Optical Dot Gain: Lateral Scattering Probabilities

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In the development of the technology of halftone imaging there has been significant interest in physically modeling the halftone microstructure. An important aspect of the microstructure is the scattering of light within the paper upon which the halftone image is printed. Because of light scatter, a photon may exit the paper at a point different from the point at which it entered the paper. The effect that this light scatter has on the perceived color of the printed image is called optical dot gain. Optical dot gain can be characterized by lateral scattering probabilities, which is the probability that a photon entering the paper through a particularly inked region exits the paper through a similar or different type inked region. In this article we explicitly calculate these lateral scattering probabilities for the case of AM and FM halftone screening. We express these probabilities in terms of the fractional ink coverage and the lateral scattering length, a quantity that characterizes the distance a photon travels within the paper before exiting.

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Introduction

Halftone imaging is a widely used technique for producing printed images. Recently there has been significant interest in physically modeling the halftone microstructure to control the tone characteristics of the halftone image better.[1–4] An important aspect of this microstructure is the scattering of light within the paper upon which the image is printed. This effect of scattering is called optical dot gain, because, for achromatic images, the ink dots are effectively larger as a result of the scattering.[5] Several authors have expressed optical dot gain in terms of lateral scattering probabilities[1,2] which is the probability that a photon having entered the paper through a particular type inked region exits the paper through a similar or different type inked region. In this article we explicitly calculate these probabilities; in particular we calculate the ink–ink probability, which is the probability that if a photon enters the paper through an inked region it also exits the paper through an inked region—a conditional probability we label P_{ii}. Knowledge of P_{ii} allows one to calculate all the other lateral scattering probabilities.[1] Although the calculation done here involves a single array of dots, our results are applicable to a chromatic halftone image.[9] We make the calculation for the case of both AM and FM halftone screening.[2]

In Ref. 1 it is shown that the ink–ink probability can be expressed in terms of an infinite series—the Z-series—involving the Fourier transforms of the dot shape and the paper's point spread function. Here, we explicity calculate P_{ii} and obtain a closed-form expression.

The model we construct to determine P_{ii} is as follows: a uniform stream of photons is incident on the paper within an area of one dot, and we calculate the fraction of the photons that exit the paper through this dot and through all the other dots. This fraction is P_{ii}. We assume that the dots are circular with radius d and that they are arranged in a square grid (screen) with screen period r. The origin of the coordinate system is at the center of the dot through which the photons enter the paper (see Fig. 2).

We define $2\pi R(\rho)\rho,d\rho$ as the probability that a photon, having entered the paper through the dot centered on the origin, exits the paper through an annulus, also centered on the origin, with radius ρ and thickness $d\rho$. $R(\rho)$ is the radial reflectance per unit area, and $R(\rho)$ integrated over the entire surface is the paper's reflectance, R_p:

$$R_p = 2\pi\int_0^\infty R(\rho)\rho d\rho. \tag{1}$$

We define the radial covering distribution $A(\rho)$, as the probability that an arbitrary point at a distance ρ from the origin is covered by ink.

Then the ink–ink probability is:

$$P_{ii} = 2\pi\int_0^\infty R(\rho)A(\rho)\rho d\rho. \tag{2}$$

In the section "Reflectance" we calculate the reflectance per unit area, $R(x,y)$, for photons that have entered the paper through the area of a single dot. In the section "Radial Covering Distribution" we calculate the covering distribution $A(\rho)$. In the section "Ink-Ink Probability" we carry out the integration of Eq. 2, making two approximations to obtain a closed-form expression for P_{ii}. The calculations carried out in these sections are for AM halftone screening in which the number of dots within a region is constant and the size of the dots is varied. In the section "FM Halftone Screen" we calculate P_{ii} for FM halftone screening: the dots are of constant size and the number of dots is varied. In the section "Ink-Ink Probability for Diffusion PSF" we give the ink-ink probability as calculated with the diffusion point spread function.

