Joint Parameter Estimation for Event-based Vision Sensor Characterization

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Abstract

This paper proposes a pixel-wise parameter estimation framework for Event-based Vision Sensor (EVS) characterization. Using an ordinary differential equation (ODE) based pixel latency model and an autoregressive Monte-Carlo noise model, we first identify the representative parameters of EVS. The parameter estimation is then formulated as an optimization problem to minimize the measurement-prediction error for both pixel latency and event firing probability. Finally, the effectiveness and accuracy of the proposed framework are verified by comparison of synthetic and measured event response latency as well as firing probability as function of temporal contrast (so-called S-curves).

Introduction

The Event-based Vision Sensors (EVS) [1–5], sometimes also referred to as Dynamic Vision Sensors (DVS), capture our world from a completely different perspective compared to classical CMOS Image Sensors (CIS). In EVS, each pixel independently detects whether the logarithmic luminance changes beyond defined relative thresholds in an asynchronous manner (Fig. 1a). This enables sparse, low-latency, and low-power image acquisition. This paradigm change requires novel models, characterization techniques as well as model parameter estimation.

As EVS sensors are novel, so is their characterization. Commonly used characterization methods measure e.g. the average time from a contrast step to an event trigger or the average event trigger probability for a given temporal contrast step (Fig. 1 b & c). These methods, unfortunately, provide no predictive value as the resulting sample averages depend largely on measurement conditions such as sensor contrast threshold, temporal contrast step, or timing aspects such as the time used to determine whether an event was observed. Therefore, a model-based approach is needed. Simulators to emulate event sensors have been published in recent years [6, 7], however, these describe sensors only phenomenologically and do lack careful calibration against simulation and characterization. Only recently, a physically based model has been proposed and reasonable resemblance to simulated and measured observations was demonstrated [8, 10]. This paper builds upon the physical model published in [10]. It analyzes the relevance of the employed model parameters, determines key parameters, and then promotes an optimization-based model parameter estimation method.

Figure 2 illustrates the proposed experimental characterization setup EVS used for parameter estimation. Our sensor is modeled based on an ordinary differential equation (ODE) pixel latency model and an autoregressive Monte Carlo noise model [10]. The model was derived using common circuit modeling techniques. Comparisons between measurements, simulations, and model demonstrated good resemblance and thus the validity of the approach [10].

In this work, we deduce that the most significant pixel model parameters are the temporal contrast threshold (\(C_{\text{TH}}\)), the coupling capacitance (\(C_C\)), and source follower bias current (\(I_{\text{BSF}}\)).
Figure 3. Simplified log-amplifier schematic a) and SF schematic b) and their respective small signal circuits in c) and d), respectively.

Other model parameters are assumed to match our prior expectations from design. Our goal in characterization is then to find the pixel-wise parameter values that can best align the model with the real sensor data for both event response latency and event firing probability. We formulate this model parameter estimation as an optimization problem and then minimize the loss function based on the relative error between the model prediction and the actual measurement weighted in the L2 norm. We jointly account for pixel latency and event firing probability. We validate the quality of the parameter extraction method using the simulation model as this provides a ground truth. Finally, we apply the method to real measurements.

Circuit Modelling

Assuming that all transistors operate in saturation and weak inversion and that the inverting amplifier in the feedback loop of the logarithmic amplifier has an idealized affine-linear behavior, and neglecting its additional pole, we can model the large signal response through simple ordinary differential equations [10]. The logarithmic amplifier follows:

\[
\frac{dV_{FE}(t)}{dt} + \frac{A \cdot I_{D1}(t)}{1 + A \cdot C_C} \exp \left( \frac{\zeta + A \cdot C_C}{A \cdot C_C} \cdot V_T \cdot \Delta V_{FE}(t) \right) = \frac{A}{1 + A \cdot C_C} \cdot I_{PD},
\]

where \( \Delta V_{FE} \) describes the relative signal change at the output of the logarithmic amplifier with respect to the signal at the last sampled reference luminance level. \( A \) is the open-loop gain of the inverting amplifier, \( I_{D1} \) is the current through the transistor whose source is connected to the photodiode operating the linear-to-logarithmic conversion. \( I_{PD} \) is the photocurrent, \( C_C \) is the feedback capacitance between the input and output of the log-amplifier, \( \zeta \) is the transistor subthreshold slope parameter and \( V_T \) is the thermal voltage.

