Analyzing CTF of Print by MTF of Paper

Shinichi Inoue*,‡
Tokyo Research Laboratories, Research Center, Mitsubishi Paper Mills Limited, Tokyo, Japan

Norimichi Tsumura*,§ and Yoichi Miyake†,§
Department of Information and Computer Sciences, Chiba University, Japan

Image quality of print is dependent on the paper characteristics. However, there are relatively few studies on the imaging characteristics of paper. In a previous report, we introduced a new method for measuring the modulation transfer function (MTF) of paper. In this report, a contrast transfer function (CTF) of print is introduced. We find that the CTF of print, $CTF_{print}(\omega)$, can be expressed by a simple function of the MTF of paper, $MTF_{paper}(\omega)$: $CTF_{print}(\omega) = [1 + MTF_{paper}(\omega)]/2$. The CTF of print predicted by the function was approximately same as the measured CTF of print.

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Introduction

Image quality of print is dependent on the paper characteristics. As is well known, paper is a turbid medium, therefore, incident light to paper causes absorption, transmission, reflection, and scattering. The modulation transfer function (MTF) is usually applied to represent the light-scattering phenomenon for the analysis of transparency films or lenses.[1–3] However, relatively little has been studied on the imaging characteristics of print and paper.[4–6]

In a previous report, we introduced a new method for measuring MTF of paper.[7] On the basis of the experimental results, in this report, we analyze a contrast transfer function (CTF) of print mathematically by the MTF of paper. Analyzed results are compared with the measured data by experiment.

Reflection Image Model

Before we introduce a CTF of print, it is useful to discuss the light intensity transfer behavior of print. For the discussion, we use a model that can predict reflection intensity (or density) of print termed the reflection image model. The reflection image means an image that is observed as reflection light intensity. For example, print, photographic print, and hardcopy are the reflection images. We use the word “print” as the reflection image in this report. In our previous report, the experimental results showed that the predicted density by this model is approximately same as the measured density in halftone image, and the estimated optical dot gain is well correlated to the measured optical dot gain.[7]

We assume that the reflection image consists of two layers, a transparent image layer and a diffuse reflection layer, as shown in Fig. 1. We may consider that the transparent image layer is ink and the diffuse reflection layer is paper. Each step in this model is explained as follows:

Step 1. Incident light is projected to the transparent image layer.
Step 2. The transparent image layer absorbs incident light.
Step 3. Incident light is scattered and diffused in the diffuse reflection layer, and this phenomenon is represented as the point spread function (PSF) or the MTF of the diffuse reflection layer.
Step 4. The scattered and reflected light in the diffuse reflection layer is absorbed in the transparent image layer again.

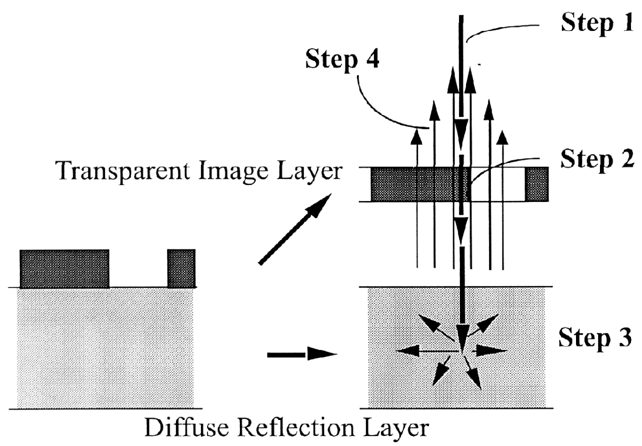

Figure 1. Diagram of reflection image model.

