

# Novel Image Super-Resolution and Denoising Using Implicit Tensor-Product B-Spline

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## Abstract

*In the dynamic realm of image processing, coordinate-based neural networks have made significant strides, especially in tasks such as 3D reconstruction, pose estimation, and traditional image/video processing. However, these Multi-Layer Perceptron (MLP) models often grapple with computational and memory challenges. Addressing these, this study introduces an innovative approach using Tensor-Product B-Spline (TPB), offering a promising solution to lessen computational demands without sacrificing accuracy. The central objective was to harness TPB's potential for image denoising and super-resolution, aiming to sidestep computational burdens of neural fields. This was achieved by replacing iterative processes with deterministic TPB solutions, ensuring enhanced performance and reduced load. The developed framework adeptly manages both super-resolution and denoising, utilizing implicit TPB functions layered to optimize image reconstruction. Evaluation on the Set14 and Kodak datasets showed the TPB-based approach to be comparable to established methods, producing high-quality results in both quantitative metrics and visual evaluations. This pioneering methodology, emphasizing its novelty, offers a refreshed perspective in image processing, setting a promising trajectory for future advancements in the domain.*

## Introduction

The dynamic realm of image processing and the rapidly evolving landscape of machine learning are currently undergoing a paradigm shift, thanks to the innovative development of neural fields and coordinate-based neural networks[1, 2, 3, 4]. These advanced computational models have significantly accelerated progress in visual computing, enabling intricate tasks such as the synthesis of three-dimensional shapes and the refinement of established image processing methodologies.[5, 7] As they meticulously parse and reconstruct complex visual data, these networks are unlocking groundbreaking applications in digital imaging, augmented reality, and virtual environments, ultimately altering the trajectory of tech innovation. However, the practical deployment of these neural networks is frequently impeded by their intensive computational demands, which encompass an insatiable need for high processing power and substantial memory allocation.[7] These stringent requirements pose challenges to scalability and versatility, particularly limiting the application of these networks in scenarios that necessitate swift, real-time processing and heightened resource efficiency. As a result, the utilization of these advanced networks is often restricted to environments with abundant resources, thus impeding their widespread adoption and curtailing the momentum of further technological breakthroughs.

In direct response to these hurdles, our study introduces



Figure 1: Example of denoised image using our proposed method.

an avant-garde approach that harnesses the potential of Tensor-Product B-Spline (TPB) technology to adeptly navigate the intricacies of image denoising and super-resolution. This innovative methodology markedly diverges from the traditional, iterative processes associated with neural network algorithms, offering a more streamlined and resource-savvy computational alternative. By fine-tuning TPB coefficients to accurately depict high-dimensional functions, our strategy significantly mitigates computational load while preserving the integrity, quality, and efficiency of the output. Our strategy is distinctively marked by its analytical focus on the input of coordinate data over mere pixel value examination. This strategic pivot allows for a more profound investigation into the complex interplay between the coordinates' spatial information and the corresponding pixel intensities, dramatically enhancing the accuracy of the model and substantially diminishing the propensity for predictive discrepancies. Empirical evidence demonstrates that the efficacy of our TPB-centric methodology in performing denoising and super-resolution operations is not only on a par with conventional neural network techniques but is also distinguished by its operational efficiency.

Throughout this document, we meticulously delineate the architecture of our innovative framework, probing into the detailed workings of each critical component and elucidating the principles that underpin their functionality. We also present a comprehensive collection of experimental findings that lend credence to the superior performance and dependability of our approach. This seminal work heralds the advent of a more democratized, economical, and scalable chapter in image processing technology, establishing a new standard for the fusion of sophisticated engineering prowess with prudential computational resource management. Building upon the foundational aspects of our framework, which sets a new benchmark in image processing technology, we

now turn our attention to the broader landscape of advancements in this domain. The following section, 'Related Work', delves into the realm of image super-resolution, a field where neural network approaches, particularly those using Multilayer Perceptrons (MLP), have shown significant promise. This exploration is crucial to understanding the context and evolution of our methodologies within the dynamic and ever-evolving field of image processing.

