Photography in the Service of Stereoscopy*

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Our gift of binocular vision, the principles of stereoscopic viewing methods, and the creation of stereoscopic images are all closely related. Here we recapitulate the geometry connecting these fields and provide information that can assist in generating effective stereoscopic image pairs.

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Introduction

Ever since its invention, photography has been a major source of image pairs to be used in a variety of stereoscopic viewing systems. The classical parlor stereograms, recent 3-D films at Disney theme parks, Imax 3-D presentations, and stereograms by scientists and engineers as well as by a vast number of amateur and professional stereographers rely largely on photographically produced images.

The aim of the present work is to enable one to determine *in advance* the necessary conditions for creating stereoscopic image pairs having proper parallactic shift for comfortable viewing and at the same time having correct framing that does not require custom cropping. Those familiar with stereoscopic photography will recognize that we refer to the lens interaxial distance and to the interaxial spacing between the respective image apertures.

General Considerations

Principles of Stereoscopic Viewing Systems. The origins of present-day stereoscopy go back to the work of Sir Charles Wheatstone[1] and Sir David Brewster[2] in mid-nineteenth century England. Devices constructed by both of these inventors share the same basic concept. These devices introduce in front of the observer's eyes a means of channeling the view seen by each eye along a separate path. At the far end of this path are located image pairs consisting of specially prepared art work or specially prepared photographs. The distinguishing characteristic of such an image pair is that they represent scenes or objects as seen from two different vantage points. In the process of stereoscopic viewing the two images are fused in the mind and merged to form a single 3-D illusion.

Stereoscopic viewing devices not only display the images selectively to our eyes but also assist in superimposing the images, using optical means such as lenses, mirrors, and prisms. Some systems physically overlay the images and rely for selective channeling on other optical techniques, such as encoding with polarizers, with complementary color filters, or with multiplexing rasters. Some observers can dispense with viewing devices by "commanding" their eyes to merge a pair of side-by-side images; this technique is known as *free vision*. This type of viewing is very simple; however, its success still requires a correct pair of stereoscopic images.

The Significance of Convergence and Accommodation in Binocular Vision and Stereoscopic Viewing. Normally we are accustomed to judging distances by subtle mental cues received from the muscular eye controls of convergence and accommodation. These two functions are *coupled*, and they operate in synchronism as we scan each detail of the scene observed. [Fig. 1(a)]. However, in stereoscopic viewing the tie between the accommodation and convergence functions of the observer's eyes must be *uncoupled*. The viewing distance to the right- and left-eye images remains constant, whereas the merged scene details of the 3-D image appear at the intersections of the individual lines of sight, as shown in Fig. 1(b).

Convergence is also important in the original capture of the stereoscopic image pair because it determines the *framing* of the subject matter within the apertures. One of the practical methods of achieving convergence is to allow the distance between the image apertures to be greater than the lens interaxial distance. The geometry and calculation of these distances are discussed in a later section.

Awareness of the Stereoscopic Window. The stereoscopic window is a concept that is represented in practically every stereoscopic viewing system. If we look into an empty stereoscopic slide viewer we see that the image apertures appear as the boundaries of a single opening, the *stereoscopic window*. A similar effect is observed by looking at two blank cards through a Brewster–Holmes parlor stereoscope. Whereas the edges of the window do not participate in the creation of the 3-D image, they provide an enclosure for the 3-D image. Similarly, the edges of a stereo print form a border for the 3-D image perceived inside.