Reflectance

The reflectance per unit area $R(x,y)$ is the probability that a photon exits the paper at the point x,y after having

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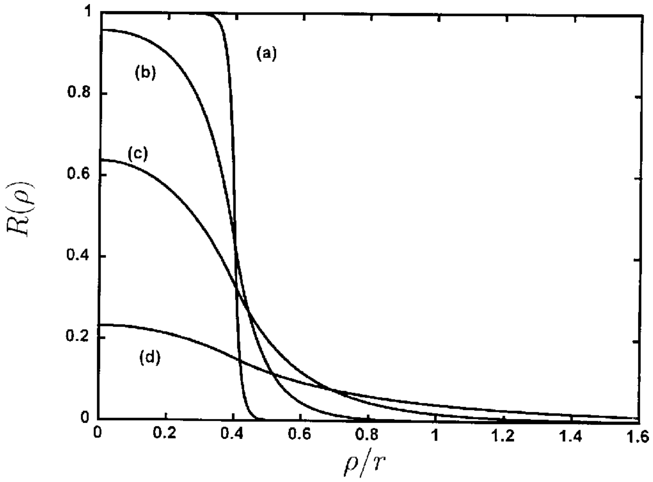

Figure 1. Radial reflectance per unit area $R(\rho)$ with $d = 0.4r$ and (a) $\bar{\rho} = 0.1r$, (b) $\bar{\rho} = 0.6r$, (c) $\bar{\rho} = 1.5r$, and (d) $\bar{\rho} = 4.0r$.

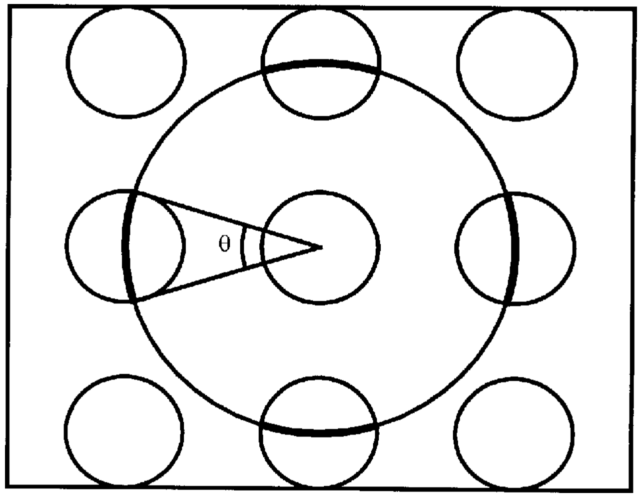

Figure 2. The small circles are dots, and the large circle has radius ρ. Light is incident through the central dot. The value $A(\rho)$ is the sum of the bold arc-lengths of the large circle divided by its radius. The value θ is the angle subtended by the bold arc-lengths.

entered the paper through the area of a dot of radius d centered on the origin and is given by:

$$R(x,y) = \frac{R_p}{S_0} \iint H(x-x', y-y') I(x', y') dx' dy', \qquad (3)$$

where S_0 is the number of photons incident on the paper per unit time, $H(x,y)$ is the paper's normalized point spread function, and $I(x,y)$ is the incident photon distribution. The value $H(x-x',y-y')$ is the probability that a reflected photon having entered the paper at x',y' exits the paper at x,y. The value $I(x,y)$ is the number of photons per unit area per unit time entering the paper at the point x,y and is given by:

$$I(x,y) = \frac{S_0}{\pi d^2} \mathrm{circ}\left[\frac{\sqrt{x^2+y^2}}{d}\right],$$

where d is the radius of the dots and circ $[\rho/d]$ is:

$$\mathrm{circ}\left[\frac{\rho}{d}\right] = \begin{cases} 1, & 0 \le \rho \le d \\ 0, & \rho > d \end{cases}.$$

The integral Eq. 3 is a convolution and can be evaluated as the inverse Fourier transform of the product of the transforms of $H(x,y)$ and $I(x,y)$. The transform of $H(x,y)$ is the paper's modulation transfer function (MTF) labeled $\tilde{H}(k)$, with k the spatial frequency (lines per unit length). Owing to the assumed isotropy of the point spread function, the MTF has circular symmetry.

