Nip Width of an On-Demand Fuser: Part II. Longitudinal Model

W. Sean Harris, Edwin M. Odom and Steven W. Beyerlein

University of Idaho, Mechanical Engineering Dept., EPB 324K, Moscow, Idaho 83844-1030

This paper examines bending of the pressure roller and heating element in an on-demand fuser, a circumstance that leads to variation in distributed load along the length of the roller. These bending effects are coupled with the deformation of the rubber coating on the pressure roller and lead to a variation in nip width along the length of the fuser. A longitudinal model has been developed using the Rayleigh–Ritz method and a cross-section model described in a companion paper. This model predicts local nip width, centerline deflection of the pressure roller, compression of the rubber layer, and deflection of the heater element. This model can be applied in designing a crowned pressure roller and/or heater element that will ensure a constant nip width.

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Introduction

On-Demand Fusers. Many processes use rollers covered with a rubber layer to apply pressure to a surface. The rubber layer in these rollers conforms to the surface, spreading the pressure over a larger area. The contact area formed by a soft roller is referred to as the nip. One example of this type of roller is the pressure roller of an on-demand fuser in a laser printer. Other examples include the application of liquid coatings, cylindrical rollers in the paper making and textile industries, and friction drives.

In a laser printer, the pressure and temperature are elevated in the nip formed between a pressure roller and a heater element. These conditions cause the toner particles to melt and fuse to the paper. The performance of the fusing process is dependent on the nip width, which is ideally uniform along the length of the roller.

Nip Width Variation. The nip width may vary over the length of the roller. Bending can occur in either the pressure roller or the heater element, resulting in a nonuniform distributed load between the roller and heater element. Because the nip width is a function of distributed load, this causes a variation in nip width.

The nip width controls the residence time available for the fusing process to be completed. A uniform nip width across the fuser will result in a constant temperature across the page. This is desirable to obtain consistent fusing. The toner temperature must be maintained within a specific temperature window.[1] If the toner temperature is too low, the fusing quality is poor. If the temperature is too high, the toner sticks to the roller instead of the paper. A variation in toner temperature reduces the operating freedom within this window.

Paper Deformations. A variation in pressure distribution is also a cause of paper curling and wrinkling. Curling can occur when passing through the fuser because of nonuniform water removal from the paper.[2,3] The elevated temperature in the nip evaporates moisture in the paper. If this temperature is nonuniform across the paper, the evaporation is nonuniform. This creates residual stresses and causes the paper to curl. Wrinkling can occur because of nonuniform compression of the rubber coating along the roller. This causes the effective radius of the roller to vary along the roller, changing the paper speed. This variation of paper speed across the page produces paper wrinkling.

Roller Crowning. Currently, the approach to minimizing paper deformation and temperature variation is to crown the pressure roller and/or heater element. Crowning in this context means the geometry is modified so the first contact between the roller and heater element occurs at the midpoint of the roller. This increases the distributed load at the midpoint of the roller and counteracts the effects of bending in the pressure roller and heater element. The amount of crowning necessary is normally determined experimentally by testing different prototypes during the design process. This can take several iterations and is quite time consuming. To reduce development time for fuser designs, a longitudinal model is presented here for predicting the variation in nip width and distributed load for a fuser with a crowned pressure roller and/or heater element. This model is based on the Rayleigh–Ritz method using a series displacement function presented by Chou and Pagano.[4]

Fuser Description

The solution method presented here is valid for a pressure roller of any size or made of any material. In this study, a pressure roller from a laser printer fuser was used to validate the Rayleigh–Ritz analysis. The material properties and dimensions of this fuser are used as input to the longitudinal model. These results are then compared

