Coating Weight Uniformity and Die Internal Design

John Chong

Atlas Consulting Engineers, Inc., 20 Longview Dr., Holmdel, New Jersey 07733

Based on the variational methods, design equations for coating dies that are widely used in the photographic emulsion coating industries have been obtained in analytical forms, and the key dimensionless parameters that affect the coating weight uniformity have been identified. The effects on coating weight uniformity of a tapered angle in the die distribution chamber have been investigated and its performance is compared with nontapered dies. The coating weight uniformity index has been defined in terms of the pressure ratio in the distribution chamber and sample calculations have been made for each die to show the effects of die internal dimensions on coating weight uniformity indexes. Other factors that could also affect the coating weight uniformity, such as the precision of machining of the die slot opening, a distorted wetting line on the lower pressure side coating bead meniscus, and turbulent eddies in the vacuum box, have been discussed briefly.

Journal of Imaging Science and Technology 41: 41–47 (1997)

Introduction

In a number of industrial coating processes, very wide extrusion dies are used to form thin, uniform layers. Good uniformity in these layers is extremely important from a product quality point of view. In the photographic film manufacturing industry in particular, nonuniformity of at most a few percent variation in coating weight across the width can be tolerated. Coating dies must, therefore, be designed carefully to meet such stringent requirements. Despite this fact, the design of coating dies used in industries, even for simple Newtonian fluids, has been very much heuristic based on trial and error.

A typical extrusion coating die used in modern photographic emulsion coatings with a cascade or bar coater consists of a distribution chamber or cavity with a very narrow slot attached to it. In a multilayer coating operation, thin fluid layers coming out of each slot join together on the slide without mixing and flow down on an inclined surface before it is coated on a moving substrate. The uniformity of the fluid layers coming out of the slots is dependent not only on the dimensions and shape of the die distribution chamber, but also on the dimensions of the slots, and on the rheological properties of each fluid. A number of theoretical equations for the layer uniformity have been published in the literature[1–5] for a Newtonian fluid for simple die distribution chambers having cylindrical and rectangular shapes. However, in practice, the industry very seldom uses coating dies with such simple shape distribution chambers in order to prevent the formation of circulating fluid pockets, which could generate bubbles or cause thermal degradation. It is the purpose of this paper to obtain design equations in closed analytical form for coating of the dies that are most widely used in the photographic emulsion coating process and to compare the results obtained with different die dimensions.

In the following sections, we present the mathematical formulations of the problem using the variational methods. This method is very powerful for solving complex engineering problems where analytical solutions are often prohibitive. Some sample calculations are made for the uniformity based on the realistic die dimensions for comparison.

Theoretical Development

Basic Fluid Mechanic Equations. We consider an isothermal two-dimensional flow of a Newtonian fluid through a coating die distribution chamber as shown in Figs. 1 and 2. The basic momentum equation governing the slow viscous flow, where the inertia terms are neglegible, is given by the following equation:

$$\frac{\partial P}{\partial x} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{1}$$

where u is the fluid velocity at a point in the die cavity, P is the pressure, z is the direction of flow, and μ is the viscosity. Equation 1 is to be solved subject to the boundary condition that at the walls of the die cavity, the fluid velocity is zero.

We use the two-step method to calculate the uniformity of the thin fluid layer coming out of the slot attached to the die distribution chamber. In the first step, we calculate the velocity field of the fluid inside the distribution chamber without a slot attached to it and use it as

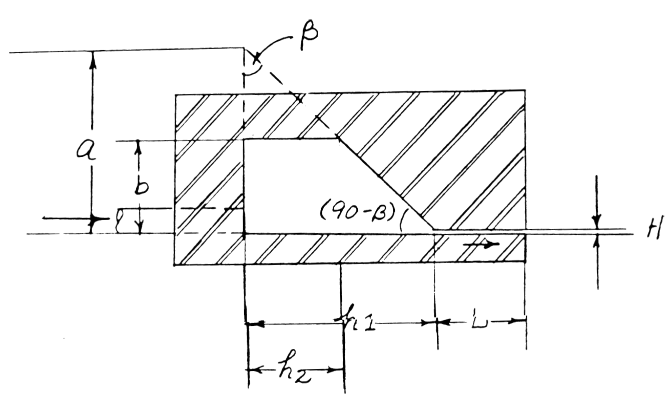

Figure 1. Cross section of die distribution chamber.

Original manuscript received February 4, 1995. Revised May 15, 1996.