The transform of the circ[ ] function is:

$$\mathcal{F}\left\{\mathrm{circ}\left[\frac{\sqrt{x^2+y^2}}{d}\right]\right\} = \pi d^2 \frac{J_1(2\pi k d)}{\pi k d},$$

where J_1 is a Bessel function.

Due to the circular symmetry, the inverse Fourier transform can be expressed as a Hankel transform, and one writes Eq. 3 as:

$$\frac{\pi d^2}{R_p} R(\rho) = 2\pi d \int_0^\infty \tilde{H}(k) J_1(2\pi k d) J_0(2\pi k \rho) dk, \qquad (4)$$

where ρ is the polar radial coordinate.

To evaluate Eq. 4, one must choose an appropriate point spread function. A widely used PSF is[6]:

$$H(\rho) = \frac{2\pi}{\bar{\rho}^2} K_0(2\pi\rho/\bar{\rho}),$$

where K_0 is a modified Bessel function of the second kind. The parameter $\bar{\rho}$ is $\bar{\rho} = 4 < \rho >$ and $< \rho >$ is the first moment of H called the lateral scattering length. It is the average lateral distance a photon travels within the paper, and its inverse, $< \rho >^{-1}$, is the approximate bandwidth of the paper. The MTF is:

$$\tilde{H}(k) = \frac{1}{1 + (\bar{\rho} k)^2}. \qquad (5)$$

Integrating Eq. 4 using Eq. 5, one finds:

$$\frac{\pi d^2}{R_p} R(\rho) = \begin{cases} 1 - (2\pi d/\bar{\rho}) K_1(2\pi d/\bar{\rho}) I_0(2\pi\rho/\bar{\rho}), & 0 \le \rho \le d \\ (2\pi d/\bar{\rho}) I_1(2\pi d/\bar{\rho}) K_0(2\pi\rho/\bar{\rho}), & d < \rho \end{cases}, \qquad (6)$$

where I_0 and I_1 are modified Bessel functions of the first kind. Figure 1 shows the radial reflectance Eq. 6, with $d = 0.4r$, for several different $\bar{\rho}$.

Radial Covering Distribution

The radial covering distribution, $A(\rho)$, is the probability that an arbitrary point at a distance ρ from the origin is covered by ink. The value $A(\rho)$ is the fraction of the circumference of a circle, centered on the origin with radius ρ, that lies on a dot. This is shown graphically in Fig. 2. The small circles are the dots, with radius d, and the large circle has a radius ρ. The variable $A(\rho)$ is the sum of the bold arc-lengths of the large circle divided by its circumference. If the dots overlap ($d > r/2$), then for some values of ρ the sum of the arc-lengths is larger than the circumference—in this case, all points of the large circle lie on a dot and $A(\rho) = 1$.

The value $A(\rho)$ is calculated as follows: We define the neighbor distribution $N(s)$ as the number of dots whose centers lie at a distance s from the origin. We define $\theta(s,\rho)$ as the angle subtended by the arc-length covering a dot whose center lies at a distance s, as shown in Fig. 2. Then, the radial covering distribution is:

$$A(\rho) = \frac{1}{2\pi} \int_0^\infty N(s) \theta(s,\rho) ds. \qquad (7)$$

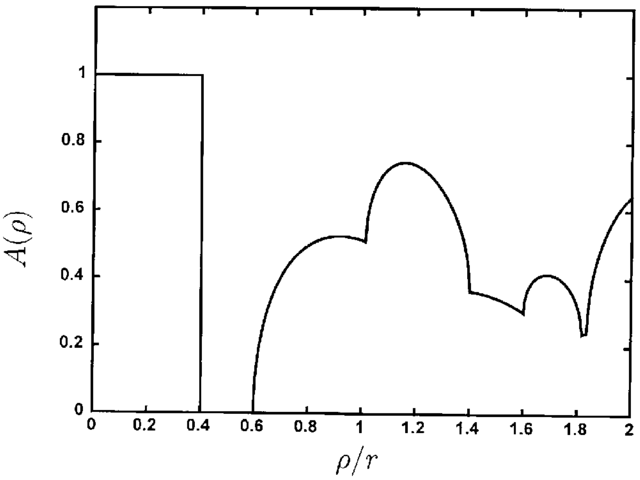

Figure 3. The value $A(\rho)$ with $d = 0.4r$.