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* IS&T Member

† IS&T Fellow

‡ inoue_shinichi@ mpm.co.jp

§ tsumura@ics.tj.chiba-u.ac.jp; miyake@ics.tj.chiba-u.ac.jp

These steps can be expressed as the following equation.

$$i_{out}(x,y) = i_{in}\, r\, \{ t(x,y) * \mathrm{PSF}_{paper}(x,y)\} t(x,y), \quad (1)$$

where $i_{out}(x,y)$ at coordinates x, and y is reflection light intensity, namely the observed image; $t(x,y)$ is transmittance of the transparent image layer, $\mathrm{PSF}_{paper}(x,y)$ is the PSF of the diffuse reflection layer and * indicates the convolution integral. We assumed that the incident light i_{in} and reflectance of the diffuse reflection layer r are uniform, then i_{in} and r are constant.

Equation 1 can be expressed in the Fourier domain

$$i_{out}(u,v) = i_{in}\, r\, \{T(u,v)\mathrm{MTF}_{paper}(u,v)\} * T(u,v), \quad (2)$$

where $I_{out}(u,v)$ and $T(u,v)$ are the spectrum of $i_{out}(x,y)$ and $t(x,y)$, respectively. $\mathrm{MTF}_{paper}(u,v)$ is the MTF of the diffuse reflection layer. We can consider that these steps and the equation are divided into two processes. The first process is that from steps 1 to 3 in the model, which can be described $i_{in}\, r\, \{T(u,v)\mathrm{MTF}_{paper}(u,v)\}$ in Eq. 2. The second process is step 4 where the light is absorbed in the transparent image layer again, and this is expressed as the convolution integral in Eq. 2. Note that the first process can be analyzed mathematically according to the MTF concept. We introduce an analyzing technique for leading a response function of print in the next part.

Definition of CTF of Print

We introduce a CTF of print instead of an MTF. We define the CTF of print as follows: When the transparent image layer has sinusoidal transmitance, contrast $c(\omega)$ is given by the difference of the maximum and minimum intensity as follows:

$$c(\omega) = i_{max} - i_{min}, \quad (3)$$

where ω denotes the spatial frequency, and i_{max} and i_{min} denote the maximum and minimum intensities of the output. We define the CTF of print $\mathrm{CTF}_{print}(\omega)$ for spatial frequency ω by

$$\mathrm{CTF}_{print}(\omega) = c(\omega)/c(0), \quad (4)$$

where $c(0)$ denotes the contrast when the spatial frequency is 0.0.

We now introduce a relation between the CTF of print and the MTF of paper mathematically. The function is set out as follows: Methods based on the use of spatial sine waves are commonly used in the practical measurement of the MTF.[1–3] Let the input projected by optical system be of a one-dimensional sine wave with an amplitude $a(\omega)$ as follows:

$$i_{out}(x) = b + a(\omega) \cdot \cos(2 \cdot \pi \cdot \omega \cdot x), \quad (5)$$

where ω denotes the spatial frequency, and $a(\omega)/b$ denotes the modulation. The MTF for the spatial frequency ω is given by the ratio of the output modulation to the input modulation. The modulation $M(\omega)$ can be expressed by

$$M(\omega) = \frac{i_{max} - i_{min}}{i_{max} + i_{min}}, \quad (6)$$

where i_{max} and i_{min} denote the maximum and minimum of $i_{out}(x)$ in Eq. 5, respectively. The $\mathrm{MTF}(\omega)$ is given by the ratio of the modulation at ω to the modulation at the spatial frequency zero, as Eq. 7:

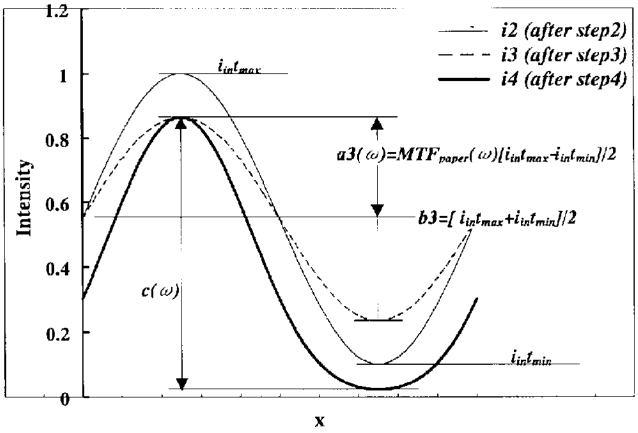

Figure 2. A schematic diagram of the signals in each step of reflection image model.

