value n could be calculated by convolution based on the point spread function derived from the MTF of the paper for the transmittance of images. Their work was limited to the conditions where the fixed fractional dot area was 50% and the solid density was held constant.

In each of these studies, the convolution and point spread functions are used to predict the reflectance of halftone images and a relationship to Yule-Nielsen n is sought. In our current work, we have focused on how to derive accurate values of n without reference to theory. Considering the many variable factors in printing, we recommend a simple and understandable correction term derived using the quadratic equation, that in turn, uses the coefficient k to mathematically describe the color images reproduced by superimposing halftone dots on a substrate.

Arney and Katsube[10] treated the case of the halftone print made from a thermal transfer printer. They paid attention to the variation in the reflectance of the paper between the dots with varying fractional dot area. Accordingly they estimated the probability of a photon that enters the paper by passing through a halftone dot and returning under the dot, and the probability of a photon that enters the paper between the halftone dots also emerging under the dot, that are both functions of the fractional dot area. If these probabilities can be determined, the reflectance of a halftone dot and the paper could be expressed as functions of the fractional dot area. By substituting these functions into the Murray-Davies equation, the reflectance of a halftone print can be expressed.

Rogers[10] reported that when the point spread function of the paper is known, the reflectance of the halftone print is numerically expressed by convoluting the round dot halftone dot distribution pattern and the point spread function. He derived the photon transport differential equation, and solved the equation by introducing the boundary conditions of the paper surface. Accordingly, he got the point spread function of the paper, and he derived the reflectance based on photon transport probability in the paper. Though the correction term is not specifically expressed in his paper, we can derive the same shape correction as ours for correction of optical dot gain by transforming his three equations.

For

$$\overline{R} = \mu\overline{R}_i(\mu) + (1-\mu)\overline{R}_n \quad (5)$$

we substitute

$$\overline{R}_i(\mu) = R_P T_0[1-(1-T_0)\mu^{1-s}] \quad (6)$$

and

$$\overline{R}_n(\mu) = R_P\left[1-(1-T_0)\frac{\mu}{1-\mu}(a-\mu)^{1-s}\right] \quad (7)$$

becomes

$$\overline{R} = R_P\left[(1-\mu)+\mu T_0^2-(1-T_0)^2\mu(1-\mu^{1-s})\right]. \quad (8)$$

By rewriting μ as a and T_0^2 as R_S, Eq. 8 becomes

$$\overline{R} = R_P\left[(1-a)+aR_S-\left(1-\sqrt{R_S}\right)^2 a(1-a^{1-s})\right]. \quad (9)$$

The correction term for optical dot gain then exhibits the same functional shape as our proposed correction term $ka(1 - a)$ as shown in Eq. 4.

However in the case of $s = 0$, light is perfectly scattered in the paper. The factor $\left(1-\sqrt{R_s}\right)^2$ corresponds to our correction coefficient k. Rogers considered paper to be a homogeneous substance and explained the effect of optical dot gain theoretically. Though that is a significant theoretical achievement, our correction method should be adequate for practical use, given the variability during printing.

Accordingly we made a print sample and compared the three methods (Yule-Nielsen's n, Rogers' correction term, and our correction term) by simulation in

Microsoft Excel™. The result was that there was almost no difference between the three methods, and the values obtained by the three methods agree within the range of variability of the measured values. Therefore, we are confident that the most simple correction method, namely ours, is adequate to correct optical dot gain in practice. Now we show that the correction of optical dot gain in the case of superimposed halftone dots on paper is possible for the first time by introducing our correction term to Pollak's equation.

Experimental

Preparation of Printing Sample. To evaluate the accuracy of our equations, we made various print samples, using both single-color and four-color superimposed prints. We prepared the original square halftone dot positive films using a scanner. The screen ruling was 175 lpi. The screen angle was 15° for the cyan print, 45° for the magenta print, 90° for the yellow print, 75° for the black print. The original image was on Fujichrome PROVIA™, using only the gray scale section and color area.