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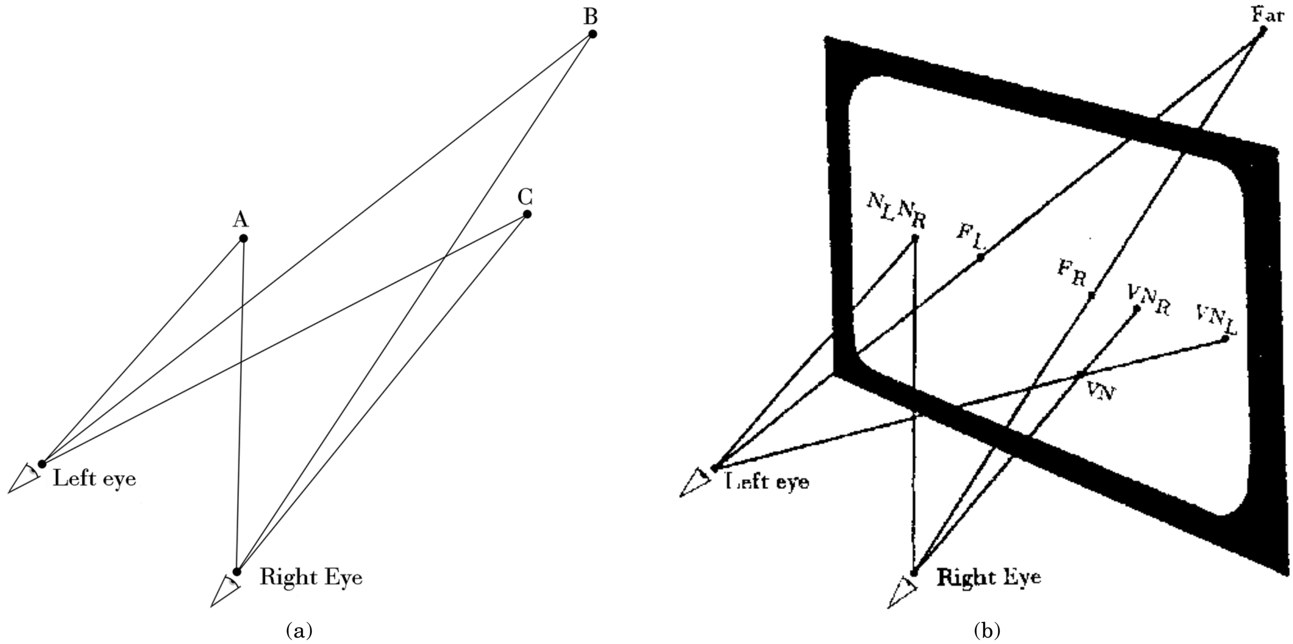

Figure 1. (a) Natural binocular vision, showing convergence at each of the three points A, B, and C; (b) viewing of a stereoscopic image pair, showing convergence at far (F), near (N), and very near (VN) points.

The stereoscopic window also forms an interface between the domains of coupling and uncoupling mentioned in the previous section. Within the domain bounded by the window the accommodation of the eyes remains unchanged, locked to the distance between the observer's eyes and the stereogram. In the case of lenticular stereoscopes the accommodation and focus are considerably relaxed, the focusing and convergence now being assisted by the lenses of the stereoscope.

What is even more important is that the stereoscopic window establishes a plane of reference for the illusionary stereo image. The image may appear in front of the window, in back of the window, or both in front and in back. The placement depends on the creative imagination of the photographer in establishing the position of the 3-D image in relation to the stereoscopic window.

Experimentation

The studies that follow originated with work by Rule,[3] who served as a consultant to Polaroid Corporation during the 1940s. Further experimental work was conducted by the author in the Polaroid Research Laboratories.[4]

The Geometry of Stereo Image Recording

Figure 2 is a perspective drawing of a typical camera setup, showing two lenses side-by-side, their image apertures, and the far and near points of the subject. To simplify the geometry, the far and near points, the center of the lens, and the center of the aperture for the left-eye camera are shown as collinear. Following is the key to the linear variables indicated:

- D = distance from lens to far point of subject or scene
- d = distance from lens to near point of subject or scene
- f = distance from lens to film plane
- p = parallax, the shift between furthest homologous points at the film plane
- T_1 = lens interaxial distance
- T_2 = center-to-center distance between image apertures
- w = image aperture width.

The parallax, p, should be known to the stereographer from the specifications of the particular imaging format and the intended viewing conditions. The focal length of the lens can be used as the lens-to-film distance except in the case of close-ups, which require the actual lens-to-film distance. The values sought are T_1 and T_2.

Establishment of Parallax, *p*. Parallax as a linear dimension is established on the basis of limitations imposed by the uncoupling of the convergence and accommodation functions of our eyes during stereoscopic viewing. Because of the tendency of favoring a level orientation during image capture, we often refer to parallax as the "horizontal displacement" or "horizontal parallax." We find that the linear size of the horizontal displacement is proportional to and limited by the viewing distance to the stereogram. Conversely, the viewing distance is a function of the size or width of the stereo image. Here we come to a point where an exact mathematical model perhaps does not exist. However, a ratio of parallax to image width of 1:24 emerges from the experience and actual practice of professional stereographers.

As an example, in the StereoRealist format the image width is 24 mm and the recommended parallax is 1 to 1.2 mm. For the 6 × 13 cm format the recommended parallax is[5a,6] approximately 2.5 mm. These specifications are stated as approximations because differences exist among observers and there is great forgiveness in the human visual system.

