Modulation Transfer Function for Development to Complete Field Neutralization*

S. Jeyadev† and H. M. Stark†
Xerox Corporation, Webster, New York 14580

The method employed by Streifer and Stark to calculate toner pile height due to the development to field neutralization of weak sinusoidal image patterns superposed on a solid area is extended to cover realistic development systems by applying more general boundary conditions on the free surface of the toner layer. We found that as long as the altered boundary conditions do not vary spatially, the effects of the new boundary conditions are propagated via a single parameter: the toner pile height corresponding to a solid area part of the image. This finding allows us to extend the realm of the analysis to the case of partial neutralization, provided some conditions are met by the development process. A theoretical development modulation transfer function (MTF) is derived from the toner pile height calculations. The variation of this MTF as a function of the solid area toner pile height is exploited to obtain the theoretical MTFs for the development of multiple layers in color xerography.

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Introduction

In charged pigment xerography, the development process involves the deposition of charged toner particles on an image receiver (usually, a photoreceptor) bearing a latent electrostatic image. The dynamics of toner delivery is determined not only by the nature of the electric fields produced by the latent image pattern, but also by the technology used to present the toner to the latent image. As the development proceeds, the charge in the developed toner layer reduces the electric field above it and thus reduces the field for subsequent development.

Depending on the material package and process parameters, the amount of toner present on the photoreceptor (PR) *after* the development process is complete may vary, from the limit where the electric field due to the latent image is not significantly altered by the developed toner layer to the limit where sufficient toner is deposited such that the charge in the developed toner layer eventually reduces the field above it to zero. In modeling the former case, one usually assumes that the amount of toner developed is simply proportional to the normal component of the electrostatic field of the bare latent image,[1] in which case it is possible to obtain the frequency response or modulation transfer function (MTF) of the development subsystem. The latter condition, usually termed *development to neutralization*, represents an upper limit to the amount of toner that can be developed in the absence of toner supply limitations. For this case, although deriving the thickness of the toner layer for solid area images is straightforward, deriving the thickness of the toner layer for a sinusoidal image pattern, and thus extracting an MTF, is a nontrivial problem. The principal difficulty is that the calculation required to solve the toner layer thickness represents a free-boundary value problem: Although the boundary condition at the interface of the developed and free toner is known (i.e., the field vanishes), the surface itself is not known until the problem has been solved.

The basic boundary value problem that gives the toner layer thickness for development to neutralization of a periodic, 1-D latent image has been stated and formally solved by Streifer and Stark.[2] They considered the following problem: a PR of thickness s and permittivity ε_s carries (see Fig. 1) a latent image given by $\sigma(x)$ such that $\sigma(x + 2a) = \sigma(x)$. The toner is described by a charge density of ρ and permittivity ε_t. The height of the toner layer assumes description by the yet unknown function $Y(x)$. The usual boundary conditions are applied at the ground plane and the PR–toner interface. Further, the potential at the upper surface of the toner, $Y(x)$, is taken to be zero. This set of boundary conditions is not sufficient to solve the problem, because $Y(x)$ is, as yet, not known. The problem is

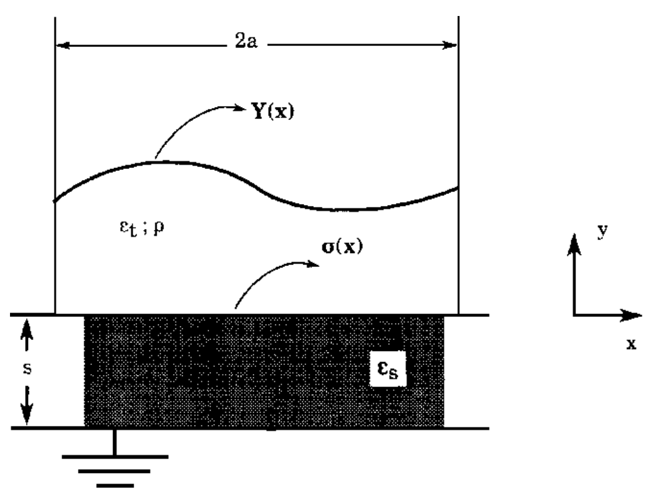

Figure 1. The model problem solved by Streifer and Stark.[2] The latent image charge pattern $\sigma(x)$ has a period $2a$. The surface of the toner layer is given by $Y(x)$. The boundary conditions at this interface are the potential $\phi(x,Y(x)) = 0$ and the field $E_y(x, Y(x)) = 0$ (see text).