$$\mathrm{MTF}(\omega) = M(\omega)/M(0). \quad (7)$$

At the point of after step 3, the output intensity, $i3_{max}$ and $i3_{min}$, can be denoted according to this MTF concept:

$$i3_{max} = b3 + a3(\omega), \quad (8)$$

$$i3_{min} = b3 - a3(\omega). \quad (9)$$

The parameter $a3(w)$ and $b3$ in Eqs. 8 and 9 can be expressed by

$$a3(\omega) = \mathrm{MTF}_{paper}(\omega)[i_{in} \cdot t_{max} - i_{in} \cdot t_{min}]/2, \quad (10)$$

$$b3 = [i_{in} \cdot t_{max} + i_{in} \cdot t_{min}]/2. \quad (11)$$

Figure 2 shows a schematic diagram of the signals in each step. The maximum output after step 3 is multiplied by maximum transmitance of the transparent image layer to calculate the final maximum intensity of output. The minimum output after step 3 is multiplied by minimum transmitance of the transparent image layer to calculate the final minimum intensity of output. Accordingly, the contrast $c(\omega)$ can be expressed by

$$\begin{aligned} c(\omega) &= i3_{max} \cdot t_{max} - i3_{min} \cdot t_{min} \\ &= [b3 + a3(w)] \cdot t_{max} - [b3 - a3(w)] \cdot t_{min}. \end{aligned} \quad (12)$$

Equation 12 is rewritten using Eqs. 10 and 11:

$$c(\omega) = \frac{{i_{in}}^2 (t_{max}^2 - t_{min}^2)(1 + \mathrm{MTF}_{paper}(\omega))}{2}. \quad (13)$$

The contrast of the spatial frequency zero, $c(0)$, is rewritten in Eq. 14. Here, we use that the $\mathrm{MTF}_{paper}(0)$ equals 1.0.

$$c(0) = \frac{i_{in}^2 (t_{max}^2 - t_{min}^2)(1 + \mathrm{MTF}_{paper}(0))}{2} = i_{in}^2 (t_{max}^2 - t_{min}^2). \quad (14)$$

The CTF of print can be calculated using Eqs. 10 and 11 according to the CTF definition in Eq. 4.

$$CTF_{print}(\omega) = \frac{c(\omega)}{c(0)} =$$

$$\frac{i_{in}^2(t_{\max}^2 - t_{\min}^2)(1 + MTF_{paper}(\omega))}{2} \cdot \frac{1}{i_{in}^2(t_{\max}^2 - t_{\min}^2)}. \tag{15}$$

Then, the CTF of print becomes the next simple function as follows:

$$\mathrm{CTF}_{\mathrm{print}}(\omega) = \frac{1 + \mathrm{MTF}_{\mathrm{paper}}(\omega)}{2}. \tag{16}$$

We find that the CTF of print is determined by the MTF of paper.

Experiments

Experiments were performed to examine the CTF function characterized mathematically in Eq. 16. The MTF of paper was measured using the method proposed by our previous paper.[7] Accordingly, the CTF of print could be predicted by Eq. 16. On the other hand, the CTF of print was measured directly by experiment. Two types of sample paper were used for these experiments: an uncoated paper and a coated paper. The uncoated paper is 64.0 g/m^2 for ppc and mat. The coated paper is 104.7 g/m^2 for printings, coated with 15 g/m^2 pigment on each side, and glossy.