Calculation of T_1. For convenience we construct the line "L" in Fig. 2. Then

$$\begin{aligned} &\frac{p}{L} = \frac{f}{d} \text{ and } L = \frac{pd}{f}. \\ &\frac{T_1}{D} = \frac{L}{D-d}. \\ &T_1 = \frac{DL}{D-d} = \frac{p}{f} \cdot \frac{Dd}{D-d}. \end{aligned} \tag{1}$$

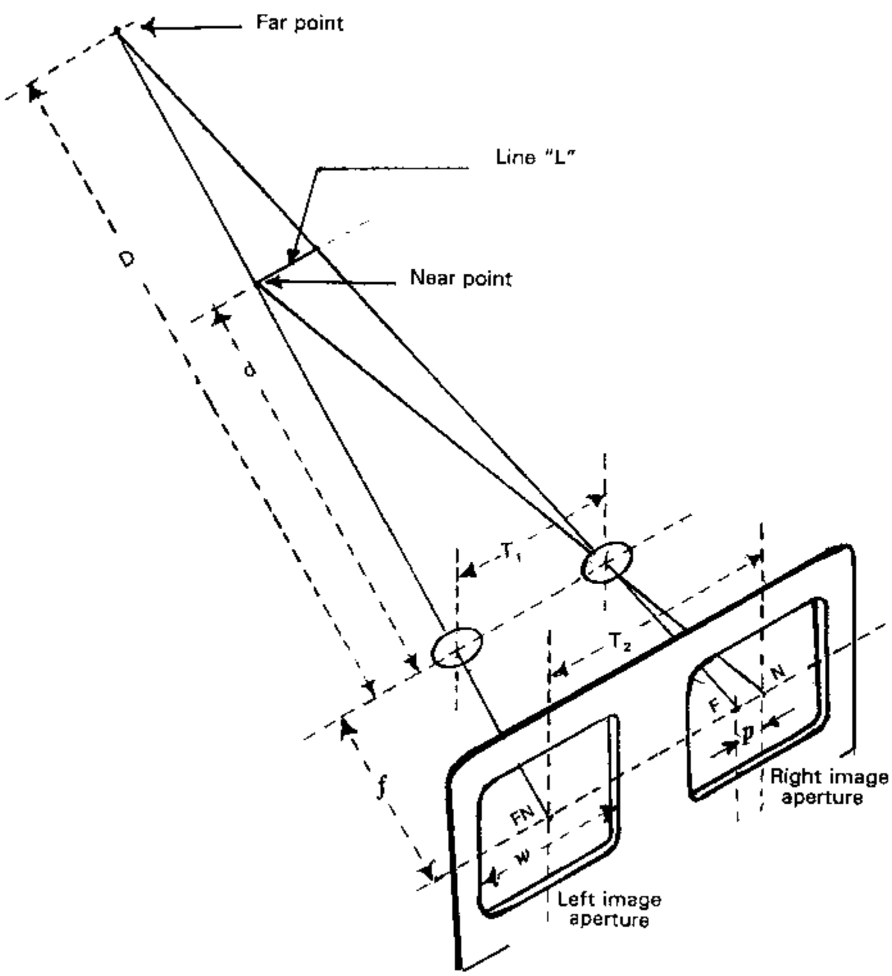

Figure 2. A typical stereoscopic camera setup, showing two side-by-side lenses of focal length f, separated by the distance T_1, and the two image apertures. The value T_2 is the center-to-center distance between the camera apertures, N is the near point of the scene, and F the far point. The value D is the distance from the lens plane to the far point, d is the distance from the lens plane to the near point, and p is parallax.

$$\text{If } D = \infty,\ T_1 = \frac{p}{f} \cdot d. \tag{2}$$

Calculation of T_2. The value of T_2 can be determined from the same diagram.

$$\frac{T_2}{d+f} = \frac{T_1}{d}.$$
$$T_2 = T_1 \frac{d+f}{d}. \tag{3}$$

Note that T_2 is only slightly larger than T_1. In some cases the construction of the stereo camera prevents adjusting the T_2 dimension—for example, on side-by-side 70-mm motion picture cameras—but still allows the adjustment of the interlens distance, T_1. This adjustment is usually small enough to achieve the desired T_1 and sufficient to provide the needed convergence between the camera units. An alternative is to use auxiliary optical devices.[7a]

A few custom cameras built for close-up work provide laterally adjustable lens mounts that allow for independent adjustment of T_2. A patent of Land, Batchelder, and Wolff describes a camera with fixed interaxial distance but with coupled convergence and focusing adjustments.[8] This feature made it possible to maintain the stereo window at the near distance.