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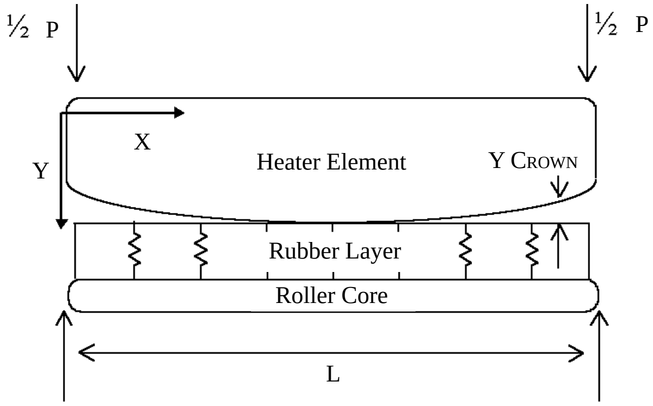

Figure 1. Schematic of longitudinal model.

to the measured nip width across the pressure roller. The material properties of this fuser are shown in Table I. The dimensions are shown in Table II.

TABLE I. Pressure Roller Material Properties

Component	Young's Modulus Symbol	Young's Modulus psi	Poisson's Ratio Symbol	Poisson's Ratio Value
Rubber	E_R	85.5	ν_R	0.5
Roller core	E_C	10.5*10^6	ν_C	0.3
Heater	E_H	10.5*10^6	ν_H	0.3

TABLE II. Pressure Roller Dimensions

Dimension	Symbol	Value
Length	L	8.69 in.
Diameter	D	0.625 in.
Rubber thickness layer	h	0.1162 in.
Roller core moment of inertia	I_C	1.166*10^{-3} in.^{4}
Heater moment of inertia	I_H	5.93*10^{-3} in.^{4}

Longitudinal Model

Variable Definition. Figure 1 shows a schematic of the longitudinal model. The roller core and the heater element are represented as beams, and the rubber coating separating the roller core and the heater is represented as a series of discrete springs. These discrete springs are shown only for illustrative purposes in this schematic. In the solution, the rubber coating is treated as a continuous spring extending over the entire length (L) of the roller. One half of the total load (P) is applied to each end of the roller.

In the design of pressure rollers, a uniform nip width along the roller is accomplished by crowning the pressure roller and/or heater element. This longitudinal model includes the effects of crowning. The amount of crowning is expressed by the variable y_{CROWN}, which is the space between the pressure roller and heater element under no load conditions. The variable y_{CROWN} is therefore a function of X and can be defined by Eq. 1. The coefficients c_n are determined by the geometry of the fuser. The compression of the rubber layer is therefore dependent on the crown of the roller as well as the deflection of the roller core and heater element. Equation 2 gives this relationship.

$$y_{\mathrm{CROWN}} = c_0 + \sum_{n=1,2,3...} c_n \cdot \sin\left(\frac{n \cdot \pi \cdot x}{L}\right), \tag{1}$$

$$\delta = y_H - y_C - y_{\mathrm{CROWN}}. \tag{2}$$

Assumptions. This model assumes the rubber layer acts as Winkler foundation, where spring deflection is proportional to the distributed load at any point. It also assumes that the spring stiffness, as well as the stiffness of the roller core and of the heater element, is uniform over the length of the roller. Equal loads are applied at each end of the roller in this model.

Rayleigh–Ritz Method. The theorem of least work states that among all the stress distributions in a system that satisfy equilibrium as well as all the boundary conditions, the true stress distribution has the least elastic energy.[5] Therefore, if it was possible to change the deflections in a system without altering the loads (by a virtual displacement), the energy would increase. This change in energy is equal to the change in internal energy (U) minus the work done by the external forces (W). This value is termed the potential energy of the system (Π) and is shown in Eq. 3. Because the potential energy is a minimum for the equilibrium position, the derivative of the potential energy is zero only at the true stress state, as shown in Eq. 4. This is known as the principle of minimum potential energy.[6]

$$\Pi = U - W, \tag{3}$$

$$\delta\Pi = \delta(U - W) = 0. \tag{4}$$

The Rayleigh–Ritz method uses this principle of minimum potential energy to calculate the deflections for the equilibrium position. This method assumes a displacement function with unknown coefficients. The potential energy of the system is calculated based on this displacement function. Because the potential energy is at a minimum for the equilibrium position, the derivative of the potential energy with respect to any of the unknown coefficients is zero. By solving this system of equations, the coefficients a_0, a_n, and b_n are determined. The displacement functions used for the roller core (y_C) and the heater element (y_H) are shown in Eqs. 5 and 6.