Predicting CTF of Print. ***Measuring MTF of Paper by Sinusoidal Test Pattern Projection Method.*** The CTF of print was predicted by Eq. 16, using the measured MTF of paper. We measured the MTF of sample papers by the sinusoidal test pattern projection method.[7] A sinusoidal test pattern was projected onto paper, and this reflection intensity distribution was measured with a modified microdensitometer. The modulation $M(\omega)$ for each spatial frequencY ω can be easily determined using Eq. 6 by experiment. [1–3] The MTF for the spatial frequency ω can be given by the ratio of the modulation to the modulation at the spatial frequency 0 as Eq. 7.

The microdensitometer used, which is a modified MPM-5 microdesitometer (UNION Optical), is shown in Fig. 3. The normal illuminating system of the microdensitometer is replaced with a special projection system that is fixed on the sample bed at an angle of 45°. This projection system consists of a chart holder with a sinusoidal test pattern film and a halogen lamp. The internal projection system is coated black to avoid the flare light. The sample bed is a black background. The function of this optical system is to project a uniform image of the sinusoidal test pattern in the chart holder onto the image plane. The projected image is focused on a line only on the sample bed, because the image is projected at the angle of 45°. Therefore scanning was done on this focused line. The effective measurement aperture of the microdensitometer is 0.01 mm width and 0.1 mm height.

The sinusoidal test pattern used is Sine Patterns, M-5-60, a variable transmission type pattern made on exceptionally high resolution film. The sinusoidal test pattern arrays contain a set of sinusoidal areas with the spatial frequencies of interest. Each sinusoidal area varies sinusoidally in transmittance. Six sinusoidal test pattern arrays are used for each sample paper. They have different spatial frequencies, 0.375, 1.0, 3.0, 6.0, 8.0, and 10.0 cycles/mm.

The input sinusoidal test pattern projected onto the sample paper is degraded by internal diffusion in the paper, and the resulting degraded image is then reflected onto the slit of the microdensitometer through the scanning optics. Measurements were made by driving the sample bed with the sample and the projection system (which remained fixed in relation to one another). Readings were taken at 0.01-mm intervals, namely, values obtained correspond to the output sinusoidal image distribution. Because this recording data is reflection density, reflectance can be obtained by

$$R_o = 10^{-D_r}, \tag{17}$$

where R_o is reflectance, and D_r is reflection density. This reflectance corresponds to the intensity of the output image. The modulation was calculated from maximum and minimum intensity reflectance of the output sinusoidal image.

The output sinusoidal images measured for the coated sample paper in this way are shown in Fig. 4. These results include the spreading of light by the instrument itself, and a correction should be made for this using the measuring system MTF, that was, in turn, measured by transmittance of the sinusoidal test pattern film shown in Fig. 5. The resulting MTF of paper was corrected using this system MTF. The MTFs of the coated and the uncoated paper are shown in Fig. 6.

It could be considered that the MTF of paper can be approximately expressed by[7]

$$\mathrm{MTF}_{\mathrm{paper}}(\omega) = \frac{1}{[1 + (2 \cdot \pi \cdot d \cdot \omega)^2]^{\frac{3}{2}}}. \tag{18}$$

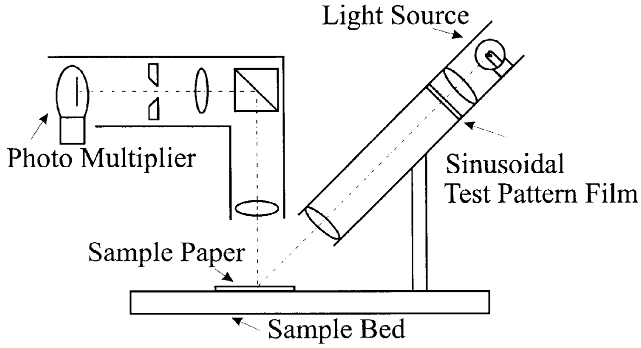

Figure 3. A schematic diagram of the sinusoidal test pattern projection method for measuring the MTF of paper.