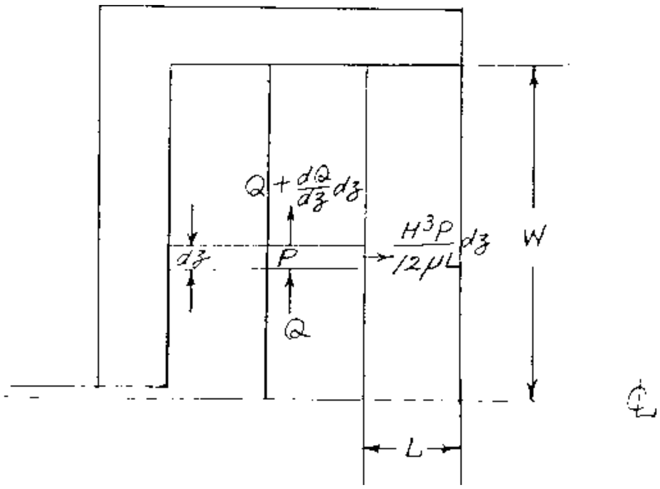

Figure 2. Top view of the die distribution chamber and slot.

an approximation for the real case where the die has a slot attached. In the second step, the resulting velocity profile is coupled with the velocity field in the slot by means of a material balance. This method was first used successfully by Carley[4] for a cylindrical die cavity to calculate the uniformity of Newtonian and low-power fluids flowing out of narrow slots. However, when the die internal geometry is complicated, obtaining analytical solutions in closed forms for Eq. 1 is almost prohibitive.

Formulation of Variational Problems. Equation 1 can be solved using the well-known variational method.[6,7] This method is very powerful and can be applied to a fairly complex engineering problem to obtain an approximate design equation where the conventional method is prohibitive because of the complexities in the internal geometries. The method is essentially based on the principle of minimum entropy production or minimum energy dissipation. The velocity field governed by Eq. 1 must be such that the total energy dissipated in the die cavity is an extremum. This principle, when applied to Eq. 1 takes the following mathematical form[6]:

$$I = \iint_D \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 + \frac{2}{\mu} \frac{\partial P}{\partial z} u \right] dx\, dy. \tag{2}$$

It can be readily shown that the Euler–Lagrange equation, which stems from requiring I to be stationary, is[6]:

$$\frac{\partial F}{\partial u} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial U_x} \right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial U_y} \right) = 0, \tag{3}$$

where F is the integrand of Eq. 2, and U_x and U_y are the velocity gradients in the direction of the subscripts. Because of the nonequilateral parallelogram shape die distribution chamber as shown in Fig. 1, it is convenient to transform the coordinates into a new coordinate system as shown in Fig. 3.[7]

A simple geometrical consideration shows that the following relationships exist:

$$\frac{a-b}{a} = {}^{h_2}\!/_{h_1} = r,\ a = h_1 (\tan\beta)^{-1}.$$

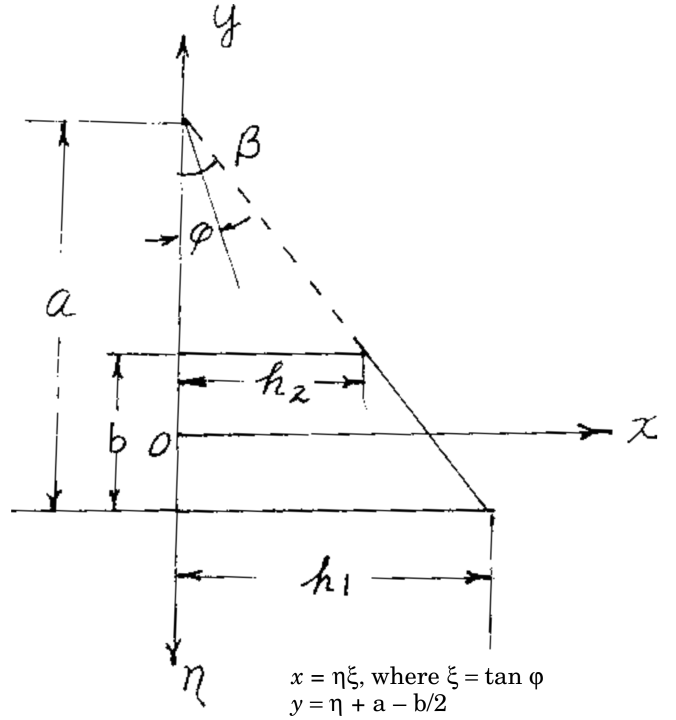

Figure 3. New coordinate system.