$$y_{\mathrm{C}} = a_0 + \sum_{n=1,2,3...} a_n \cdot \sin\left(\frac{n \cdot \pi \cdot x}{L}\right), \tag{5}$$

$$y_{\mathrm{H}} = \sum_{n=1,2,3...} b_n \cdot \sin\left(\frac{n \cdot \pi \cdot x}{L}\right). \tag{6}$$

These displacement functions were suggested by Chou and Pagano[4] to satisfy the boundary conditions of the problem. At $x = 0$ and $x = L$, the deflection boundary conditions of the problem dictate that the deflection of the heater (y_H) must be zero. Additionally, the loading boundary condition requires that the moment, which is proportional to the second derivative of the displacement function, is zero at $x = 0$ and $x = L$.

The strain energy in the roller core and heater due to bending can be expressed in terms of their displacement functions. The internal energy (U) of a beam expressed in terms of its deflections is shown[7] in Eq. 7.

$$U = \int_0^L \frac{EI}{2}\left(\frac{d^2}{dx^2} y\right)^2 dx. \tag{7}$$

By substituting the displacement functions for the roller core and heater element from Eqs. 5 and 6 into Eq. 7, the internal energy of the roller core (U_C) and heater (U_H) is found.

$$U_C = \int_0^L \frac{EI}{2}\left(\frac{d^2}{dx^2}\left(a_0 + \sum_{n=1,2,3...} a_n \cdot \sin\left(\frac{n*\pi*x}{L}\right)\right)\right)^2, \quad (8)$$

$$U_H = \int_0^L \frac{EI}{2}\left(\frac{d^2}{dx^2}\left(\sum_{n=1,2,3...} b_n \cdot \sin\left(\frac{n*\pi*x}{L}\right)\right)\right)^2, \quad (9)$$

From Part I of this paper, the compression of the rubber layer (δ) was predicted as a function of pressure roller size, rubber material, and load.[8] This correlation is given in Eq. 10. Values for α_0, α_1, and α_2 are reported in Part I of this paper.[8] The compression of the rubber layer is used to calculate the internal energy in the rubber layer (U_R).

$$\left(\frac{\delta}{D}\right) = \alpha_0\left(\frac{h}{D}\right)^{\alpha_1} \cdot \left(\frac{w}{DE_R}\right)^{\alpha_2}. \quad (10)$$

To simplify the longitudinal model, the nonlinear behavior in Eq. 10 is approximated by an equivalent linear spring. This is done by linearizing the deflection about the nominal load (w_n). This nominal load is defined as the total load applied to the roller (P) divided by the length (L). The resulting linear spring is described by Eq. 11. The coefficients k_0 and k_1 for the linear spring can be calculated from calculus principles[9] and the results are shown in Eqs. 12 and 13.

$$w = k_0 * (\delta - k1), \quad (11)$$

$$k_0 = \frac{(DE_R)^{\alpha_2}}{D \cdot \alpha_0 \cdot \alpha_2 \cdot \left(\frac{h}{D}\right)^{\alpha_1} w_n^{(\alpha_1 - 1)}}, \quad (12)$$

$$k_1 = D \cdot \alpha_0 \cdot \left(\frac{h}{D}\right)^{\alpha_1}\left(\frac{w_n}{DE_R}\right)^{\alpha_2} - \frac{w_n}{k_0}. \quad (13)$$