The printing was done using a lithographic proof press. The pre-sensitized plates used were "FPP-B" made by Fuji Photo Film Co., Ltd. (Tokyo, Japan). The paper used was art paper "Tokubishi Art" made by Mitsubishi Paper Mills Co., Ltd. (Tokyo, Japan). The ream weight of the paper was 135kg per 1000 sheets, where each sheet was 788 mm by 1091 mm. The printing inks used were "TK For4 CIL" made by Toyo Ink MFG. Co., Ltd. (Tokyo, Japan). The order of printing was black, cyan, magenta and finally yellow.

Measurement of Light Density of Print and Fractional Dot Area. The optical density of the printing sample was measured with a Macbeth RD914 densitometer. When the reflectance value was required, the density value was converted by setting $R = 10^{-D}$. The fractional dot areas were measured using image processing software made by the Mitani Corporation, using a CCD camera incorporated with a microscope.

First we calibrated the dot pitch by inputting the image scale to the system. Next a frame size that contains an array of nine halftone dots (3 vertically and 3 horizontally) was determined. This required a separate step because the shapes of the halftone dots were not exactly the same. The image processing was then done using this frame size. The halftone dot images of the prints were magnified to 100× with the microscope and fed to the CCD camera. On display, the frame position was then manually set to the proper location. The color images of the halftone dots were then converted to the gray images in the frame. The brightness histogram of the gray image was used for converting the binary images using a threshold that we set by studying the brightness value distribution. There are two peaks in the histograms for the cyan, magenta, and black prints, and we used the minimum value between the peaks as the threshold value. Because the contrast of the yellow print was relatively low, there is only a single peak in the distribution of brightness values, so we used color separation to convert the binary image. By counting the number of elements corresponding to halftone dots and paper, the fractional dot areas were derived. We measured the fractional dot areas of the halftone dots, and because both the halftone dot images and the binary images were simultaneously visible on the Macintosh computer monitor, we were able to make sure that the shapes of both images corresponded accurately.

Results and Discussion

Optical dot gain expressed by reflectance is the differential reflectance between the value calculated by the Murray-Davies equation and measured reflectance. Measured optical reflectance is smaller or equal to the value calculated by the Murray-Davies equation. Now the differential reflectance corresponding to optical dot gain is

$$D.R. = (1 - a + aR_S) - \text{Measured Reflectance.} \quad (10)$$

Our correction term is

$$D.\ R. = ka(1 - a) \quad (11)$$

Yule-Nielsen's correction term is

$$D.R. = (1 - a + aR_S) - (1 - a + a10^{-D_s/n})^n. \quad (12)$$

Rogers' correction term is

$$D.R. = \left(1 - \sqrt{R_S}\right)^2 a(1 - a^{1-s}). \quad (13)$$

Using both the measured value of fractional dot area at various parts of the gray scale on the color target at C, M, Y, and BK of each progressive proof, and the measured optical density through R, G, and B filters on the same location, the values of optical dot gain could be estimated as a function of light reflectance. Three measurements were made for each patch. These values were plotted versus fractional dot area as shown in Figs. 1 and 4. The scattering of the measured values is seen for the patches of fractional dot area 0.08, 0.59, 0.61, but it is not appreciably seen at the patches of the other fractional dot areas. There is more scatter between patches than in the same patch.

In Fig. 1, the broken line with one dot expresses our correction curve that is symmetrical, and the broken line with two dots expresses Rogers' correction curve, and the solid line expresses the Yule-Nielsen correction curve.