## Related Work

In the rapidly advancing field of image super-resolution, neural network approaches have demonstrated remarkable adaptability and performance. A study employing a Multilayer Perceptron (MLP)[8] framework has made significant strides in this area. By training MLPs on various image categories, researchers have dissected the network's behavior and efficiency in enhancing image details. The application of standard evaluation metrics like Mean Squared Error (MSE), Peak Signal-to-Noise Ratio (PSNR), and Structural Similarity Index (SSIM) has allowed a comprehensive assessment of image quality post-enhancement. Notably, the findings reveal that MLPs, when trained on specific categories, can achieve results on par with more traditional super-resolution methods, thus positioning MLPs as a formidable approach in the realm of low-level image processing. Complementing the MLP framework, there has been a significant development using the SinGAN[9] model for super-resolution. Initially, while SinGAN offered a novel approach to creating high-resolution images from a single input image, it was hampered by suboptimal image quality. Addressing this, a novel network structure evolving from SinGAN has been put forth, which learns exhaustively from a single image to enhance resolution. This improved structure discards the need for noise input and integrates dense connections coupled with attention mechanisms, considerably enhancing the network's learning efficiency. The impact of these alterations is evident in the superior performance and efficiency evidenced in super-resolution tasks, showcasing the potential of generative models in producing high-quality super-resolution images. Another innovative contribution to this domain is the development of Neural Knitwork[10], a unique architecture that employs a coordinate-based MLP for neural implicit representation learning. This technique diverges from the traditional pixel-based approach, instead of using image patches to train the network. This adversarial optimization of image patch distribution ensures consistency and coherence in the image reconstruction process. Significantly, Neural Knitwork heralds a new direction in super-resolution, image inpainting, and denoising by achieving high fidelity in results with a substantial reduction in the number of required parameters. The architecture achieves an 80% reduction in parameters compared to convolutional neural network (CNN) solutions while delivering comparable performance and efficiency.

The quest for effective denoising techniques in image processing has led to the exploration of both traditional and cutting-edge methodologies. The innovative concept of Deep Image Prior (DIP)[11] posits that the architecture of generator networks innately embodies critical image statistics, which can be harnessed without relying on an extensive learning process. Employing a neural network that is randomly initialized, DIP effectively tackles a spectrum of inverse problems, notably denoising, as well as super-resolution and inpainting. This method not only offers

high-quality image restoration from flash-no flash input pairs but also provides insights into the inductive biases inherent in conventional generator network architectures. In doing so, DIP forms a conceptual bridge linking learning-based techniques with those that operate without explicit learning, thus enriching the toolkit available for image restoration. In the realm of medical imaging, particularly the denoising of PET images, Noise2Void (N2V)[12] has emerged as a notable unsupervised technique. It is distinguished by its blind-spot network architecture, which requires merely a single noisy input image, aligning perfectly with the constraints of clinical environments. By incorporating group-level pretraining along with individualized fine-tuning, and by leveraging anatomical images, the adapted N2V method enhances its denoising capabilities. Through empirical evidence, it is shown that N2V surpasses traditional denoising algorithms, demonstrating consistency and reliability in both simulated and clinical scenarios, thereby advocating for its adoption in practical medical applications. Adding to these advancements, Noise2Self (N2S)[13] presents a general framework for the self-supervised denoising of high-dimensional data. This framework is predicated on the assumption that noise is statistically independent across different dimensions, while the signal itself exhibits correlation. With this foundation, N2S can calibrate an array of denoising algorithms, from simple computational filters to complex deep neural networks. Its efficacy is validated across various data types, including natural imagery, biological microscopy, and gene expression datasets. N2S extends the principles underpinning the training of neural networks with noisy images and the application of cross-validation in matrix factorization. This framework sets a new standard for blind denoising and self-supervision, indicating a significant potential for the refinement and application of self-supervised learning models in denoising and beyond.

Weller et al.'s work[14] represents a significant advancement in tensor-product B-spline surface methodology. By introducing internal knots, they provide improved control over local surface properties, crucial for intricate surface modeling, especially in applications requiring high precision in localized details. This development opens up new avenues for complex modeling tasks in electrical engineering, where the accurate rendering of surfaces is often essential. Building on the theme of precision and flexibility in image processing, the study by Pradeep[15] and others delves into the application of B-spline functions in this domain. They offer a framework for image reconstruction and manipulation that stands out for its efficiency and versatility. This research extends the utility of B-splines beyond traditional applications, presenting a toolset that can adapt to various challenges in image analysis and manipulation in electrical engineering. Margolis et al.'s[16] contribution of "ndsplines," a Python library for implementing tensor-product B-splines in arbitrary dimensions, is a testament to the evolving computational landscape in image processing. This tool significantly enhances the capabilities of researchers and professionals in the field, providing a versatile and powerful resource for tackling complex image processing tasks. It reflects the growing trend of integrating advanced computational tools with traditional engineering methodologies. The synthesis of these investigations delivers a robust synthesis of the current landscape and the prospective advancements in the application of Tensor-Product B-Spline within the domain of Image Processing. Collectively, these studies expand the horizons of achievable out-

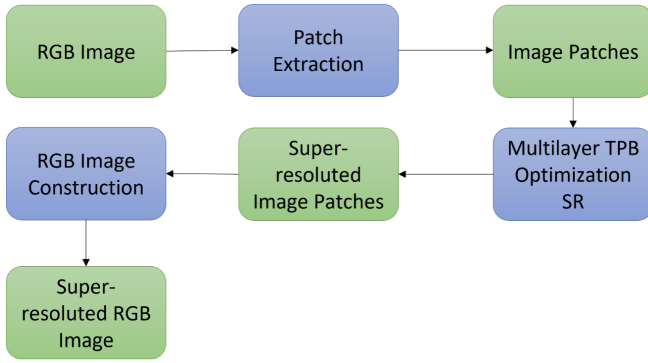


Figure 2: Proposed method for Super-resolution

comes, introducing novel methodologies that not only augment the granularity and efficacy of image processing techniques but also catalyze the evolution of electrical engineering practices. By enhancing current methodologies and catalyzing the inception of groundbreaking applications, these advancements are pivotal in charting new territories for innovation in the sector.