The energy per unit length of this approximate linear spring is then $1/2*w*(\delta - k_1)$. The total energy of the rubber coating is found by integrating this over the length of the roller, as done in Eq. 14.

$$U_R = \int_0^L \frac{1}{2} \cdot w \cdot (\delta - k_1) \cdot dx. \quad (14)$$

Replacing the distributed load (w) in this equation with Eq. 11 and replacing the rubber layer compression (δ) with Eq. 2 results in Eq. 15. The internal energy of the rubber layer is now expressed in terms of the unknown displacement function coefficients a_n and b_n.

$$U_R = \frac{k_0}{2} \cdot \int_0^L (y_B - y_C - y_{CROWN} - k_1)^2 \cdot dx. \quad (15)$$

The deflection at each of the loading points on the roller is simply a_0 due to the form of the displacement function. Therefore the virtual work (W) done by the applied loads is $P*a_0$. The potential energy equation for this problem is, therefore, $\Pi = (U_C + U_H + U_R) - P*a_0$. The coefficients of a_n and b_n are calculated by taking the derivative of the potential energy (Π) with respect to each individual coefficient and setting the results equal to zero.

Results

The values for the coefficients a_0, a_n, and b_n are displayed below in Eqs. 16 through 20.

From the calculated values of a_0, a_n, and b_n, the displacement of the roller core and heater element can be calculated by summing the sine terms in Eqs. 5 and 6. Summing the first five terms of these series is sufficient to converge within 1% of the exact deflection. The distributed load (w) along the length of the roller can be calculated directly from these displacement functions using Eq. 21.

From Part I of this paper, the nip width can be predicted as a function of pressure roller size, rubber material, and load. This correlation is given in Eq. 22.

For the Term a_0

$$a_0 = \frac{\frac{-P}{k_0 \cdot L} + \frac{2}{\pi}\sum_{(n=1,3,5)}\left[\frac{E_C \cdot I_C \cdot L^4 \cdot k_0 \cdot (4 \cdot k_1 + 4 \cdot c_0 + c_n \cdot \pi \cdot n) + E_H \cdot I_H \cdot L^4 \cdot k_0 \cdot (4 \cdot k_1 + 4 \cdot c_0 + c_n \cdot \pi \cdot n)}{E_H \cdot I_H \cdot \pi^5 \cdot n^6 \cdot E_C \cdot I_C + (E_C \cdot I_C + E_H \cdot I_H) \cdot L^4 \cdot k_0 \cdot \pi \cdot n^2}\right] - \frac{2}{\pi} \cdot \sum_{(n=1,3,5...)} \frac{c_n}{n} - c_0 - k_1}{\left[1 - \frac{2}{\pi}\sum_{(n=1,3,5)}\left[\frac{4 \cdot E_C \cdot I_C \cdot L^4 \cdot k_0 + 4 \cdot E_H \cdot I_H \cdot L^4 \cdot k_0}{E_H \cdot I_H \cdot \pi^5 \cdot n^6 \cdot E_C \cdot I_C + (E_C \cdot I_C + E_H \cdot I_H) \cdot L^4 \cdot k_0 \cdot \pi \cdot n^2}\right]\right]}. \quad (16)$$

For Even Terms of *n*, the Coefficients are

$$a_n = \frac{-E_H \cdot I_H \cdot L^4 \cdot c_n \cdot k_0}{E_H \cdot I_H \cdot \pi^4 \cdot n^4 \cdot E_C \cdot I_C - E_H \cdot I_H \cdot L^4 \cdot k_0 - L^4 \cdot k_0 \cdot E_C \cdot I_C}, \quad (17)$$

$$b_n = \frac{E_C \cdot I_C \cdot L^4 \cdot c_n \cdot k_0}{E_H \cdot I_H \cdot \pi^4 \cdot n^4 \cdot E_C \cdot I_C - E_H \cdot I_H \cdot L^4 \cdot k_0 - L^4 \cdot k_0 \cdot E_C \cdot I_C}. \quad (18)$$