These curves are derived by adjusting the parameter of each equation so as to yield the same value at the point of about 0.5 fractional dot area. Rogers' curve is skewed to the left and the Yule-Nielsen curve is skewed to the right, while our curve is symmetrical. But the differences among these three curves are far smaller than the scatter of measured values, and the results

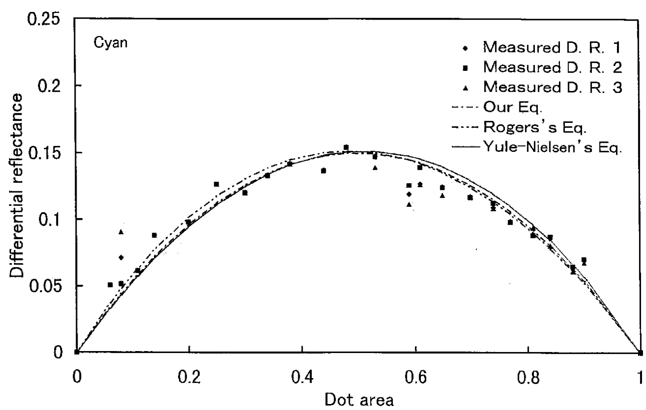

Figure 1. Comparison of three kinds of correction methods for optical dot gain in the case of C print.

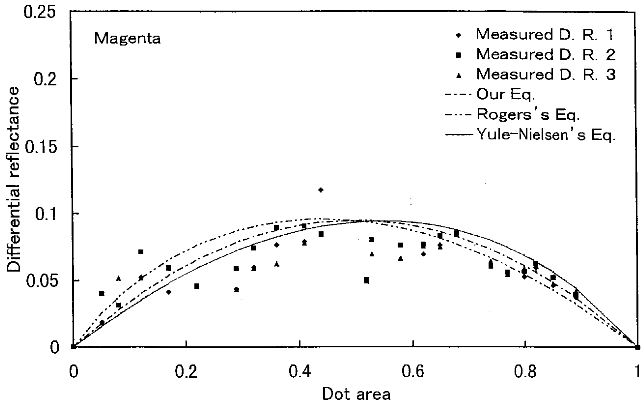

Figure 2. Comparison of three kinds of correction methods for optical dot gain in the case of M print.

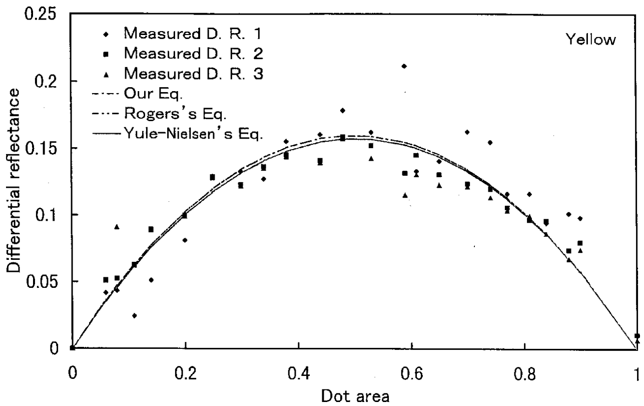

Figure 3. Comparison of three kinds of correction methods for optical dot gain in the case of Y print.

are considered equivalent whichever correction method is used.

In Fig. 2, scattering of measured values for the magenta halftone print is large within patches, and between patches in the case of dot areas smaller than 0.65. As for the case of the cyan halftone print, we normalize the three correction curves at dot area 0.5. In this case the correction values are smaller than the values for cyan. The characteristic skewing of Rogers' curve and the Yule-Nielsen curve becomes remarkable. In this case the scatter of measured values is large, so it can be predicted that the approximations are effectively equivalent whichever method is used.

In Fig. 3 significant scatter of measured values in the case of the yellow halftone print is again seen within the same patch. Before normalizing the three curves at dot area 0.5, the three curves become nearly the same. Therefore, whichever curve is used, the correction is essentially the same.