The source-follower is designed to avoid kick-back from the switched capacitor circuit performing the difference detection. It is modeled as:

\[
\frac{dV_{O1}(t)}{dt} = \frac{1}{C_{LSF}} \left[ I_{BSF} \right. - \left. I_{SF}(t_0) \cdot \exp \left( \frac{\zeta \cdot \Delta V_{O1}(t) - \Delta V_{FE}(t)}{\zeta \cdot V_T} \right) \right],
\]

with \( \Delta V_{O1} \) being the relative output signal change, \( I_{SF} \) is the current through the source-follower transistor and \( I_{BSF} \) is its bias current. \( C_{LSF} \) is the load seen by the source-follower.

It can be shown that the logarithmic amplifier has an analytical solution to a step-response in photocurrent \( I_{photo-0} \to I_{photo-1} \):

\[
\Delta V = \frac{\Delta V_{FE}}{V_T \cdot \frac{A \cdot \zeta}{\zeta + A}} = \ln \left( \frac{I_{photo-1}}{I_{photo-0}} \frac{1 + C_{scene} \cdot \exp \left( -V_T \cdot \frac{t - t_0}{\tau} \right)}{1 + C_{scene}} \right),
\]

with the time-constant \( \tau = \frac{1 + A \cdot C_C}{\frac{A \cdot \zeta}{\zeta + A} \cdot V_T \cdot C_C} \). Assuming that the bandwidth limitation of the log-amplifier is more significant than the source-follower, an analytical response can be found as:

\[
\Delta V_{O1}(t) \approx \frac{1}{\zeta} \cdot \Delta V_{FE}(t)
\]

Figure 3 depicts the small signal equivalent circuits used to derive the noise model. For small photocurrents corresponding to low-light conditions, the shot noise of the photocurrent and transistors dominates over e.g. flicker noise assuming the operating range of interest is not at very low frequencies. We thus approximate the noise to follow shot noise processes resulting in the following autocorrelation functions \( R_{FE, FE} \) and \( R_{O1, O1} \) which are injected at the input and output of the source-follower circuit [10]:

\[
R_{FE, FE}(\Delta t) = \zeta \cdot \frac{k_B \cdot T}{C_C} \cdot e^{-\frac{|\Delta t|}{C_C \cdot V_T}} + \zeta \cdot \frac{k_B \cdot T}{C_{PD} \cdot C_C \cdot \frac{C_C + C_{PD}}{C_C + C_{PD}} + e^{-\frac{|\Delta t|}{C_{PD} \cdot V_T}}}
\]

\[
R_{O1, O1}(\Delta t) = \frac{k_B \cdot T}{C_{LSF}} \cdot e^{-\frac{|\Delta t|}{C_{LSF} \cdot V_T}}
\]

Here, \( k_B \) is the Boltzmann constant, \( C_{PD} \) is the photodiode capacitance and \( C_L \) is the load capacitance seen by the inverting amplifier. \( g_{mO1} = \frac{k_B \cdot V_T}{C_C} \), \( g_{mG2} = \frac{k_B \cdot V_T}{C_{PD}} \), and \( g_{mG-SF} = \frac{k_B \cdot V_T}{C_{LSF}} \) are the small-signal gains of the log-amplifier transistor, the transistor of the inverting amplifier driven with bias \( b \) and the source follower. In our event vision simulator, these autocorrelation functions are used to create Monte Carlo trials using autoregressive processes. This allows the synthesis of training data for algorithm development from ground truth high-speed video data. Furthermore, this enables the evaluation of parameter extraction methodologies as conversely to actual measurements, the underlying model parameters are known apriori.