Equation 2 now becomes in the new coordinate system[7]:

$$I = \int_0^{\tan\beta} \int_{a-b}^{a} \left[\frac{1+\xi^2}{\eta^2} \left(\frac{\partial u}{\partial \xi} \right)^2 + \left(\frac{\partial u}{\partial \eta} \right)^2 - \frac{2\xi}{\eta} \left(\frac{\partial u}{\partial \xi} \right) \left(\frac{\partial u}{\partial \eta} \right) - Cu \right] \eta\, d\xi\, d\eta,$$

where

$$C = \frac{2}{\mu} \frac{\partial P}{\partial z}. \tag{4}$$

We now postulate that the velocity field in the die cavity can be approximated by the following equation:

$$U = A_0 (\eta^2 - a'^2)(\eta^2 - a^2)\xi^2(\xi^2 - \tan^2\beta), \tag{5}$$

where $a' = a - b$, and A_0 is a constant to be determined later. It is clear that the velocity field given by Eq. 5 satisfies the appropriate boundary conditions which are given below:

$$U = 0 \text{ at } \eta = a,\ \eta = a' \text{ for all } \xi,$$
$$U = 0 \text{ at } \xi = 0,\ \xi = \tan \xi \text{ for all } \eta.$$

≠ Equation 5 shows that the velocity distribution inside the die cavity is dependent on the angle, β. We show later that for a given die cavity dimension, there is a certain range in angle, β, where the assumed velocity distribution given by Eq. 5 is a good approximation. Differentiation of Eq. 5 for each velocity gradient and its square products yields:

$$\left(\frac{\partial u}{\partial \xi} \right)^2 = A_0^2 \left(\eta^2 - a^2 \right)^2 \left(\eta^2 - a'^2 \right)^2 \left(4\xi^2 - 2\xi \tan\beta \right)^2,$$

$$\left(\frac{\partial u}{\partial \eta} \right)^2 = A_0^2 \left[3\eta^3 - 4\eta\left(d^2 + a^2 \right) \right]^2 \left[\xi^2 \left(\xi^2 - \tan^2\beta \right)^2 \right].$$

The algebra involved in evaluation of the integration given by Eq. 4 is quite lengthy and tedious. We present only the final results for each integration, respectively, from left to right. Let $I = I + I + I + I$, then we have:

$$I_1 = A_0^2(\tan\beta)^7\left(\frac{44}{105}+\frac{156}{315}\tan^2\beta\right)a^4 \bullet M, \tag{6}$$

where M is a dimensionless parameter solely dependent on die geometries and is equal to

$$M = \left(\frac{7}{24}\right)\left(1-r^8\right)+\frac{3}{4}r^4\left(1-r^4\right)+r^4\ln r, \tag{6'}$$

$$I_2 = -\left(\frac{19}{215}\right)A_0^2(\tan\beta)^9 a^4 \bullet N, \tag{7}$$

where

$$\mathrm{N} = 2\left(1-r^8\right)-\left(\frac{8}{3}\right)\left(1-r^6\right)\left(1+r^2\right)+\left(1-r^4\right)\left(1+r^2\right)^2; \tag{7'}$$

$$I_3 = -\left(\frac{4}{15}\right)A_0^2(\tan\beta)^5 a^4 \bullet S, \tag{8}$$

where

$$\mathrm{S} = \left(\frac{1}{2}\right)\left(1-r^8\right)-\left(1-r^6\right)\left(1+r^2\right)+\frac{1}{2}\left(3r^2-1\right)\left(1-r^4\right); \tag{8'}$$

$$I_4 = -\left(\frac{2}{15}\right)CA_0 a^4(\tan\beta)^5 \bullet R, \tag{9}$$

where

$$\mathrm{R} = \left(\frac{1}{6}\right)\left(1-r^2\right)-\frac{1}{4}\left(1-r^4\right)\left(1+r^2\right)+\frac{1}{2}r^2\left(1-r^2\right). \tag{9'}$$

The coefficient A_0 in Eq. 5 can now be evaluated by the Ritz method[7] to satisfy the following condition:

$$\frac{\partial I}{\partial A_0} = 0. \tag{10}$$

When the preceding condition is applied to Eq. 4, we find that the coefficient A_0 is:

$$A_0 = \frac{CR}{15(\tan\beta)^3\left[\left(\frac{44}{105}+\frac{156}{315}\tan^2\beta\right)M-\left(\frac{19N}{315}\tan^2\beta+\frac{4}{15}\frac{S}{\tan^2\beta}\right)\right]}. \tag{11}$$