In Fig. 4 the differential reflectance derived from the densities measured through the red filter is shown. For the case of the black halftone print, the measured values of density through B, G, and R filters are nearly the same, therefore only the differential reflectance derived from the density through the red filter is shown to introduce of the correction item to Pollak's equation. Again, the scatter of measured values is observed rather strongly. Normalizing the three kinds of correction values at the dot area 0.5, it seems that Rogers' curve

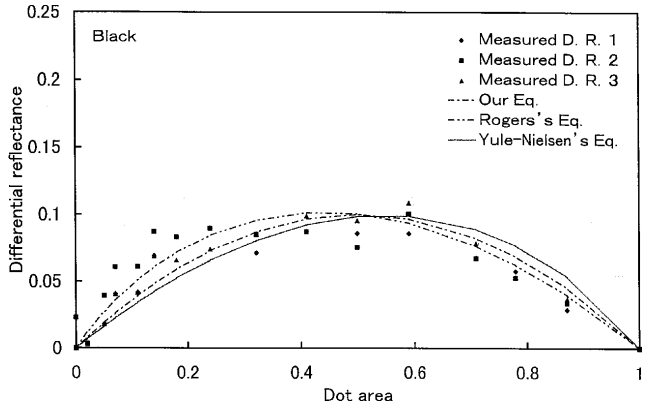

Figure 4. Comparison of three kinds of correction methods for optical dot gain in the case BK print.

TABLE I. Summary of Correction Parameters

	k	$(1 - T_D)^2$	1 – s	*n*
C	0.60	0.68	0.85	1.82
M	0.38	0.67	0.48	1.42
Y	0.64	0.64	1.00	1.95
bK	0.40	0.71	0.48	1.42

skewed to the left is the best match to the distribution of measured values. The Yule-Nielsen curve is skewed too much to the right. But considering the scatter of the measured value, we infer that there is little difference in the extent of correction whichever correction curve is used.

Table I summarizes the coefficient k for our correction curve, the parameter $1 - s$ of Rogers' correction curve and the parameter n of Yule-Nielsen's correction curve. The value of each parameter has almost the same value for C and Y, M and BK. In Table I the value of $(1 - T_0)^2$ in the Rogers' equation is also shown, which corresponds to k of our equation. As the value $1 - s$ becomes nearer to unity, the value of $(1 - T_0)^2$ becomes nearer to the value of k.

Introducing Our Correction Term to Pollak's Equation

In Figs. 5 through 8, the measured optical reflectances through R, G, and B filters and the values derived from Murray-Davies' equation at the regions of C, M, Y, and bK prints corresponding to regions of the gray scale of the original color target are shown. The values are estimated for the case of the paper reflectance being taken as unity.

At each fractional dot area, the difference between the optical reflectance derived from the Murray-Davies' equation and the measured value expresses the effect of optical dot gain. In the case of C, M, and Y, each progressively printed, the optical reflectance through the complementary color filter is smaller than the value calculated by the Murray-Davies equation, but the optical reflectance through now complementary color filters, nearly agrees with the Murray-Davies' equation. In Fig. 8, for the case of the black print, the effect of optical dot gain through R, G, and B filters is nearly the same, and a correction is required to obtain the optical reflectance through each color filter.

Here, we mention our approach to introducing the correction term for the effect of optical dot gain into Pollak's equation. The effect of optical dot gain is shown

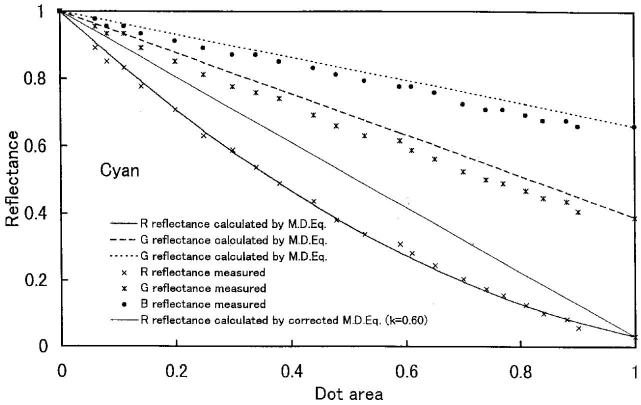

Figure 5. The values measured through R, G, and B filters and values calculated by the Murray-Davies' equation and the modified equation in the case of C print.