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† IS&T Member

one other condition is required. The electric field everywhere above the toner layer should vanish. Streifer and Stark argue that it is sufficient to require the y component to vanish.

The formal solution consists of a set of coupled nonlinear equations, which must be solved to obtain $Y(x)$, as well as the usual Fourier coefficients of the solution for the potential in the PR and the toner layer. Streifer and Stark outline an approximate, iterative numerical procedure to find $Y(x)$ (at specified points in $-a \le x \le a$) and a corresponding set of Fourier coefficients to expand the potential.

Streifer and Stark also illustrate how a perturbative solution is obtained when $\sigma(x)$ consists of a uniform component that has small amplitude sinusoids superposed—the "small signal" case. In this case, the result for $Y(x)$ is very interesting: Essentially, it consists of a uniform part [corresponding to the uniform component in $\sigma(x)$], components with sinusoidal frequencies in the spatially varying part of $\sigma(x)$, *and higher harmonics of these frequencies.* The last part of the response is a result of the inherent nonlinear nature of the problem. *Thus although a part of the toner pile height can be described by the usual MTF, there are portions of the response that cannot be captured by the standard MTF.*

In Ref. 2, the potential boundary condition applied at the upper surface of the toner layer is that the potential vanishes at this surface (and also in the region above the toner layer, because the electric field also vanishes in this region). However, development subsystems are usually deployed with a bias voltage (or cleaning field) to keep toner away from the background areas of the image; for example, in magnetic brush development a bias voltage is applied to the development roller.

We also note that the present calculation is more relevant to development technologies that, in the process of depositing toner to the image, do not disturb toner that may have been previously deposited on the PR surface. The well-known scavenging effects of the magnetic brush development subsystems will dominate over those trying to achieve field neutralization on local and global length scales and wash out the subtle structures in $Y(x)$ that would be produced by pure field neutralization.

As mentioned earlier, we must solve the problem with the more general boundary condition

$$\Phi(x,y) = V_b,\ y \ge Y(x) \tag{1}$$

to obtain solutions relevant to practical development subsystems. Because of the inherent nonlinear nature of the problem, linear superposition of a uniform electric field—corresponding to V_b—cannot be used to obtain the new solution. We will restrict ourselves to charge densities of the form

$$\sigma(x) = \sigma_0 + \sum_{n=1}^{\infty} \sigma_n \cos k_n x, \tag{2}$$

because it sufficiently illustrates how the results of Ref. 2 should be modified. In Eq. 2

$$k_n = nk = (2\pi / 2a)n = \pi n / a, \tag{3}$$

where $2a$ is the period of $\sigma(x)$. The modified problem can be solved by following the same process used in Ref. 2. Although the solution procedure is rather long and complex, the *mathematical* modification to the formulas of Streifer and Stark is minimal. In this study we present only the formulas relevant to the toner pile height, $Y(x)$, and the MTF *in the weak signal limit.* A more complete presentation will be published elsewhere.

Perturbative Analysis of the Toner Pile Height

For the perturbation analysis we take the expansion parameter to be

$$\delta = \max\{\sigma_n\} / \sigma_0. \tag{4}$$

Assume the perturbative form for the toner pile height to be

$$Y(x) = g_0(x) + \delta g_1(x) + \delta^2 g_2(x) + \ldots, \tag{5}$$

where the functions $g_n(x)$ are determined using the method in Ref 2. The first term on the right-hand side of Eq. 5 is the toner pile height corresponding to the uniform component of the image represented in Eq. 2 by σ_0 and is given by

$$g_0(x) = -\frac{\varepsilon_t}{\varepsilon_s} s + [(\varepsilon_t s / \varepsilon_s)^2 - 2\frac{\varepsilon_t}{\rho_t} V_{dev}]^{1/2}, \tag{6}$$

where

$$V_{dev} = \frac{\sigma_0 s}{\varepsilon_s} - V_b = V_0 - V_b \tag{7}$$

is the development voltage corresponding to a PR initially charged to a voltage V_0. To facilitate comparison with Ref. 2, we have implicitly assumed charged area development in Eqs. 6 and 7. However, the analysis is also applicable to discharged area development after the requisite changes, such as substituting the background voltage V_{bg} for V_0 in Eq. 7, are made.