Therefore, the velocity field inside the die cavity is given by Eqs. 5 and 11. The volumetric flow rate is obtained by integrating the velocity profile given by Eq. 5. Thus, we have:

$$\begin{aligned} Q &= \int_0^{\tan\beta}\int_{a'}^{a} u(\xi,\eta)\eta\, d\xi\, d\eta \\ &= -\left(\frac{2}{15}\right)A_0(\tan\beta)^5 a^4 R = -C\alpha, \end{aligned} \tag{12}$$

where

$$\alpha = \frac{2a^4(\tan\beta)^3 R^2}{255\left[\left(\frac{44}{105}+\frac{156}{315}\tan^2\beta\right)M-\left(\frac{19N}{315}\tan^2\beta+\frac{4}{15}\frac{S}{\tan^2\beta}\right)\right]}. \tag{12'}$$

We should point out that the fluid velocity profile inside the die cavity, which is given by Eq. 5, assumes that there is no side flow into a narrow slot attached to it. Under this condition, the pressure gradient is constant. Based on the velocity distribution, we calculate the volumetric flow rate at any cross section of the die as is given by Eq. 12. In the formulation of the uniformity index for the flow coming out of a narrow slot, we couple Eq. 12 with the flow rate through the die slot by means of a material balance on a differential element of the die cavity as shown in Fig. 2. This means that the flow rate at any cross section of the die cavity is no longer constant because of the small amount of side flow through the slot. Consequently, the pressure gradient has to change in the direction of flow.

Flow Uniformity Index Formulation. Let us consider a material balance on a differential element, dz, of the die distribution chamber as shown in Fig. 2. The extrusion flow rate through the slot at that point where the pressure in the die chamber is equal to P is given by the following equation[3]:

$$q(z) = \frac{H^3 P}{12\mu L}, \tag{13}$$

where H is the opening of the slot and L is the slot length. Because the flow is in a creeping motion, the entrance effect is neglected. The material balance yields the following differential equation for the pressure:

$$\begin{aligned} &-\frac{d}{dz}(Q)dz + q(z)\; dz = 0 \\ &or\ \frac{d^2P}{dz^2} - K^2P = 0, \end{aligned} \tag{14}$$

where

$$K = \sqrt{\frac{H^3}{24L\alpha}} \tag{14'}$$

and α is defined by Eq. 12′.

We now consider a center-fed die so that the boundary conditions for the pressure at the center and at the end of the die cavity are, respectively:

$$P = P(0),\ \left(\frac{\partial P}{\partial z}\right)_{z=W} = 0,$$

where W is one-half of the die cavity width.

The solution for Eq. 14 subject to the boundary conditions is:

$$\frac{P(z)}{P(0)} = e^{-Kz} + e^{-KW}\left[\frac{\sinh K^z}{\cosh KW}\right]. \tag{15}$$

TABLE 1. UI and Re • f for Different Die Internal Dimensions) (die dimensions in inches)

$r = (h_2/h_1)$	H	L	w	h_1	tan β	UI	Re • f
0.40	0.01	1	25	1	0.64	0.9972	15.66
0.45	0.01	1	25	1	0.62	0.9976	16.45
0.5	0.01	1	25	1	0.62	0.9970	16.34
0.5	0.01	1	25	1	0.63	0.9964	19.04
0.6	0.008	1	25	1	0.57	0.9981	17.02
0.6	0.01	1	25	1	0.57	0.9963	17.02
0.6	0.015	1	30	1.25	0.57	0.9928	17.02
0.6	0.01	1.25	25	1	0.57	0.9971	17.02

Since the minimum and maximum flow rates occur at the far end and center of the cavity, the pressure ratio given by Eq. 15 is precisely equal to the volumetric flow rate ratio through the die slot for Newtonian fluids, which is the desired measure of the flow uniformity for the coating die at any point accross the width of the die. Therefore, the overall uniformity index (UI), is :

$$UI = \frac{P(W)}{P(0)} = e^{-KW}\left[1+\tanh(KW)\right] = \frac{1}{\cosh KW}. \qquad (16)$$

Equation 16 can be approximated to a simple form given by the following equation if UI is greater than 0.8[3]:

$$UI \cong \frac{1}{1+\frac{1}{2}(KW)^2+\cdots} \cong 1-\frac{1}{2}(KW)^2. \qquad (17)$$

Substituting K given by Eq. 14′ into Eq. 17, we finally obtain a desired design formula for the coating weight uniformity in a closed form:

$$UI = 1-\frac{75}{32}(\tan\beta)^3\left(\frac{H}{L}\right)\left(\frac{HW}{h_1^{\,2}}\right)^2 \cdot Z, \qquad (17')$$

where Z =

$$\frac{1}{R^2}\left[\left(\frac{44}{105\tan^2\beta}+\frac{156}{315}\right)M-\left(\frac{19N}{315}+\frac{4}{15}\frac{S}{\tan\beta^4}\right)\right] \qquad (18)$$

and M, H, S, and R are dimensionless and are only functions of die internal geometries. It is interesting to note that the uniformity index is only a function of die internal geometries and is independent of the fluid properties for Newtonian fluids under an isothermal condition. Also to be noted in the equation are the dimensionless groups, (H/C) and $(HW/h_1^2)^2$, which seem to be common parameters and are independent of the die cavity geometry.