Calculation of D and d. To calculate D and d we rewrite Eq. 1 as follows:

$$\frac{p}{f} \cdot \frac{1}{T_1} = \frac{1}{d} - \frac{1}{D}. \tag{4}$$

Then, for a given value of D,

$$\frac{1}{d} = \frac{1}{T_1} \cdot \frac{p}{f} + \frac{1}{D} \quad \text{and} \quad d = \frac{T_1 f D}{pD + T_1 f}. \tag{5}$$

Similarly, for a given value of d,

$$\frac{1}{D} = \frac{1}{d} - \frac{p}{T_1 f} \quad \text{and} \quad D = \frac{T_1 f d}{T_1 f - pd}. \tag{6}$$

The Design of a Lens Interaxial Calculator. To simplify and speed up the calculation of T_1 we designed the Polaroid Interocular Calculator.[9] The reciprocal form of Eq. 1, as shown in Eq. 4, enabled us to design this calculator in the form of a circular slide rule. An original calculator is shown in Fig. 4. A convenience of such a calculator is the rapid evaluation of any of the variables in cases where the equipment does not allow full control of needed adjustments. We provided several hundred of these calculators to stereo

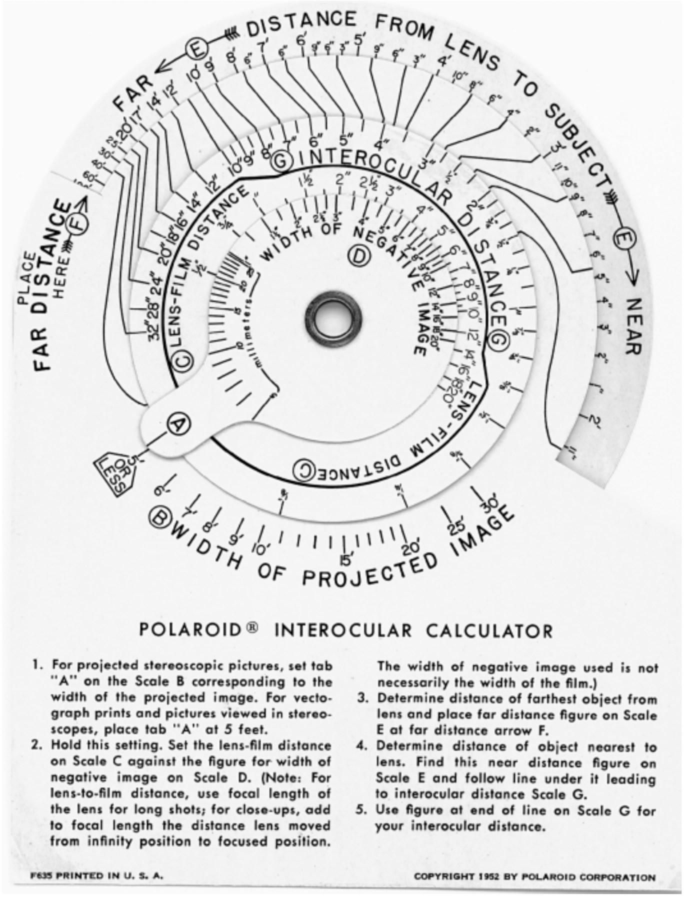

Figure 3. The Polaroid Interocular Calculator, a circular slide rule for determining appropriate lens interaxial distance, given the intended final width of the image to be viewed, the lens-film distance, the width of the negative image, and the near and far distances, (Reprinted with permission of Polaroid Corporation).

photographers here and abroad. The calculator has been cited in several publications on stereoscopic photography.[7b,10]

Before going further in our discussion it is appropriate to note the compatibility of the above calculations with traditional rules and recommendations.

The 1/30 Rule. Ferwerda[5b] and other writers on stereoscopy[11] often recommend an interaxial value of 1/30 the near distance. The origin of this rule lies in the assumptions that the subject extends to infinity and that the focal length of the lens is the conventional 1.25 times the image width. The rule is valid under these conditions and it is useful, but it does not apply to medium shots and close-ups. There it does not make full use of the available depth effect. Following is the rationale for the 1/30 rule.