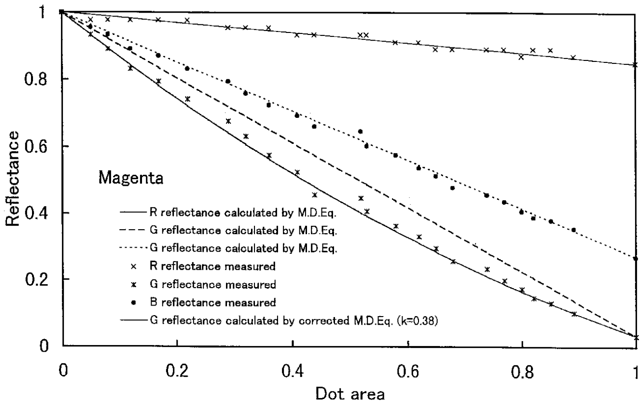

Figure 6. The values measured through R, G, and B filters and values calculated by the Murray-Davies' equation and the modified equation in the case of M print.

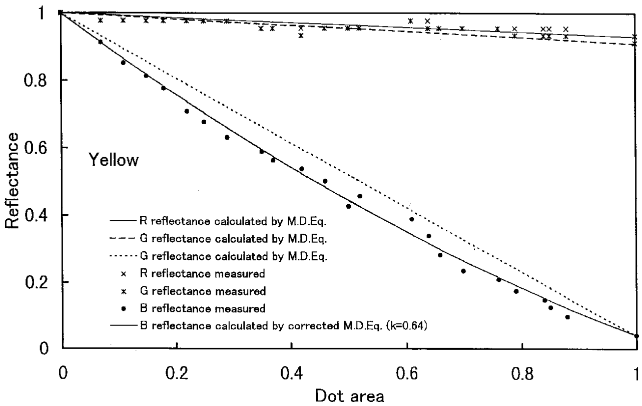

Figure 7. The values measured through R, G, and B filters and values calculated by the Murray-Davies' equation and the modified equation in the case of Y print.

in Figs. 1 through 4 for the case of halftone dots with only one color printed on the paper, but an actual full color halftone print is constructed by superimposed halftone dots. Therefore, we are not able to introduce each correction parameter separately into the corresponding term of Pollak's equation.

The halftone print comprising four color inks superimposed is divided into a total of seventeen color areas containing the first color, the second color, and white of

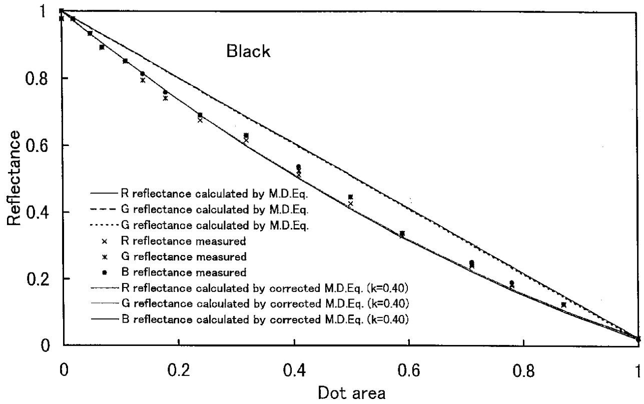

Figure 8. The values measured through R, G, and B filters and values calculated by the Murray-Davies' equation and the modified equation in the case of BK print.

paper. Observing the print through R filter, the parts of the area printed by C ink and BK ink are seen as black area. Fractional dot area of these black areas can be derived by Demichel's method[11] that uses the multiplication theorem of probability. The area where C is not printed and bK is not printed corresponds to the white area of the paper. The expression describing that area is $(1 - c)(1 - BK)$, as c is the fractional dot area of C and bK is the fractional dot area of bK.

By this method, we can approximately correct the effect of optical dot gain compounded by C and BK. If we substitute the halftone dot of BK with one of C, and still suppose that the fractional dot area of C should increased by $[1 - (1 - c)(1 - bK)]$, we introduce the correction value for the effect of optical dot gain at that fractional dot gain of C into Pollak's equation, as shown in Fig. 9.