Before we calculate the uniformity index of a coating die with given internal dimensions using Eq. 17′, we need to discuss the validity of the equation. The basic question is "How good is the assumed fluid velocity profile given by Eq. 5?" A detailed discussion is given in the Appendix. Here we calculate the product of the Reynolds number and the Fanning friction factor, Re • f, for each UI and find out whether the Re • f factor is close to 16. The Appendix shows that this factor is only a function of die internal geometries. We have chosen some realistic die internal dimensions and the resulting UI and Re • f factors are given in Table I.

The figures in Table I show that for a cascade coating die with the internal dimensions given in the table, the calculated UI is quite realistic when compared with equivalent rectangular cross-section dies. (Compare, for example, the figures in the second row of Table I with those in the last row of Table II, shown in a later section). We should point out that this is for coating a single fluid layer. For a multilayer coating bar of say, eight layers, the overall UI will be the same as for the single layer if all the dies had the same geometry and dimensions.

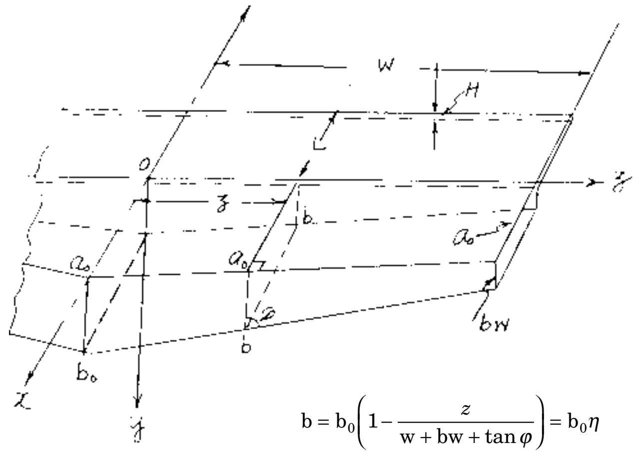

Figure 4. Tapered die and dimensions.

Tapered Rectangular Cavity Die. We now consider another popular die internal geometry that is used widely in coating photographic emulsions. Shown schematically in Fig. 4, this is a tapered rectangular cross-section die and the area decreases toward the end of the die. Normally, the industrial designer creates additional complications by adding a small exit angle to the slot opening to prevent the formation of a circulating fluid pocket. We neglect that angle for mathematical simplicity, but consider the fluid motion in three-dimensional spatial coordinates. For the die geometry considered here, the basic momentum equation governing the slow viscous flow of a Newtonian fluid is given by the following equation:

$$\frac{\partial P}{\partial z} = \mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right). \qquad (19)$$

The variational equation for Eq. 19 now takes the following form:

$$I = \int_0^W\int_0^{b_0}\int_0^{a_0}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2+\frac{2}{\mu}\frac{\partial P}{\partial z}u\right]dx\,\partial y\,dz. \qquad (20)$$

The Euler–Lagrange equation for the integral equation is

$$\frac{\partial F}{\partial U}-\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial Ux}\right)-\frac{\partial}{\partial y}\left(\frac{\partial F}{\partial Uy}\right)-\frac{\partial}{\partial y}\left(\frac{\partial F}{\partial U_z}\right) = 0, \qquad (21)$$

where U_x, U_y, and U_z are the velocity gradients in each subscript direction and F is the integrand of Eq. 20.

We now postulate that the fluid velocity in the die distribution chamber is given by the following equation:

$$U = y(y-b)\frac{1}{\eta^3}\Psi(x), \qquad (22)$$

where $b = b_0\eta$, and $\eta = 1 - Z/(w + bw\tan\varphi)$.

In Eq. 22, the function $\Psi(x)$ is unknown and it has to be evaluated later. The velocity given by Eq. 22 must satisfy the necessary boundary conditions; namely, that it is zero at $Y = 0$, and $Y = b$. Another required boundary condition—that velocity must be zero at $X = 0$ and $X = a_0$—has to be applied to the unknown function $\Psi(X)$. It must also satisfy the condition that the volumetric flow rate at any cross section of the convergent rectangular cavity be constant and independent of the direction of the flow.