If we solve Eq. 2 for the interaxial distance, T_1, given $f = 1.25\,w$ and $p = w/24$,

$$T_1 = \frac{w}{24} \cdot \frac{1}{1.25w} \cdot d = \frac{1}{30} \cdot d.$$

Sidestep Landscapes and Aerial Photographs. Both Wing[12] and Weiler[13] provide guidance for producing stereo image pairs with a single handheld camera. In these cases we again assume a camera with a conventional focal length lens. For the sidestep method described by Wing, it may be useful to assume that a 1-ft sidestep is good for a near distance of 30 ft, 2 ft for 60 ft, and so on.

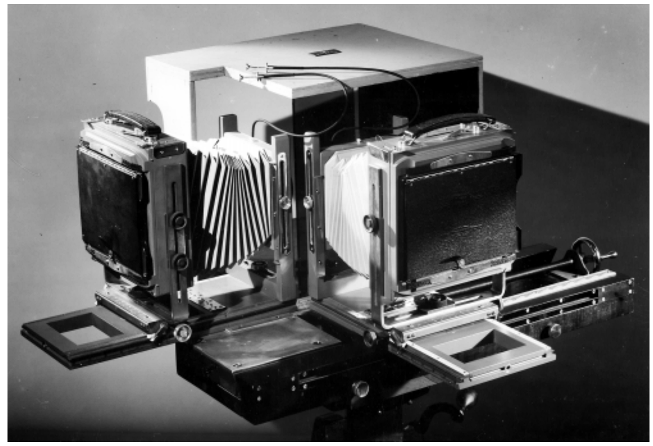

Figure 4. The experimental 5″ × 7″ studio stereo camera.

In the case of cloud pictures from an airliner window, here is some arithmetic. At 800 km/h, the plane travels 220 m/s. A rapid sequence stereo pair a half-second apart would make $T_1 = 110$ m. Multiplication by 30 gives a distance of about 3.3 km, a fair distance for pictures of cloud formations. Oblique shots may also be practical. At an altitude of 10,000 m, 1/30 is 333 m of travel, equivalent to about 1.3 s interval between shots.

Stereoscopic Cameras. Stereoscopic cameras follow the pattern of our natural binocular vision by recording a pair of images from two different vantage points. The traditional amateur stereoscopic camera is a side-by-side two-lens unit with adjustable focusing. The fixed lens interaxial distance is smaller than the aperture interaxial distance, resulting in a convergence that establishes the stereoscopic window at about 2 m from the camera.

Using a camera with fixed lens interaxial we can still obtain correct parallax if we balance the subject composition within proper limits of far and near distances, D and d, as indicated by Eqs. 5 and 6.

A number of stereo photographers use single or double cameras on a mechanical slide bar or have two cameras modified by permanent coupling. The greatest progress in stereo camera construction has been made in the field of professional motion picture equipment. Here we find single track, side-by-side format on 70-mm film and double-track 35-mm cameras, in both cases with elaborate optical and mechanical refinements.[7c]

For much of our research at Polaroid we used stereoscopic image pairs made with cameras such as the split-field 5″ × 7″ Speed Graphic and aerial reconnaissance photographs from government sources.

A 5" x 7" Studio Stereo Camera. We also constructed an experimental studio-type stereo camera (Fig. 5). We coupled two 5″ × 7″ Burke James view cameras on a slide-bar-type bed at 90° to one another, separated by a 45° semitransparent mirror. The large negatives and color transparencies were easy to evaluate, and both enlargements and reductions were made as needed. The lens interaxial distance could be adjusted from zero to about 20 cm, and the T_2 distance could be adjusted by the "sliding back" mechanism. Two 30-cm Wollensak Velostigmats were operated simultaneously by twin cable releases, and the whole assembly balanced well on an Ansco studio tripod.

Experimental Stereoscopic Images. We used the 5″ × 7″ camera and the Polaroid Interocular Calculator to generate a series of studio photographs that included long shots, medium shots, and close-ups. We also made photographs of tabletop subjects. We obtained very gratifying results confirming the validity and usefulness of a reference parallax-to-image width ratio of 1:24. We also found that a ratio of 1:50 was of limited practical use because it provided insufficient depth effect. A ratio of 1:100 gave still less parallax and approached the limits of stereo acuity. We concluded that the maximum parallax afforded by the ratio 1:24 was more satisfactory, even when it resulted in Lilliputism or other distortions. Our observations during recent years have been consistent with these findings.