This correction is expressed in Eq. 14.

$$c1 = 1 - (1 - c)(1 - bk)$$
$$R = R_P[1 - c + cR_{SC} - k_C c1(1 - c1)](1 - m + mR_{SM})(1 - y + yR_{SY}).(1 - bk + bkR_{SBK}) \quad (14)$$

Similarly, observing the full color halftone print through green filter, the area printed by M and bK halftone dots are seen as black area. By using the method of Demichel, the fractional dot area that is not printed by M and bK halftone dots becomes $(1 - m)(1 - Bk)$. Therefore, the fractional dot area that is covered by M

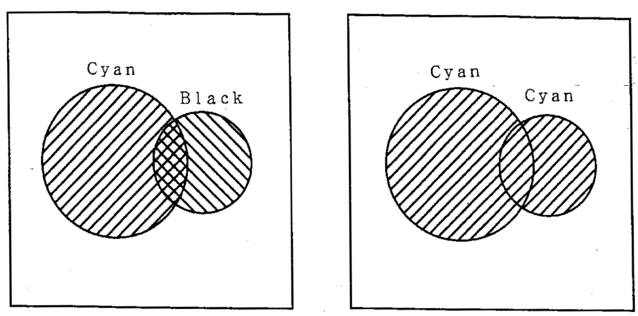

Figure 9. Schematic of C and BK halftone dot superimposed on paper, and schematic of C halftone dots in the case where the BK halftone dot is converted to a C halftone dot for approximately the correction for optical dot gain.

and BK halftone dots becomes $1 - (1 - m)(1 - BK)$. We considered this area as all M halftone dots, and introduced the correction value at that fractional dot area to the magenta term of Pollak's equation. By this method, we can approximately correct the effect of optical dot gain compounded by M and bK as expressed in Eq. 15.

$$m1 = 1 - (1 - m)(1 - bk)$$
$$G = G_P(1 - c + cG_{SC})\,[1 - m + mG_{SM} - k_M m1(1 - m1)] \cdot (1 - y + yG_{SY})\,(1 - bk + bkG_{SBK}) \quad (15)$$

Similarly, observing the full color halftone print through a blue filter, the areas printed by Y and BK halftone dots are seen as black area. By using the method of Demichel, the fractional dot area that is not printed by Y and BK halftone dots becomes $(1 - y)(1 - bk)$. Therefore, the fractional dot area that is covered by Y and BK halftone dots becomes $1 - (1 - y)(1 - bk)$. We considered this area as all Y halftone dots, and introduced the correction value at that fractional dot area to the yellow term of Pollak's equation. By this method, we can approximately correct the effect of optical dot gain compounded by Y and BK, as expressed in Eq. 16.

$$y1 = 1 - (1 - y)(1 - bk)$$
$$B = B_P(1 - c + cB_{SC})(1 - m + mB_{SM}) \cdot [1 - y + yB_{SY} - k_Y y1(1 - y1)](1 - bk + bkB_{SBK}) \quad (16)$$

To verify how well our modified Pollak's equation incorporating our correction term for optical dot gain corresponded to the measured value of the print comprising superimposed halftone dots constructed by various combination of C, M, Y, and BK fractional dot areas, we selected the 12 color areas of the color target and measured optical density through R, G, and B filters. Next, by converting the optical density into the reflectance, we compared the calculated values to the measured values. These selected colors are 12 colors of an equally divided hue circle; their lightness is L*1, and their saturation is C*3. Table II summarizes the measured fractional dot areas of these patches in the C, M, Y, and BK progressive proofs.