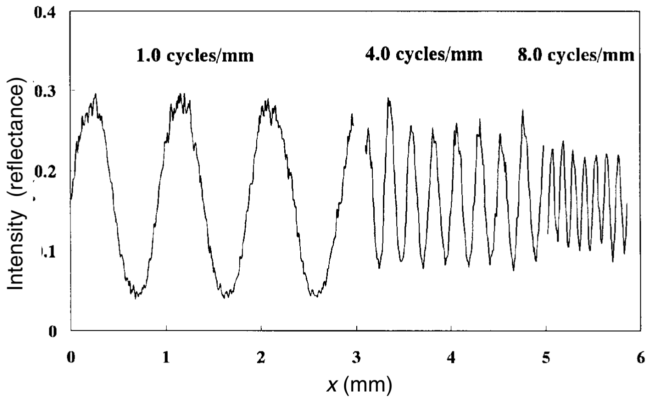

Figure 4. Projected sinusoidal test pattern traces for the coated paper.

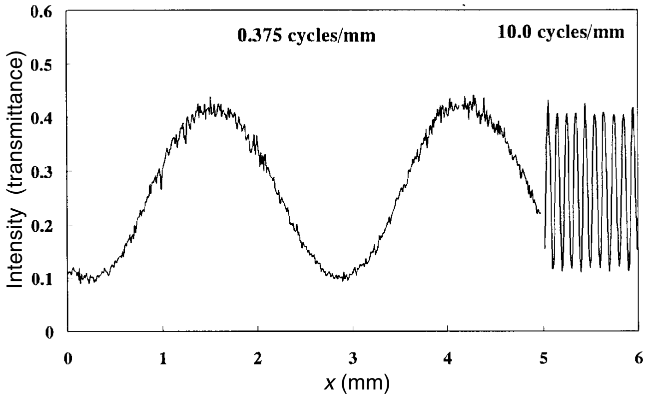

Figure 5. The sinusoidal test pattern film transmittance traces.

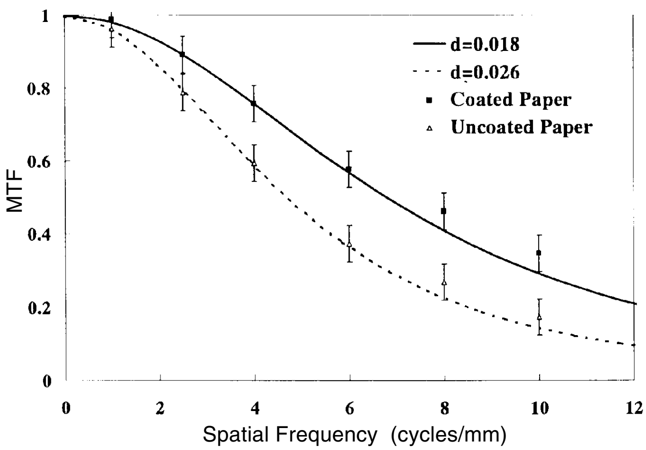

Figure 6. MTF curves in Eq. 18 and measured MTFs. The error bars show the standard deviations for the measurement noise.

Figure 6 shows the measured MTF data (dots) and the MTF fitting curves using this expression (lines). The coefficient d was calculated as 0.018 for the coated paper and 0.026 for the uncoated paper, respectively.

The CTFs of print for two sample papers were predicted using Eq. 16. The lines in Fig. 9 in the next section show the predicted CTF curves for the coated paper and the uncoated paper.