The next term on the right-hand side of Eq. 5 is the first-order contribution from the sinusoidal components of the image to the toner pile height. Before giving the expression for this term, we first define the quantities

$$\psi_n = \varepsilon_s \coth(k_n s) - \varepsilon_t, \tag{8}$$

$$\omega_n = \varepsilon_s \coth(k_n s) - \varepsilon_t, \tag{9}$$

$$\theta_n = \omega_n \exp(k_n g_0) - \psi_n \exp(-k_n g_0). \tag{10}$$

In terms of these quantities, we now write the first-order term as

$$\delta g_1(x) = -2\frac{\varepsilon_t}{\rho_t} \sum_{n=1}^{\infty} \frac{\sigma_n}{\theta_n} \cos k_n x. \tag{11}$$

Comparing the expression for the solid area portion of the response, given by Eq. 6, with that in Ref. 2, we see the difference is that the surface potential of the PR is replaced by the actual development potential V_{dev}, due to the modified boundary condition given in Eq. 1. Comparing Eqs. 8–11 with their counterparts in Ref. 2 shows that their mathematical form remains identical. This remains true for the second-order terms, as well. *Thus the only explicit change required to the formulas of Streifer and Stark is the modification of the expression for g_0; all of the effects of the new boundary condition are propagated via g_0.* We can use this fact to extend the realm of the analysis to specific cases of partial neutralization.

In principle, the solid area toner pile height g_0 is a measurable quantity and we can use it as a basic parameter in describing the MTF. However, in practice, one measures the developed mass per unit area (DMA) on the PR, rather than on g_0. Knowledge of the mass density of the developed toner on the PR allows us to parameterize the MTF, using the solid area DMA rather than the *calculated* solid area response in Eq. 6, at least for a given development technology and toner package: If ρ_{mass} is the mass density of developed toner on the PR, then

$$g_0 = \frac{DMA}{\rho_{\text{mass}}} = \frac{DMA}{\rho_t} < q / m >, \qquad (12)$$

where $<q/m>$ is the tribo of the toner. Equation 12 can be used to parameterize the MTF in terms of the DMA on the PR. For the sake of convenience we shall continue to use g_0 in our expressions.

It is evident that the first-order response given by Eq. 11, each component of which corresponds to a component in Eq. 2, will produce the traditional MTF. But, due to the inherent nonlinear nature of the problem, the second-order response involves coupling between the various input frequencies, resulting in the appearance of "sum" and "difference" frequencies.

MTF in the Weak Signal Limit

Let the only nonzero amplitude in Eq. 2 be that of the *m*'th component:

$$\sigma_n = \sigma_n \delta_{n,m}. \qquad (13)$$

In this case the solid area component is still given by Eq. 6. For the first-order response only the $n = m$ term survives in Eq. 11. Because there is only one nonzero wave vector, we will henceforth drop the subscript on k (i.e., we set $k_m = k$) and use k itself as the index for quantities such as σ, θ, ω, etc. We now express the first-order response as

$$\delta g_1(x) = -2 \frac{\varepsilon_t \sigma_k}{\rho_t \theta_k} \cos kx, \qquad (14)$$

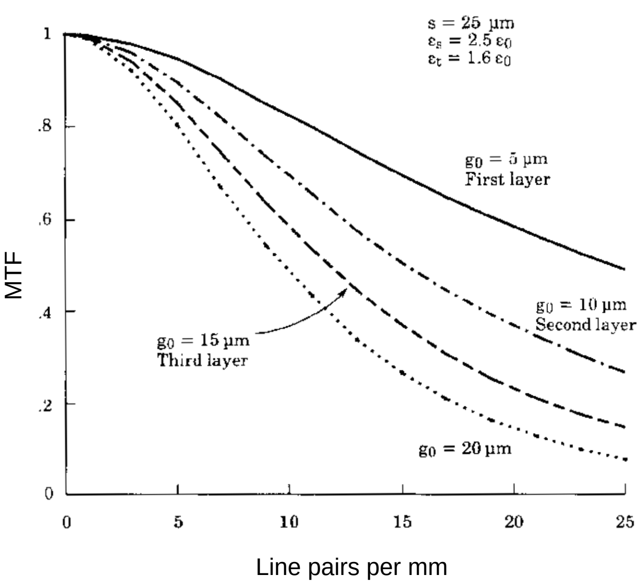

Figure 2. The MTF as a function of spatial frequency for four different solid area responses. In multiple layer color xerography the curves could be interpreted as the MTFs for different colors, depending on the number of layers of toners already present on the PR (see text for discussion).