In Figs. 10 through 12 the measured reflectance through each filter, the reflectance calculated by Pollak's equation, and the reflectance calculated by our modified Pollak's equation wherein our correction term has been introduced are shown. Except for patch 3-I, black halftone dots exist in each patch. Therefore, the correction of optical dot gain is required except where the patch is solid, and the combined effect for correction of the light of complementary color to each print color and for the light of corresponding color can be verified. To the reflectance through red filter, except for green

TABLE II. Fractional Dot Areas: Superimposed Primaries

Patch	Dot area (C)	Dot area (M)	Dot area (Y)	Dot area (bK)
3-A	0.47	1.00	0.85	0.24
3-B	0.43	0.94	0.90	0.21
3-C	0.36	0.80	1.00	0.09
3-D	0.61	0.71	1.00	0.20
3-E	0.75	0.58	1.00	0.18
3-F	0.88	0.45	0.88	0.21
3-G	0.90	0.63	0.77	0.29
3-H	1.00	0.67	0.51	0.21
3-I	1.00	0.75	0.02	0.00
3-J	1.00	1.00	0.00	0.08
3-K	0.67	1.00	0.21	0.17
3-L	0.53	1.00	0.56	0.25

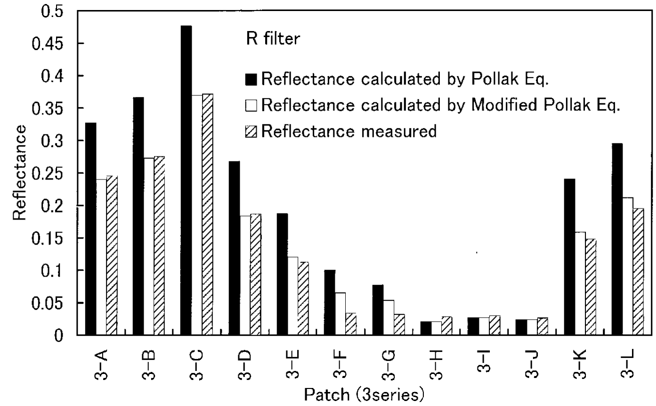

Figure 10. Comparison of measured reflectance, reflectance calculated by Pollak's equation and reflectance calculated by our modified equation for each patch of the color target print using R filter.

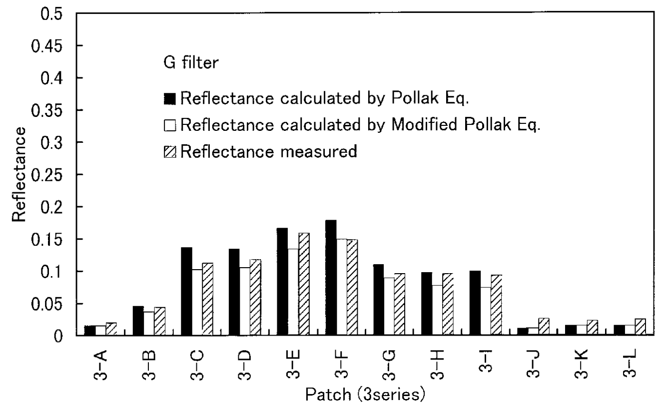

Figure 11. Same as Fig. 10, using G filter.

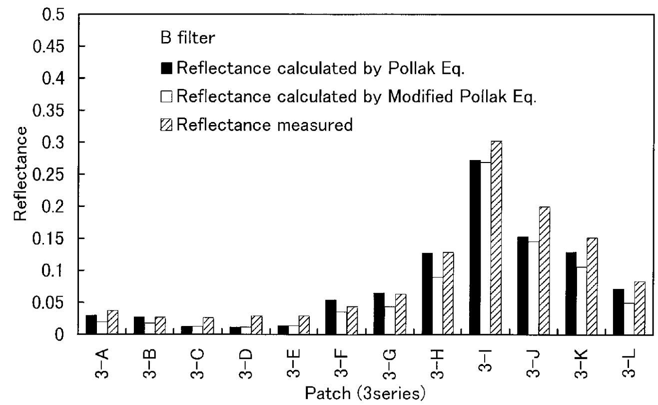

Figure 12. Same as Fig. 10, using B filter.

patches 3-F and 3-G the correction of optical dot gain has been done perfectly. For reflectance through the green filter with the yellow and blue patches of 3-E, 3-H and 3-I the effect of the correction is a little excessive but for the rest of the patches the effect is good. For reflectance through the blue filter, the blue and violet patches 3-I and 3-J are not correctly estimated because of the lack of halftone dots of Y and BK. For reddish violet patches 3-K and 3-L, the correction value is larger than the value needed.