Both $N(s)$ and $\theta(s, \rho)$ are derived in Ref. 2 and are given by:

$$\theta(s,\rho) = \begin{cases} 2\pi, & 0 \le \rho \le d - s \\ 2\arccos\left[\left(s^2+\rho^2-d^2\right)/(2s\rho)\right], & |s-d| \le \rho \le s+d \\ 0, & \rho \le s-d \text{ or } \rho \ge s+d \end{cases} \tag{8}$$

and

$$N(s) = \sum_{k=0}^{\infty} p_k \delta(x_k - s), \tag{9}$$

where $\delta(x)$ is a Dirac delta function and x_k is $r\sqrt{k}$ with k a natural number, and p_k is the number of combinations of integers n and m such that $k = n^2 + m^2$. The quantity x_k is the distance to the kth "set" of dots, and p_k is the number of dots in the "set"; i.e., the number of dots at a distance x_k. The first few x_k/r with nonzero p_k are 0, 1, $\sqrt{2}$, 2, $\sqrt{5}$, $\sqrt{8}$; and the corresponding p_k are 1, 4, 4, 4, 8, 4.[7]

Carrying out the integration in Eq. 7 and defining:

$$A_0(\rho) = \begin{cases} 1, & 0 \le \rho \le d \\ 0, & \rho > d \end{cases}$$

and for $k \ge 1$:

$$A_k(\rho) = \begin{cases} (p_k/\pi)\arccos\left[\left(x_k^2+\rho^2-d^2\right)/(2x_k\rho)\right], & x_k - d \le \rho \le x_k + d \\ 0, & \rho < x_k - d \text{ or } \rho > x_k + d \end{cases},$$

one obtains:

$$A(\rho) = \sum_{k=0}^{\infty} A_k(\rho). \tag{10}$$

Figure 3 shows $A(\rho)$ for dot radius $d = 0.4r$.

Equation 10 is correct for $d \le r/2$. If $d > r/2$, the right side of Eq. 10 is greater than 1 for some values of ρ, in which case one sets $A(\rho) = 1$.

Ink–Ink Probability

Inserting the expressions for $R(\rho)$, Eq. 6, and $A(\rho)$, Eq. 10, into Eq. 2, one obtains:

$$\frac{\pi d^2}{R_p} P_{ii} = 2\pi \int_0^d \left[1 - \frac{2\pi d}{\bar\rho} K_1(2\pi d/\bar\rho) I_0(2\pi\rho/\bar\rho\right] \rho d\rho + (2\pi)^2 \frac{d}{\bar\rho} I_1(2\pi d/\bar\rho) \sum_{k=1}^{\infty} \int_d^{\infty} K_0(2\pi\rho/\bar\rho) A_k(\rho) \rho d\rho. \tag{11}$$

Integrating the first term and dividing by πd^2 one obtains:

$$1 - 2K_1(2\pi d/\bar\rho) I_1(2\pi d/\bar\rho). \tag{12}$$

This expression is the probability that a reflected photon exits the paper through the same dot as that through which it entered the paper.

Integrating the second term, one obtains a sum of integrals of the form:

$$\int_{x_k-d}^{x_k+d} K_0(2\pi\rho/\bar\rho) \arccos\left[\frac{x_k^2+\rho^2-d^2}{2x_k\rho}\right] \rho d\rho. \tag{13}$$

These integrals can be evaluated numerically with little-trouble, however it is possible to get a very accurate closed-form expression by making two approximations. The first is an approximation to the arccos [ ]:

$$\arccos\left[\frac{x_k^2+\rho^2-d^2}{2x_k\rho}\right] \to \frac{1}{\rho}\sqrt{d^2-(x_k-\rho)^2}.$$

The second approximation is:

$$K_0(2\pi\rho/\bar\rho) \to K_0(2\pi x_k/\bar\rho)\exp\left[-2\pi(\rho - x_k)/\bar\rho\right],$$

for $x_k - d \le \rho \le x_k + d$.