Discussion

Geometric Characteristics of a Stereoscopic Image Pair. We have described the principles of stereoscopic viewing devices and the geometry of stereoscopic photography, and we have briefly discussed stereoscopic cameras and their use to achieve optimal stereoscopic effects. Now we can delineate the most important characteristics of a stereo image pair—first, the *parallax*, which will determine the visualized depth of the 3-D image, and second, the *framing*, which will locate the image within the 3-D space.

Methods of Measurement and Evaluation. Original stereoscopic negatives or transparencies are usually too small for superimposition over a light table and must be compared under magnification. Parallax is measurable if we superimpose in register the homologous points of the nearest detail and measure the shift of the homologous points of the farthest detail. Assuming that the images are presented in correct orientation, the shift will be displayed in the right-eye image. We can also observe and evaluate the coincidence of the image borders, which will give a clue about the correct framing.

Spatial Presentation. We can separate the presentation of the subject matter within the 3-D space into three groups: Group A, all of the image to appear beyond the stereo window; Group B, part of the image beyond the stereo window and part extending forward from the stereo window; and Group C, all of the image in front of the window . We could also add a subgroup to C for images that “float” in front of the window.

Group A. This is the most conventional case. A majority of stereo cameras are constructed with this type of presentation in mind. To have all of the image appear beyond the stereo window and be comfortable to view, the recommended maximum parallax is approximately 1/24 of the width of the image, as discussed earlier. With stereo prints to be viewed directly and transparencies projected onto screens, the 1:24 ratio will allow a maximum width of 1.5 m, at which point the lateral shift on the screen will reach its limit of 65 mm. For very large screens viewed at appropriate distances, the eyes will tolerate greater separation. Correct framing is achieved when the near homologous points are in register and the right and left borders of the stereo image coincide.

Group B. To obtain the effect of a partial image forward of the stereo window, we will need a total parallax greater than 1/24 of the image width. If we double this amount, we can use 1/24 for the portion behind the window and another 1/24 for the portion of the stereo image in front of the window. The borders of the image must coincide when the zero parallax homologous points are superimposed in register. For viewing comfort it is also desirable that the subject matter be composed fully within the borders of the stereo window. When viewed on a 1.5-m screen, such an arrangement should bring the near part of the image halfway between the observer and the screen. In other presentations the image's placement will vary according to the viewing distance.

Group C. When the entire image is created in front of the stereoscopic window, the observer's eyes will be in a convergent position. This situation seems to be more easily tolerated than looking at an image beyond the window, and the uncoupling of accommodation and convergence has a wider tolerance. In observing drawings of geometric solids that appear placed on a tabletop, a lateral shift of 1/10 of the image width is easy to accept. If the entire image is to appear in front of the stereo window, then the edges of the image pair should be in coincidence when the far homologous points are in register.

Summary

Properly applied, the tools we have described facilitate the generation of effective stereo images and permit the full enjoyment of stereoscopic presentations. The system may be summarized as follows:

1. Select the value of the parallax, p, according to the format of the camera and the requirements of the viewing method, including the choice of location of the stereo window.
2. Establish the near and far distances from lens to the subject or scene to be photographed.
3. Note the focal length of the taking lens or, for extreme close-ups, the lens-to-film distance.
4. Calculate T_1, using Eq. 1 or using an interocular calculator such as the one shown in Fig. 4.
5. According to the location of the stereo window in relation to the reconstructed image, calculate T_2, using Eq. 3.
6. Given a camera with fixed interaxial distance, calculate D and/or d to determine suitable stereo composition.

Alternative Methods for Achieving Parallax. Stereoscopic pairs generated by means other than paired camera exposures and intended for viewing by conventional stereo methods must follow similar guidelines for parallax and framing in the image output. The geometry of recording the stereo pairs will be specific to the technology of the image capture source.

Conclusion

Stereoscopic imaging provides an interface between the real-life scene and the creative presentation possibilities offered by various stereoscopic viewing methods. Parallax and convergence have been discussed as major contributing factors in the achievement of satisfactory stereoscopic results. We recognize the lack of suitable stereoscopic equipment for the full implementation of the material presented here. However, the information is still of value to any stereoscopist, whether a photographer with a conventional twin-lens stereo camera, a computer user generating stereoscopic images, an SEM microscopist, or a painter creating stereoscopic artwork.

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