and hence the normalized MTF is given by

$$M(k) = \frac{\lim k \to 0\{\theta_k\}}{\theta_k}. \qquad (15)$$

Using Eq. 10 for θ_k, we express the MTF as

$$M(k) = \frac{\varepsilon_s g_0 / s + \varepsilon_t}{\varepsilon_s \coth(ks)\sinh(kg_0) + \varepsilon_t \cosh(kg_0)}. \qquad (16)$$

Because single-pixel lines at 600 spi correspond to a wave vector of the order of 10^5 m^{-1} (or roughly 12 line pairs/mm) we will confine our attention to wave vectors less than 25 line pairs/mm. Figure 2 shows the MTF, as calculated from Eq. 16 for g_0 = 5, 10, 15, and 20μm. We have used s = 25 μm, $\varepsilon_s = 2.5\ \varepsilon_0$ for the PR parameters, and ε_t = 1.6 ε_0 for the permittivity of loosely packed toner.

Conclusions and Remarks

The formalism set up by Streifer and Stark to calculate the neutralization limit toner pile height for sinusoidally varying line patterns has been extended to the case where bias fields are present in the development nip. We find that the presence of the bias alters the responses in a nonlinear manner, but the *formulas* describing the spatially varying portion of the response remain identical, as all information about uniform boundary conditions is propagated via the solid area response. The MTF was obtained by considering the case of a small sinusoidal modulation on top of a solid area. The analysis can be extended to the case of partial neutralization by using the fact that *all* the effects of spatially uniform changes to the boundary conditions are propagated via the uniform component of the response.

Figure 2 shows that the MTF begins to roll off at about 12 lines/mm—the spatial frequency corresponding to single-pixel lines at 600 spi—in the case of development to neutralization. This has some interesting consequences for development systems that deposit multiple layers of color toner on the PR (e.g., the Konica 9028/Hewlett-Packard Color Laser Jet and the Panasonic FPC1 copier/printers) if the development is allowed to get close to this limit. Let us assume that a monolayer of toner corresponds to a pile height of 5 μm. Then the solid line in the diagram corresponds to the MTF for the first color. Now consider trying to develop lines of the second color on top of a solid area of the first color. The MTF for this process is given by the dot–dash curve. This is because we can mimic this two-step two-color development by a single step, with the first color corresponding to the solid area part that we represent by σ_0 and the lines of the second color by σ_k. The analog represented for the third layer is given by the long–short dashed line in the diagram. By comparing the curves labeled g_0 = 5, 10, and 15 μm, we can see that this model predicts that the developability of single-pixel lines on top of previously developed solid areas will be greatly affected in marking engines that deposit multiple layers of toner on the PR. This effect is another manifestation of the Gundlach rule for resolution loss of overcoated photoreceptors.

In deriving an expression for the MTF, we have worked within the "weak signal" limit. In Ref. 2, Streifer and Stark have shown that this approximation gives a very good description for values of δ as high as 0.7 by comparing the perturbative solution with the exact one obtained by solving the full nonlinear coupled equations. However, if one wishes to calculate the toner pile height for an *isolated* line, one should work with the full nonlinear equations, because the "uniform" component σ_0 would vanish in this case. However, the general formalism remains valid.

The geometry used both here and in the original paper addresses the case of infinite line patterns (along the third dimension). However, the same basic formalism can be used to solve for toner pile height for the case of charge patterns with azimuthal symmetry (i.e., circular "dot" patterns)—the case of primary interest when imaging is done using dot screens—by setting up the equations in cylindrical coordinates.

References

1. See, for example, M. E. Scharfe, *Electrophotography Principles and Optimization,* Research Studies Press Ltd., John Wiley and Sons, New York, 1983.
2. W. Streifer and H. Stark, *Applied Optics, Supplement 3: Electrophotography,* (149) 1969.