The errors in these approximations tend to cancel each other for all d and $\bar\rho$ so that the expression

$$K_0(2\pi x_k/\bar\rho)\int_{-d}^{d} \exp\left[-2\pi u/\bar\rho\right]\sqrt{d^2-u^2}\,du \tag{14}$$

is a very accurate approximation to Eq. 13. The integral is easily evaluated, and one obtains for Eq. 14:

$$\frac{\bar\rho d}{2} I_1(2\pi d/\bar\rho) K_0(2\pi x_k/\bar\rho). \tag{15}$$

Inserting Eqs. 12 and 15 into Eq. 11, one obtains:

$$R_p^{-1}P_{ii} = 1 - 2K_1(2\pi d/\bar\rho)I_1(2\pi d/\bar\rho) + 2[I_1(2\pi d/\bar\rho)]^2 S(\bar\rho), \tag{16}$$

where we define:

$$S(\bar\rho) = \sum_{k=1}^{\infty} p_k K_0(2\pi x_k/\bar\rho). \tag{17}$$

The second term in Eq. 16, $2[I_1(2\pi d/\bar\rho)]^2 S(\bar\rho)$, is the probability that reflected photons exit the paper through dots other than the one through which they entered the paper.

It is convenient to express P_{ii} in terms of the fractional ink coverage rather than the dot radius. The percent area covered by ink, μ, is:

$$\mu = \begin{cases} \pi(d/r)^2, & 0 \le d \le r/2 \\ (\theta + \cos\theta)/(1+\sin\theta), & r/2 \le d \le r/\sqrt{2} \end{cases} \tag{18}$$

where

$$\theta = \frac{\pi}{2} - 2\arccos\left(\frac{r}{2d}\right).$$

The expression Eq. 16 is correct for $0 \le \mu \le \pi/4$. Numerical integration of Eq. 2 indicates that linear extrapolation of Eq. 16 for $\pi/4 \le \mu \le 1$ is an excellent approximation.

One then obtains for the ink-ink probability:

$$R_p^{-1}P_{ii}(\mu) = \begin{cases} 1 - \xi(\mu), & 0 \le \mu \le \pi/4 \\ 1 - \left[(1-\mu)/(1-\mu_0)\right]\xi(\mu_0), & \pi/4 \le \mu \le 1 \end{cases} \tag{19}$$

where

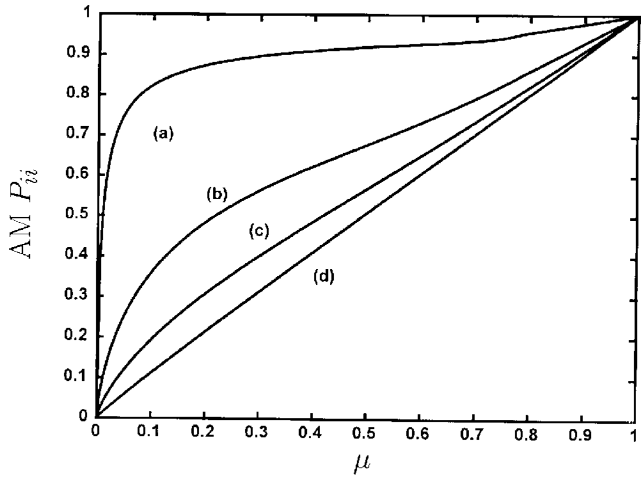

Figure 4. The value P_{ii} as a function of μ for various $\bar{\rho}$. (a) $\bar{\rho}$ = 0.2*r,* (b) $\bar{\rho}$ = 1.0*r,* (c) $\bar{\rho}$ = 2.0*r,* and (d) $\bar{\rho}$ = 6.0*r.*