The reader should realize that the equation we are developing is a relationship between volumetric flow rate and pressure gradient for a fluid flowing through a convergent distribution channel that has no slot opening. Once this relationship is developed, then we take into account the portion of fluid flowing into the slot from the distribution channel by means of the material balance.

The volumetric flow rate is readily obtained by intergrating Eq. 22:

$$Q = \int_0^b \int_0^{a_0} U \; dx \; dy = -\frac{b_0^3}{6} \int_0^{a_0} \Psi(x) \; dx. \tag{23}$$

Equation 23 shows that the functional form chosen for the velocity field is indeed the correct one, because it shows that the volumetric flow is constant and independent of the Z-direction. Let I_1, I_2, I_3, and I_4 be the integration of each term for Eq. 20, respectively, from left to right, such that we have:

$$I = I_1 + I_2 + I_3 + I_4. \tag{24}$$

$$I_1 = \frac{b_0^5 (W + b_W \tan\varphi)}{3_0} \ln\left(\frac{W + b_W \tan\varphi}{b_W \tan\varphi} \right) \int_0^{a_0} \Psi'^2(x) \, dx, \tag{25}$$

where $\Psi'(x)$ represents differentiation of $\Psi(x)$.

$$I_2 = \frac{b_0^3 (W + b_W \tan\varphi)}{6} \left[\left(\frac{W + b_W \tan\varphi}{b_W \tan\varphi} \right)^2 - 1 \right] \int_0^{a_0} \Psi^2(x) \, dx, \tag{26}$$

$$I_3 = \frac{b_0^5 (W + b_W \tan\varphi)}{20 \tan\varphi} \left[\left(\frac{1}{b_W \tan\varphi} \right)^2 - \left(\frac{1}{W + b_W \tan\varphi} \right)^2 \right] \int_0^{a_0} \Psi^2(x) \, dx, \tag{27}$$

$$I_4 = \frac{C b_0^3 W}{6} \int_0^{a_0} \Psi(x) \; dx. \tag{28}$$

When Eqs. 25 through 28 are substituted into Eq. 24, we obtain the following integral differential equation:

$$I = \int_0^{a_0} \left[K_1 \Psi'^2(x) + (K_2 + K_3) \Psi^2(x) - K_4 \Psi(x) \right] dx, \tag{29}$$

where

$$K_1 = \frac{b_0^5 (W + b_W \tan\varphi)}{30} \ln\left(\frac{W + b_W \tan\varphi}{b_W \tan\varphi} \right), \tag{30}$$

$$K_2 = \frac{b_0^3 (W + b_W \tan\varphi)}{6} \left[\left(\frac{W + b_W \tan\varphi}{b_W \tan\varphi} \right)^2 - 1 \right], \tag{31}$$

$$K_3 = \frac{3 b_0^5 (W + b_W \tan\varphi)}{20 \tan\varphi} \left[\left(\frac{1}{b_W \tan\varphi} \right)^2 - \left(\frac{1}{W + b_W \tan\varphi} \right)^2 \right], \tag{32}$$

$$K_4 = \frac{C b_0^3 W}{6} \tag{33}$$

The unknown function $\Psi(x)$ is now evaluated by solving the differential equation resulting from application of the Euler–Lagrange equation to Eq. 29. The resulting differential equation is:

$$\Psi''(x) - \left(\frac{K_2 + K_3}{K_1} \right) \Psi^2(x) - \frac{K_4}{2K_1} = 0. \tag{34}$$

A solution for the above equation is elementary and is given below:

$$\Psi(x) = C_1 \sinh \sqrt{K_0}\, x + C_2 \cosh \sqrt{K_0}\, x - \frac{K_4}{2(K_2 + K_3)}, \tag{35}$$

where $K_0 = (K_1 + K_2)/K_1$.

The boundary conditions require that:

$$\Psi(0) = 0, \; \Psi(a_0) = 0$$

and the two constants, C_1 and C_2, are evaluated readily using the above boundary conditions. The resulting solution for $\Psi(x)$ becomes finally:

$$\Psi(x) = \frac{K_4}{2(K_2 + K_3)} \left[\frac{\cosh \sqrt{K_0}\, a_0}{\sinh \sqrt{K_0}\, a_0} \sinh \sqrt{K_0}\, x + \cosh \sqrt{K_0}\, x - 1 \right]. \tag{36}$$

When Eq. 36 is substituted into Eq. 22, we finally obtain an equation for the fluid velocity in the distribution channel. Thus we have:

$$u = \frac{y(y - b)}{2\eta^3 (K_2 + K_3)} \left[\frac{\cosh \sqrt{K_0}\, a_0 - 1}{\sinh \sqrt{K_0}\, a_0} \sinh \sqrt{K_0}\, x + \cosh \sqrt{K_0}\, x - 1 \right], \tag{37}$$

where $\eta = [1 - z/(W + b_W \tan\varphi)]$.