Joint Parameter Estimation

From Eq. 1 it can be seen that for \( A \gg \zeta \) changes in \( A \) and \( C_C \) are indistinguishable, so we assume \( A \) to follow the expectation from design and focus on estimating \( C_C \). Assuming a settled starting condition \( I_{SF}(t_0) = I_{BSF} \) such that there is no displacement current flowing through \( C_{LSF} \), one can see from Eq. 2 that the impact of \( I_{BSF} \) and \( C_{LSF} \) are indistinguishable. Thus we focus on estimating \( I_{BSF} \). We empirically determined that the impact of
figure 4. Pixel response latency vs. luminance as function of \( C_C \), \( I_{BSF} \) and \( C_{TH} \).

\( \zeta \) is minor resulting in \( C_{TH} \), \( C_C \), and \( I_{BSF} \) being the most significant parameters which we aim to estimate. Figure 4 shows latency variability as function of these three key parameters.

Given the effective 1.5bit output of an EVS pixel (‘increase’, ‘decrease’, or ‘no event’), we can essentially only observe triggering probability distributions or time-stamp distributions as well as moments of these distributions. Reference [10] focused on utilizing average time stamps to derive model parameters. Assuming that the log amplifier is the latency bottleneck, the analytical step-response solutions can be rearranged to determine the noise-free time-stamp for a temporal contrast step:

\[
t_{\text{event}} = \tau \cdot \ln \left( \frac{C_{\text{scene}} \cdot [1 + C_\infty]}{C_{\text{scene}} - C_\infty} \right),
\]

with \( C_{\text{scene}} = \frac{I_{\text{photo-1}} - I_{\text{photo-0}}}{V_{\text{threshold}}} \) and \( C_\infty = \exp \left( \frac{V_{\text{threshold}}}{V_{T} G} \right) - 1 \), the comparator threshold \( V_{\text{threshold}} \) and the difference detector gain \( G \). It was shown in [10] that at sufficient temporal contrast, this noise-free time-stamp is close to the noise-affected time-stamp.

From Eq. 7, it can be seen that in order to derive \( \tau \) and \( C_\infty \) a combination of contrast changes \( C_{\text{scene}} \) as well as reference level changes \( I_{\text{photo-0}} \) are required to yield independent measurements.

Conversely to [10] in this work we not only utilize the trigger latency but also the event trigger probability to extract model parameters. This helps to balance the matching of extracted parameters for both key observables. However, this requires the model to consider the stochastic properties of the EVS pixel which we model using Monte-Carlo trials.

In this work we used grid search optimization per pixel:

\[
C_{C_{i,j}, I_{BSF_{i,j}}, C_{TH_{i,j}}} = \arg \min_{C_C, I_{BSF}, C_{TH}} J_{i,j},
\]

with the per-pixel cost function \( J_{i,j} \):

\[
J_{i,j} = \left\{ w \sum_{k} \left( \frac{t_{k,i,j} - \tau_{\text{model}}(C_C, I_{BSF}, C_{TH})}{\tau_{k,i,j}} \right)^2 + (1 - w) \sum_{k} \left( \frac{NCT_{k,i,j} - NCT_{\text{model}}(C_C, I_{BSF}, C_{TH})}{NCT_{k,i,j}} \right)^2 \right\}.
\]

\( w \) denotes the weight balancing the contributions to the loss function between pixel response latency and NCT. We empirically found that \( w = 0.5 \) yields good results for our range of interest. \( \tau_{\text{model}} \) and \( NCT_{\text{model}} \) are the mathematical models for latency and NCT estimated through Monte-Carlo trials. \( k \) are all measurements mutually distinct in scene contrast and/or nominal luminance level and \( i, j \) are the pixel coordinates. The cost function uses normalized latency and NCT in order to achieve good performance for a large dynamic range.