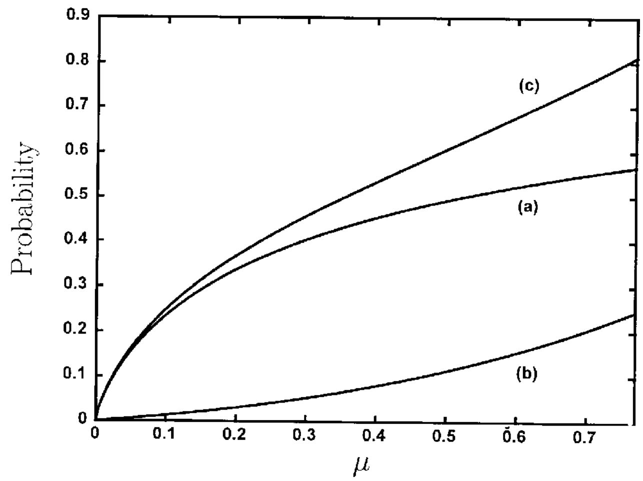

Figure 6. Comparision of the first and second terms of Eq. 16 with ρ = 1.5*r*. (a) Probability that photon exits incident dot. (b) Probability that photon exits any of the other dots. (c) Total probability that photon exits a dot, sum of (a) and (b).

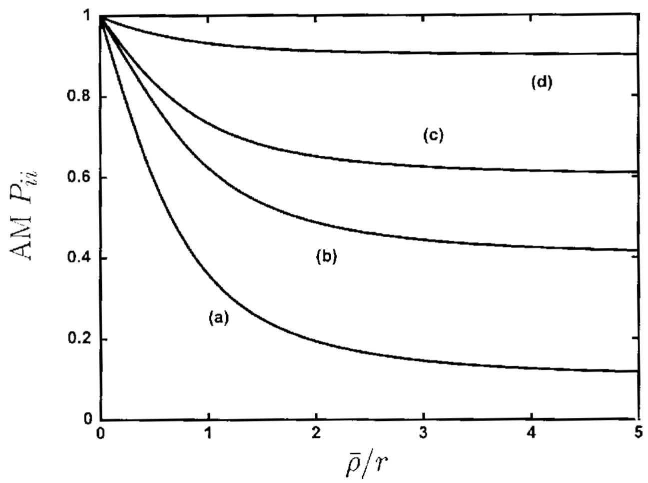

Figure 5. The value P_{ii} as a function of $\bar{\rho}$ for (a) μ = 0.1, (b) μ = 0.4, (c) μ = 0.6, and (d) μ = 0.9.

$$\xi(\mu) = 2I_1\left(2r\sqrt{\mu\pi}/\bar{\rho}\right)\left[K_1\left(2r\sqrt{\mu\pi}/\bar{\rho}\right) - I_1\left(2r\sqrt{\mu\pi}/\bar{\rho}\right)S(\bar{\rho})\right]$$

and $\mu_0 = \pi/4$. Note that ξ(μ) is the probability that a reflected photon exits the paper through a nonink region after entering through an inked region.

Figure 4 shows P_{ii} versus μ for several $\bar{\rho}$ and Fig. 5 shows P_{ii} as a function of $\bar{\rho}$ for several μ. In the figures, $\bar{\rho}$ is in units of *r*. As indicated by the curves in Fig. 5 and as can be shown by Eq. 16, if $\bar{\rho} >> r$, then $P_{ii} \approx \mu$. This corresponds to the case of "complete scattering".[1] Figure 6 shows the first and second terms in P_{ii} separately (as a function of μ) for $\bar{\rho}$ = 1.5. Curve (a) is the probability that the light exits through the incident dot, (b) is the probability it exits through the other dots, and (c) is the sum of (a) and (b). For convenience, we have set the paper reflectance equal to unity in all the figures.

FM Halftone Screen

In this section we calculate the ink-ink probability for an FM halftone screen. In such a method, all the dots have the same size and are square with dot area equal to a cell area and the number (or frequency) of dots is varied. There are a number of techniques for determining the exact placement of the dots.[8] The calculation done here is general in that our final result depends only on the average number of dots within a given region; we assume that within a region of constant tone, the dots are uniformly distributed. For ease in notation we assume the paper reflectance R_p is unity; for $R_p < 1$, the final expression for P_{ii} is multiplied by R_p.