Measuring CTF of Print by Sinusoidal Test Pattern Contact Method. The CTF of print with sample paper was measured by a contact method. A sinusoidal test pattern film was contacted onto sample paper, and this reflection intensity distribution was measured with a microdensitometer. The contrast for each spatial frequency can be easily determined by Eq. 3. The CTF of print is given by Eq. 4. The apparatus used, which is the MPM-5 microdesitometer (UNION Optical), is shown in Fig. 7. The illuminating system of the microdensitometer is the two light guides. The light source is a halogen lamp. The light guides are fixed at the near of the objective lens at an angle of 45°, from the sample bed. The direction of the light guides are on the line of the measurement aperture height. One illuminates the sample from an upper position; the other from a lower position. The effective measurement aperture of the microdensitometer is 0.01 mm width and 0.1 mm height. The sinusoidal test pattern film used is Sine Patterns, M-5-80, where a variable transmission type pattern made on exceptionally high resolution

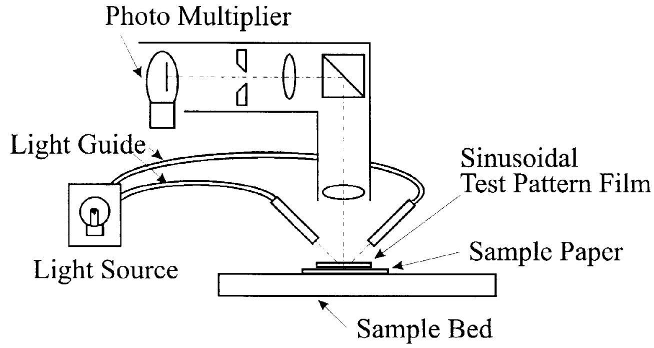

Figure 7. A schematic diagram of the sinusoidal test pattern contact method for measuring the CTF of print.

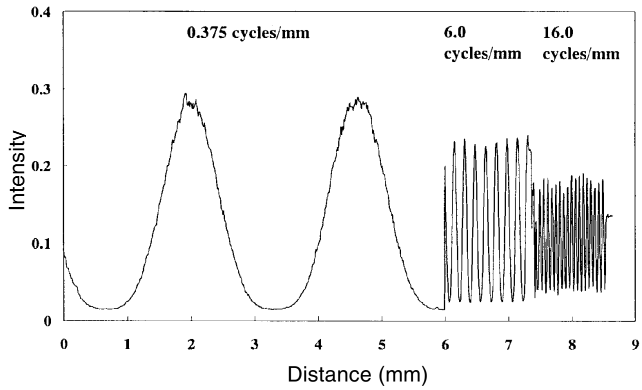

Figure 8. Contacting sinusoidal test pattern traces for the coated paper.

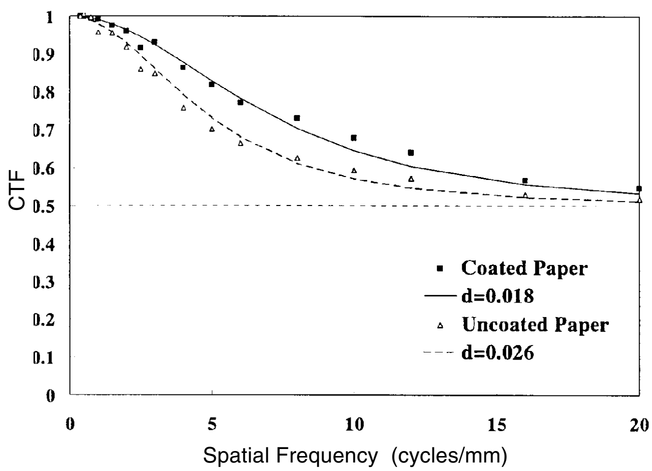

Figure 9. CTF curves in Eq. 14 and measured CTFs.

film. The sinusoidal test pattern arrays contains a set of sinusoidal areas with the spatial frequencies of interest. Each sinusoidal area varies sinusoidally in transmittance. The 15 sinusoidal test pattern arrays used for each sample paper have spatial frequencies 0.375, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0, 5.0, 6.0, 8.0, 10.0, 12.0, 16.0, and 20.0 cycles/mm.