**Method Validation**

To validate the effectiveness of the proposed cost function, we employ camera simulation for which we have apriori knowledge of the ground truth parameters. Figure 5 shows the loss curves in which for each curve only one of three parameters is...
swept from the optimality condition. In general, we have no guarantee of convexity. Thus we empirically validated convergence in the parameter range of our interest. Furthermore, global optimization techniques help to avoid getting stuck in a local optima. We do employ grid search. It can be observed that the slope of the cost function can be very small. Thus noise can have a considerable impact on the parameter estimation and a sufficient amount of Monte-Carlo samples ought to be used to mitigate this issue. Again this can be empirically validated using camera simulation for a given expected parameter range of interest.

**Experimental Results**

The proposed parameter extraction methodology (Eq. 9) is now applied to measurements using the sensor published in [9]. A region of interest (ROI) of $100 \times 100$ in the center of the image plane is selected for characterization. In order to yield independent measurements, we generate various temporal contrast steps as well as varying reference luminance levels $L_{\text{ref}}$:

$$C_{\text{scene}} \in \left\{\left\{100\%, 110\%, 120\%, \ldots, 200\%\right\} \text{ for pixel latency}\right.$$  
$$\left\{2\%, 4\%, 6\%, \ldots, 40\%\right\} \text{ for NCT}\right.$$  

and

$$L_{\text{ref}} \in \{15\text{lx}, 130\text{lx}, 310\text{lx}, 700\text{lx}, 910\text{lx}\}. \quad (10)$$

Figure 6 shows the estimated parameter distributions. $C_C$ and $C_{\text{TH}}$ roughly resemble normal distributions, whereas $I_{\text{BSF}}$ exhibits large variability and shows an almost bimodal distribution. This is because $I_{\text{BSF}}$ has a limited impact in the $L_{\text{ref}}$ range. The latency of the source follower only dominates the overall latency at $\geq 1000\text{lx}$ [9].

The extracted model parameters were used to plot expected latency curves and sample trigger probability curves vs. temporal scene contrast. The resemblance of the model and measurements is depicted in Figure 7 for selected pixel. For low luminance levels, the latency model slightly overestimates the latency. This can be improved either by adjusting the cost function to place more emphasis on this region or by introducing more degrees of freedom than the simplified single-pole approximation given by the ordinary differential equation model. The firing probability curves also show reasonable resemblance vs. measurements.

**Figure 6.** Pixel-wise distributions of estimated parameters $C_C$, $I_{\text{BSF}}$, $C_{\text{TH}}$. The parameters here are normalized to the expected value.

**Figure 7.** Pixel latency comparison between the measurement and our model using the estimated parameters under different illuminances and temporal contrast for two sampled pixels.

**Figure 8.** S-curves comparison between the measurement and our model using the estimated parameters for 200 sampled pixels.

**Figure 9.** NCT comparison between the measurement and our model using the estimated parameters for 20 sampled pixels.
as can be seen in Figure 8 where a population of 200 measured curves is compared to 200 model curves. A slight change in slope of the S-curve can be observed which can be attributed to the simplistic noise model. We compute the NCT values of 20 pixels based on the extracted model parameters and overlay these with their measured values and also find a good resemblance in Figure 9.

**Conclusion**

This work presents perspectives on parameter extraction of a physical EVS pixel model. Conversely to [10] a joint optimization approach to account for pixel latency and firing probability is employed. The optimization incorporates the stochastic nature of the pixel operation using Monte Carlo trials. The model was validated using the device simulator as the ground truth model parameters are known apriori. The model was then used to extract parameters from measured devices and a comparison of latency and estimated firing probabilities is given for a range of measurement conditions proving that the model has predictive value.

**References**