We assume the dots are potentially located on a square grid array with period *r*. The dots are labeled by their coordinates *n*, *m*, with the photons entering the paper through the *n* = 0, *m* = 0 dot. We define P_{nm} as the probability that a photon having entered the paper through the dot 0, 0 exits the paper through the dot *n, m*. We also define the stochastic variable p_{nm} as:

$$p_{nm} = \begin{cases} 1, & \text{if there is a dot at } m,n \\ 0, & \text{if there is no dot at } m,n \end{cases} \tag{20}$$

subject to the constraint:

$$\lim_{N\to\infty} \frac{1}{N^2} \sum_{n,m=-N/2}^{N/2}{}' p_{nm} = \mu, \tag{21}$$

where the ′ on Σ indicates that the *n* = *m* = 0 term is excluded from the sum ($p_{00} \equiv 1$) and μ is the fractional ink coverage. The left side of Eq. 21 is the average p_{nm} so that:

$$\langle p_{nm} \rangle = \mu \tag{22}$$

(excepting the *n* = *m* = 0 term).

The ink–ink probability is obtained by first summing the probability that a photon exits the paper through the *n*, *m* cell, P_{nm}, over all cells that contain a dot (p_{nm} = 1), then averaging over all realizations of the p_{nm} consistent with Eq. 21:

$$P_{ii} = \left\langle \sum_{nm} p_{nm} P_{nm} \right\rangle = \sum_{nm} \langle p_{nm} \rangle P_{nm}. \tag{23}$$

For a uniform distribution, the average over all possible realizations of the p_{nm} is equivalent to the average defined by the left side of Eq. 21, so one can write:

$$P_{ii} = \mu\left[\sum_{nm} P_{nm} - P_{00}\right] + P_{00}. \tag{24}$$

As we assume R_p = 1, the sum is unity:

$$\sum_{nm} P_{nm} = 1, \tag{25}$$

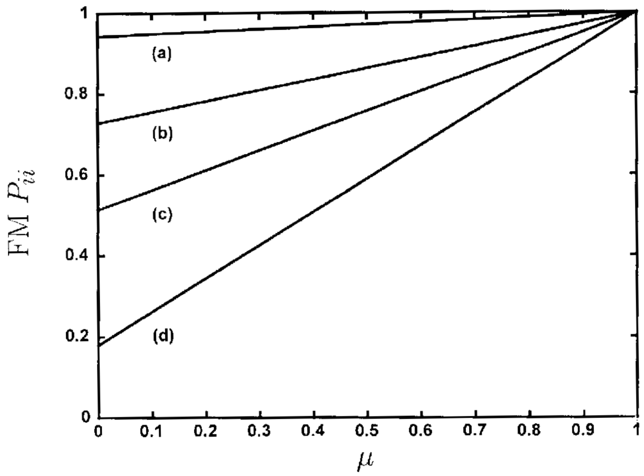

Figure 7. The value FM P_{ii} as a function of μ for (a) $\bar{\rho} = 0.2r$, (b) $\bar{\rho} = 1.0r$, (c) $\bar{\rho} = 2.0r$, and (d) $\bar{\rho} = 6.0r$.

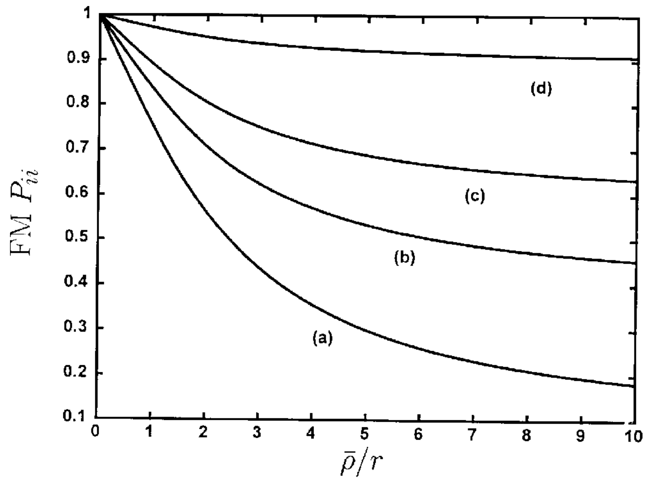

Figure 8. The value FM P_{ii} as a function of $\bar{\rho}$ for (a) μ = 0.1, (b) μ = 0.4, (c) μ = 0.6, and (d) μ = 0.9.