By introducing our correction term for optical dot gain to Pollak's equation separating the reflectance of paper, we can fairly approximate the values measured through R, G, and B filters.

The equation comprises the product of the terms for paper, C, M, Y, and BK. If the correction for optical dot gain and the correction for additivity-law failure are introduced to the equation, we conclude that the reflectance value calculated could approximate the measured value. This should show that the principle of color reproduction in printing by superimposing halftone dots of each color is subtractive color mixing.

Conclusions

1. For predicting the reflection optical density of color on paper printed with the combination of C, M, Y, and bK fractional dot areas, we chose Pollak's equation. This equation is constructed as the product of reflectance of each ink film and paper combination. It indicates that color reproduction by the printing of superimposed halftone dots basically corresponds to subtractive color mixing.
2. The value calculated by Pollak's equation does not agree with the measured value. We assigned the principal cause for nonagreement to the peculiar effect of optical dot gain in halftone dot printing. We derived the following method where the amount of correction for optical dot gain was approximated.

On the graph having fractional dot area as variable, we plotted the measured value of optical dot gain, and approximated the value by the symmetrical quadratic function $a(1 - a)$ that shows maximum value at fractional dot area 0.5. We multiplied the quadratic function by the empirical coefficient k, to obtain $ka(1 - a)$. By adjusting k we approximated the measured value with the calculated value.

On examining the degree of agreement of our quadratic equation, Yule and Nielsen's correction, and Rogers' correction with measured values, we found that these three methods had few differences from each other. Therefore we adopted our correction equation as the most simple and easiest in which to determine the empirical coefficient.

3. We introduced our correction term for optical dot gain to Pollak's equation. On comparison of measured values of R, G, and B reflectance at each fractional dot are of C, M, Y, and BK with the values calculated by the Murray-Davies equation we found that optical dot gain occurred under complementary color light in the case of C, M, Y, and occurred under R, G, and B light to the same degree in the case of BK.

Therefore, using Demichel's equation that expresses the state of superimposed halftone dots introduced into Pollak's equation and taking C, for example, the area superimposed by C and BK should be considered as C area. By using total fractional dot area, the correction of the C term to Pollak's equation can be made, then the correction of optical dot gain for the case of halftone dots superimposed becomes possible.

4. To verify the effect of our modified Pollak's equation, the equation was applied to the reflection optical densities of the print of 12 patches of color areas from a color test target. The calculated values approached the measured values fairly well.

The strong point of our equation is that its structure expresses well the actual color print, and therefore the estimates obtained do not differ substantially from the measured values. If the amount of trapping or additivity law failure of optical density of a solid print would become clear in the future, we should be able to introduce these effects to our modified Pollak's equation, as well.

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References

1. H. E. Neugebauer, *J. Opt. Soc. Am.* **42,** 740 (1952).
2. F. Pollak, *J. Photogr. Sci.* **3,** 112 (1952).
3. J. A. C. Yule, *Principles of Color Reproduction*, John Wiley and Sons, New York, 1967, p. 263.
4. A. Murray, *J. Franklin Inst.* **211,** 721 (1936).
5. F. R. Ruckdeshel and O. G. Hauser, *Appl. Opt.* **17,** 3376 (1978).
6. T. Nagayama, T. Yuasa and H. Mishina, *Bulletin of the Japanese Society of Printing Science and Technology,* **28,** 366 (1991).
7. M. Teraoka and S. Taguchi, *Bulletin of the Japanese Society of Printing Science and Technology,* **29,** 502 (1992).
8. S. Inoue, N. Tsumura and Y. Miyake, *Bulletin of the Japanese Society of Printing Science and Technology* **35,** 189 (1998).
9. J. S. Arney, *J. Imag. Sci. Technol.* **41,** 633 (1997).
10. G. L. Rogers, *J. Imag. Sci. Technol.* **41,** 643 (1997).
11. M. E. Demichel, *Proédé* **26,** 17 (1924).