Measurements were carried out by driving the sample bed with the sinusoidal test pattern film on the sample paper. The sinusoidal test pattern film on a sheet of sample paper was set on the sample bed in order to contact well. The emulsion side of the sinusoidal test pattern film contacted the sample paper. The sample bed

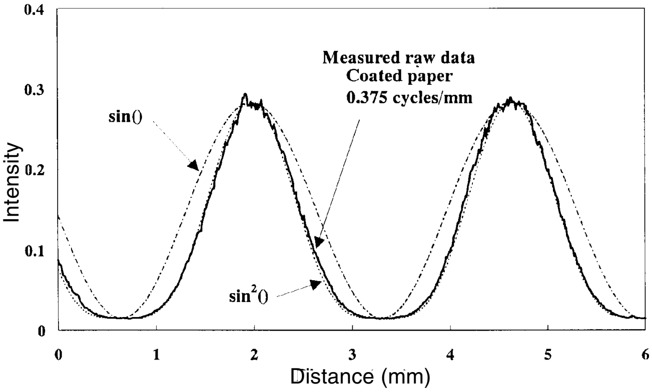

Figure 10. Contacting sinusoidal test pattern traces for the coated paper (line) and $sin^2(\ )$ curve (dotted line) for the reference.

was a black background. Readings were taken at 0.01 mm intervals, namely, values obtained corresponded to the output intensity distribution. Because these recording data are reflection density, intensity (reflectance) can be obtained by Eq. 17. The contrast was calculated from maximum and minimum intensity of the output image using Eq. 3. The output images measured for the coated paper in this way are shown in Fig. 8. The resulting CTF of paper was corrected using the system MTF. The measured CTFs (dots) of the coated and the uncoated papers are shown in Fig. 9. Figure 9 shows that the predicted (lines) and the measured (dots) CTFs of print are approximately the same for each sample paper.

Discussion

Features of the Input-Output Relation in Print. Note two features of the input–output relationship in print. The first feature is that the final output signal was not sinusoidal if the input signal was sinusoidal. For example, if the MTF of paper becomes 1.0, the output signal becomes $\sin^2(x,y)$ as

$$i_{\text{out}}(x,y) = i_{\text{in}}\, r \sin^2(x,y). \tag{19}$$

This event occurs in the low-spatial-frequency area and this phenomenon was observed in our experiment. The input intensity in which spatial frequency is 0.375 is sinusoidal in Fig. 5. Figure 10 shows the measured (line) output intensity where spatial frequency is 0.375. The dotted line in Fig. 10 shows $\sin^2(x,y)$ for the reference.

The second feature is that the macrointensity is reduced according to the increase of spatial frequency. This phenomenon was also observed in our experiment in Fig. 8, for example. The same phenomenon has been known as the optical dot gain effect in halftone images.

Predicting MTF of Paper by the CTF of Print. We introduced that the CTF of print can be expressed by the simple function using the MTF of paper. It is clear that the MTF of paper can be predicted by the CTF of print, and vice versa, as

$$\text{MTF}_{\text{paper}}(\omega) = 2 \bullet \text{CTF}_{\text{print}}(\omega) - 1. \tag{20}$$

It is much easier to measure the CTF of print by the sinusoidal test pattern contact method than to measure the MTF of paper by the sinusoidal test pattern projection method. This method could be a practical way to measure the MTF of paper.

Conclusions

In this report, the response function of print was analyzed mathematically. We introduced that the CTF of print can be expressed by the simple function using the MTF of paper. We showed that the CTF of print predicted by the proposed equation was approximately the same as the measured data. When multistage imaging systems are analyzed, it is desirable to have a quicker method of evaluating the system as a whole. The cascading of response function has been found very useful in linear systems.[2] In the cascading of MTFs, the CTF of print that is introduced here, can be used as an alternative function. It remains to apply this method to the evaluation of the hardcopy image quality.

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