which simply states that the number of photons is conserved. The probability that the photons exit the same dot as that through which they entered the paper, P_{00}, is given by Eq. 12 (where we approximate the square dot with a circular dot with area equal to cell area) with $d = r/\sqrt{\pi}$, so the ink–ink probability is:

$$P_{ii} = 1 - (1 - \mu)\chi, \tag{26}$$

with:

$$\chi = 2K_1\left(2\sqrt{\pi}r/\bar{\rho}\right)I_1\left(2\sqrt{\pi}r/\bar{\rho}\right). \tag{27}$$

Unlike with the AM halftone screen, the probability here is linear with μ for all $\bar{\rho}$. The P_{ii} is shown as a function of μ for several different $\bar{\rho}$ in Fig. 7, and as a function of $\bar{\rho}$ for several different μ in Fig. 8. For $\bar{\rho} \gg r$, the AM $P_{ii}(\mu)$ is equal to the FM $P_{ii}(\mu)$.

Note that χ is the probability that a photon having entered the paper through a dot exits the paper *outside* the dot. The different terms of P_{ii} can be interpreted by writing Eq. 26 as $P_{ii} = 1 - \chi + \mu\chi$. In other words: [the probability that the photon exits through a dot (P_{ii})] = [the probability it exits within the dot through which it entered the paper $(1 - \chi)$] + [the probability there is a dot located at an arbitrary point (μ)] × [the probability the photon exits the paper outside the dot through which it entered (χ)].

Ink–Ink Probability for Diffusion PSF

The MTF of the diffusion point spread function is:[1]

$$\tilde{H}(k) = \frac{1}{R_p}\sum_{n=1}^{\infty}\frac{q_n/\sigma_n^2}{1+(2\pi k t/\sigma_n)^2}, \tag{28}$$

where q_n and σ_n are defined in Ref. 1 and t is the paper's thickness. The paper's reflectance is:

$$R_p = \sum_n q_n/\sigma_n^2,$$

and the lateral scattering length is:

$$\langle\rho\rangle = \frac{t\pi}{2R_p}\sum_n q_n/\sigma_n^3.$$

The diffusion ink–ink probability for AM screening has the same form as Eq.19 with ξ(μ) given by:

$$\xi(\mu) = \frac{2}{R_p}\sum_{n=1}^{\infty}\frac{q_n}{\sigma_n^2}I_1\left(r\sqrt{\mu/\pi}\sigma_n/t\right)\left[K_1\left(r\sqrt{\mu/\pi}\sigma_n/t\right) - I_1\left(r\sqrt{\mu/\pi}\sigma_n/t\right)S_n\right], \tag{29}$$

were S_n is given by:

$$S_n = \sum_{k=1}^{\infty} p_k K_0(\sigma_n x_k/t).$$

For FM halftone screening, P_{ii} has the same form as Eq. 26 with χ given by:

$$\chi = \frac{2}{R_p}\sum_{n=1}^{\infty}\frac{q_n}{\sigma_n^2}I_1\left(r/\sqrt{\pi}\sigma_n/t\right)K_1\left(r/\sqrt{\pi}\sigma_n/t\right). \tag{30}$$

Conclusion

In this article, we explicitly calculate the probability that a photon exits the paper through an inked region after originally entering the paper through an inked region, and we obtain a simple closed-form expression. This conditional probability completely contains the effects of optical dot gain; i.e., knowledge of this probability allows one to account for the effects of optical dot gain in a halftone print completely. We calculate the probability for both AM and FM halftone screening.

The results reported here also allow a simple calculation of the $\mathcal{Z}$ that appear in the theory of the multi-ink halftone image.[9]

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