The volumetric flow rate can now be obtained by intergrating Eq. 37 as is given below:

$$Q = \frac{-C b_0^6 w a_0}{72(K_2 + K_3)} \left[1 - \frac{2\left(1 - \cosh \sqrt{K_0}\, a_0\right)}{\sqrt{K_0}\, a_0 \sinh \sqrt{K_0}\, a_0} \right]. \tag{38}$$

Equation 14 is still valid for the die under consideration, but the constant is now different. The pressure distribution equation is given by Eq. 39:

$$\frac{d^2 P}{dz^2} - K_0^2 P = 0, \tag{39}$$

where

$$K_0 = \left[\frac{7}{4} \left(\frac{H^3}{L b_0^4 W} \right) \frac{(K_2 + K_3)}{\left(1 - \frac{2\cosh\sqrt{K_0}\,a_0}{\sqrt{K_0}\,a_0 \sinh\sqrt{K_0}\,a_0} \right)} \right]^{1/2}.$$

The solution of Eq. 39, which is the pressure distribution in the die cavity, now becomes:

$$\frac{P(z)}{P(0)} = e^{-K_0 z} + e^{-K_0 W} \left[\frac{\sinh K_0 z}{\cosh K_0 W} \right]. \tag{40}$$

Since the uniformity index is defined as the ratio of the pressure at the end of the die cavity to that at the center for a center-fed die, it can be obtained readily based on the formula given by Eq. 17. The resulting final formula is given below:

$$UI = 1 - \frac{7}{8} \left(\frac{H}{L} \right) \left(\frac{H^2 W^2}{a_0 b_0^3} \right) \left(1 + \frac{b_W \tan\varphi}{W} \right) F(\varphi), \tag{41}$$

where

$$F(\varphi) = \left\{ \frac{1}{6} \left[\left(\frac{1 + \frac{b_W}{W}\tan\varphi}{\frac{b_W}{W}\tan\varphi} \right)^2 - 1 \right] + \frac{3}{20} \left[\left(\frac{b_0}{b_W \tan\varphi} \right)^2 - \left(\frac{b_0}{W + b_W} \right)^2 \right] \right\}.$$

Table II shows some sample calculations for UI for a number of different tapered angles,and the depth of the die cavity at the end, while other dimensions are kept constant. The resulting uniformity indexes can be compared with the dies whose tapered angle approaches 90 degrees. This will make the die cavity approaching a straight rectangular cross-section channel.

The figures in Table II show clearly that the coating weight uniformity becomes poor as the tapered angle becomes smaller, or the die cavity channel becomes shallower toward the end. This is not suprising because the pressure drop required for a given flow rate through a convergent channel should be larger than the straight channel with the same dimension without tapering.

Key Factors for Good Coating Weight Uniformity. Comparison of the UI indexes for the two different dies shows that there is a unique dimensionless parameter which contributes to improving coating weight uniformity. It is the dimensionless group given below:

$$G = \left(\frac{H^3 W^2}{L h^4} \right) = \frac{\text{(slot opening) (die width)}}{\text{(slot length) (cavity size)}}$$

$$= \left(\frac{H^3 W^2}{L a_0 b_0^3} \right).$$

These dimensionless groups should be as small as possible in order for the uniformity index to be close to unity. The effect of die cavity exit angle or tapering is not important as far as the uniformity of the coating weight is concerned. Equation 41 states that a long a narrow slot opening with a large die cavity is a key factor. For the dies considered here in calculating UI the dimensionless parameter ranged approximately from 6×10^{-4} to 3×10^{-3}. Since it is proportional to the square of the die width, for a center-fed die, the dimensionless group is one-quarter of the end-fed dies. Therefore the center-fed dies should produce a better coating weight uniformity than the end-fed ones.