For Odd Terms of *n*, the Coefficients are

$$a_n = \frac{-E_H \cdot I_H \cdot L^4 \cdot k_0 \cdot 4 \cdot a_0 - 4 \cdot k_1 - 4 \cdot c_0 - c_n \cdot \pi \cdot n}{E_H \cdot I_H \cdot \pi^5 \cdot n^5 \cdot E_C \cdot I_C - E_C \cdot I_C - E_H \cdot I_H \cdot L^4 \cdot k_0 \cdot \pi \cdot n}, \quad (19)$$

$$b_n = \frac{-E_C \cdot I_C \cdot L^4 \cdot k_0 \cdot (4 \cdot a_0 - 4 \cdot k_1 - 4 \cdot c_0 - c_n \cdot \pi \cdot n)}{E_H \cdot I_H \cdot \pi^5 \cdot n^5 \cdot E_C \cdot I_C - (E_C \cdot I_C - E_H \cdot I_H) \cdot L^4 \cdot k_0 \cdot \pi \cdot n}. \quad (20)$$

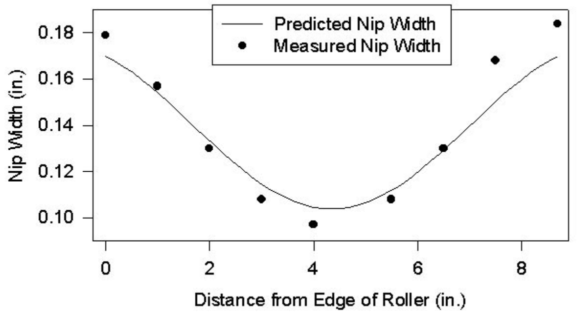

Figure 2. Nip width measurements. (For pressure roller material properties from Table I, dimensions from Table II, and a load of 15 lb.)

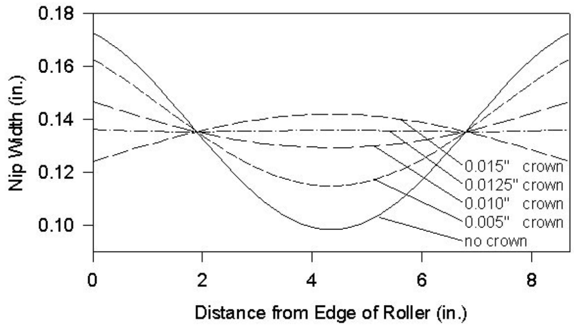

Figure 3. Nip width variation as a function of roller crown. (For pressure roller material properties from Table I, dimensions from Table II, and a load of 15 lb.)

$$w = k_0 \bullet [(y_H - y_C - y_{\mathrm{CROWN}}) - k_1]. \tag{21}$$

$$\left(\frac{S}{D}\right) = \beta_0 + \beta_1\left(\frac{h}{D}\right) + \beta_2\left(\frac{w}{DE_R}\right) + \beta_3\left(\frac{h}{D}\right)^2 + \beta_4\left(\frac{w}{DE_R}\right)^2 + \beta_5\left(\frac{h}{D}\right)\left(\frac{w}{DE_R}\right). \tag{22}$$

The coefficients for this function are reported in the companion paper.[8] This relationship, along with the calculated distributed load, can be used to predict the nip width along the length of the roller.

Model Verification

A fixture was designed and fabricated to measure the nip width of a pressure roller pressed against a flat surface. This fixture presses a pressure roller against an acrylic window by applying an equal load at each end of the roller. The nip can be viewed through the window on the bottom of the fixture, and precisely measured using a direct view microscope.

A total load of 15 lb was applied to a pressure roller with the material properties and dimensions shown in Tables I and II. The nip width was measured at 1 in. intervals across the roller. These measurements are compared with the predicted values from the longitudinal model in Fig. 2. Both methods show a significant variation of nip width along the length of this roller. This is because the pressure roller was pressed against a flat surface. Use of a crowned surface would change the nip width distribution. The correct amount of crown will cause a uniform nip width distribution.

Design for Uniform Nip Width

As shown before, a rubber-coated roller pressed against a flat surface can have a considerable variation in nip width. However, it is possible to prevent this by crowning the roller. This increases the distributed load and the nip width at the center of the roller. The example below shows how selecting the correct crown for a roller can result in a uniform nip width across the roller. These calculations account for the fact that the heater element is not perfectly rigid and will bend under a load. The roller dimensions and material properties for this calculation are the same as in the previous example and are found in Tables I and II. However, in this case the roller is crowned so the middle of the roller is closer to the base than the roller ends. This crown was varied, and Fig. 3 shows that a crown of 0.0125 in. would result in a uniform nip width along the length of the roller.

By this method, a roller can be quickly designed that will have a uniform nip width. In the case of a fuser in a laser printer, this roller configuration would result in uniform heat transfer from the heater element to the paper and consistent fusing quality across the page. It would also result in uniform water removal from the page, reducing paper wrinkling.

Conclusion

A longitudinal model has been developed for predicting variation in nip width across the entire length of an on-demand fuser. A series displacement function was used in conjunction with a Rayleigh–Ritz method and the cross-section model presented in the companion paper[8] to predict the deflections of the pressure roller core, the rubber layer, and heater element. From these deflections, the local nip width for the fuser examined in this work was found to be within 10% of values measured with a nip viewing fixture.

A uniform nip width can be achieved by crowning either the pressure roller or the heater element. A uniform nip width occurs when there is a uniform distributed load. From the standpoint of the longitudinal model, no distinction is made as to whether the crown is in the pressure roller or the heater element. Crowning the pressure roller, however, will lead to variation in roller speed across the paper. If this is excessive, problems with paper wrinkling may result. Crowning the heater element does not have this effect. For this reason, it may be preferable to crown the heater element rather than the pressure roller.

It is desirable to print on media such as thick paper, rough paper, or multiple types of envelopes. These types of media can greatly affect the load distribution and nip width across the fuser. With an envelope, for example, the pressure roller would not be pressed against a flat surface. It would be pressed against a surface that is raised by the thickness of the envelope. This would concentrate load on the envelope, increasing the nip width. The model presented in this paper could be directly applied to predict these effects. Because this model treats the surface of the roller as a series function, then any surface profile can be accommodated by this model if enough terms are included. The raised profile of an envelope could be modeled this way, as well as an idealized model of rough paper.

Nomenclature

Symbol		Description	Units
D	=	Roller cross-section diameter	in.
h	=	Rubber coating thickness	in.
L	=	Length of roller	in.
P	=	Total load applied to the roller	lb
w	=	Applied load per unit length on roller	lb/in.
w_n	=	Nominal load (average load along the roller)	lb/in.
S	=	Nip width (width of contact patch between roller and surface)	in.
δ	=	Compression of rubber layer	in.
E_R	=	Young's modulus of rubber layer	psi
E_C	=	Young's modulus of roller's solid core	psi
E_H	=	Young's modulus of heater	psi
I_C	=	Moment of inertia of roller's solid core	in.^{4}
I_H	=	Moment of inertia of heater	in.^{4}
y_C	=	Deflection of the roller core	in.
y_H	=	Deflection of the heater	in.
y_{CROWN}	=	Crown of the roller (distance separating the roller and heater)	in.
k_0,k_1	=	Coefficients for equivalent linear spring	lb/in.^{2}, in.
U_C	=	Internal energy of the roller core	in.-lb
U_H	=	Internal energy of the heater	in.-lb
U_R	=	Internal energy of the rubber layer	in.-lb
U	=	Total internal energy	in.-lb
W	=	Work done by external forces	in.-lb
Π	=	Potential energy	in.-lb

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