TABLE II. UI For Tapered Rectangular Cross Section Dies (Die Dimensions In Inches)

H	W	L	B_0	A_0	B_W	tan	UI
0.02	35	1	1	1.5	0.4068	59	0.99199
0.02	35	1	1	1.5	0.4262	61	0.99252
0.02	35	1	1	1.5	0.30	50	0.98623
0.02	35	1	1	1.5	0.3396	53	0.98893
0.02	35	1	1.5	2.0	1.496	10002	0.99971
0.01	25	1	0.887	0.725	0.8845	10003	0.99970

We should point out that in the UI calculation, it has been assumed that the die slot opening is constant and there is no variation accross the width. Since the extrusion flow rate is proportional to the cubic power of the slot opening, small local variations due to poor machining could cause significant variations in coating weights. For example, if we are going to control the coating weight within ±5% of the target value with a slot opening of, say, 0.01 in., its variation in slot opening must be better than $\pm 2 \times 10^{-4}$ in. It is not an easy task to fabricate a 60-in.-wide coating die with this kind of machining precision.

Other factors could also cause coating weight variations in a multilayer bar coating. For example, a nonuniform wetting line of the lower meniscus of the coating bead and vacuum fluctuations due to turbulent eddies in the vacuum box could also generate coating weight variations. But these are outside the scope of this work.

Conclusions

Design equations for coating dies used widely in photographic film manufacturing have been obtained in closed analytical forms, and the key design parameters, which affect the coating weight uniformity, have been identified. Despite the complexities in fancy design and high cost of fabrications for tapered dies, it contributes little to improve coating weight uniformity. The results of this study indicate that a center-fed die with a simple and easily machinable distribution chamber and a long narrow slot opening are key factors to achieve a good coating weight uniformity.

Appendix

The best way to check the validity of the assumed velocity profile in the die distribution chamber given by Eq. 5 is to compare it with experimentally measured values. Because such data are rare, an alternative way is to assume that the cross section of the die cavity is rectangle and calculate the product of the Reynolds number and the Fanning friction factor using the equivalent diameter. In a laminar flow, it has been firmly established that this product should be equal to 16, and according to Knudsen and Katz,[8] this is true for other noncircular conduicts, such as triangular or trapezoidal cross sections. Therefore we first develop a formula for the product of the Reynolds number and the Fanning friction factor based on the equivalent diameter.

The eqivalent diameter is defined as four times the hydraulic radius, which in turn is defined as a ratio of the cross-section area to the wetted perimeter. Thus we have:

$$D_e = \frac{4bh}{2(b+h)}, \text{ where } h = \frac{1}{2}(h_1 + h_2). \qquad (42)$$

The average fluid velocity inside the die cavity with no slot opening is:

$$V_{av} = \frac{Q}{(\pi/4)D_e^2}. \qquad (43)$$

The Reynolds number and the Fanning friction factor are defined as:

$\mathrm{Re} = \dfrac{D_e V_{av} \rho}{\mu}$, where ρ is the fluid density,

$$f = -\left(\frac{\partial P}{\partial z}\right)\left(\frac{D_e}{2\rho V_{av}^2}\right).$$

The product of the above two dimensionless factors is given below:

$$\mathrm{Re} \cdot f = \left(\frac{225}{2}\right) \frac{\pi Z (\tan\beta)^3}{\left[\dfrac{2}{(1+R)} + \dfrac{\tan\beta}{(1-R)}\right]^4}, \qquad (44)$$

where Z is given by Eq. 18.

Equation 3 states that the Re·f factor is only a function of die internal geometries. This means that we cannot assign any die dimensions to calculate the coating weight uniformity by Eq. 17. In other words, we have to choose die internal dimensions such that the Re·f factor can yield approximately 16. Under this condition, the assumed fluid velocity profile may be very close to the actual case. It is very fortunate that, by using realistic die dimensions, the calculated UI shown in Table I seem to be reasonably good. Many photographic emulsion coating operations require multilayer coats, and dies for each layer must be stacked together. Therefore it is desirable to have each die cavity be as small as possible in order to minimize the total size of a coat, and yet the significance of the die internal cavity size cannot be overemphasized.

References

1. E. P. Urahopoulou, A model for fluid flow in dies, *Chem. Eng. Sci.*, **46:** 629 (1991).
2. J. M. McKelvey, *Polymer Processing*, John Wiley & Sons, Inc., New York, 1962.
3. J. Carley, Flow of melts in “crosshead” slit dies, *J. Appl. Phys.* **25:** 1118 (1954).
4. E. B. Gutoff, Simplified design of coating die internals, Presented at A. I. Ch. E Spring Meeting, March 30–April 2, 1992, New Orleans, LA.
5. R. S. Schechter, *The variational method in engineering*, McGraw Hill, New York, 1967.
6. L. V. Kantrovich, and V. I. Krylov, *Approximate methods of higher analysis*, John Wiley & Sons, New York, 1964.
7. J. G. Knudsen, and D. L. Katz, *Fluid dynamics and heat transfer*, McGraw-Hill, New York, 1958.