## Method

Moving into the practical steps, the suggested method for enhancing image detail and reducing noise features a layered approach using specialized mathematical functions known as Tensor Product Basis (TPB). For Super-resolution, we start by breaking down the original RGB image into overlapping pieces as shown in Figure 2, each piece finely shaped by a specific TPB function. The heart of this process, termed "Multilayer TPB Optimization SR," layers these TPB functions to polish the image step by step. The first layer carefully adjusts each piece to closely resemble the original image, while the following layers aim to reduce any remaining discrepancies, gradually improving the image's clarity. The crucial "RGB Image Construction" step then brings these enhanced pieces together. This step is carefully planned, using specific weighting for each pixel to ensure the pieces blend smoothly, reducing any mismatch, and leading to a crisp, high-quality image.

In the domain of image denoising, the same patch-based, multi-layer TPB function approach is applied, with the distinction that patches are non-overlapping. The "Multilayer TPB Optimization I and II" stages sequentially refine each patch as shown in Figure 3, reducing noise by focusing on the residuals left after each reconstruction layer. This is complemented by a Guided Filter after the first optimization layer to eliminate visual artifacts, thereby enhancing the quality of each denoised patch. Finally, the "YCbCr Image Construction" stage pieces together the denoised patches to form a cohesive, clean image. This layered, iterative approach allows for the effective reduction of noise while maintaining image detail.

In the subsequent part, we shall present an in-depth analysis of the Implicit Tensor Product Basis (TPB) functionality, an advanced mathematical construct pivotal in the field of electrical engineering for image processing applications. This comprehensive examination will not only articulate the conceptual framework of the TPB but will also expound upon its practical implementation within the scope of image enhancement techniques.

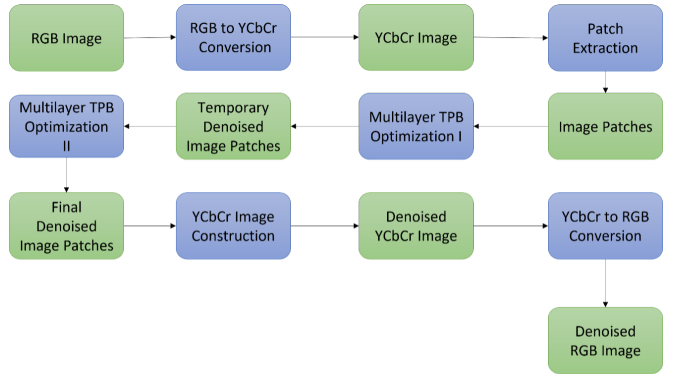


Figure 3: Proposed method for Denoising

## Utilizing Implicit TPB for Image Modeling

Transitioning to the utilization of Implicit TPB for Image Modeling, we consider a different input paradigm where coordinates form the basis of the input data rather than pixel values. The TPB receives these coordinates and processes them using ground truth data through a non-iterative, least-squares approach to problem-solving. This methodological choice for image modeling with implicit TPB is computationally efficient and expeditious. It significantly contributes to the enhancement of image resolution and the diminution of noise, which are paramount in improving the clarity and fidelity of the images processed within this framework.

The input for the Implicit TPB function consists of coordinate points, with the output being a single value. The mapping can be denoted as  $v = f_{\text{TPB}}((x, y))$ . The model consists of coefficients  $m_{\text{TPB}}$ , which are solved during the training function to obtain the TPB model  $f_{\text{TPB}} = f_{\text{TPB\_fitting}}(v, (x, y))$ .

Coordinates are fundamental as input values for implicit functions. The coordinates used in denoising and super-resolution tasks are denoted as  $\Phi_p = \{(x, y)\}$ , normalized within  $[0, 1]$ . The normalization process within each patch is performed using:

$$\hat{x} = \frac{(x - x_{\min}^p)}{(x_{\max}^p - x_{\min}^p)} \quad (1)$$

$$\hat{y} = \frac{(y - y_{\min}^p)}{(y_{\max}^p - y_{\min}^p)} \quad (2)$$

The normalized coordinates are  $\Phi_p = \{(\hat{x}, \hat{y})\}$ .

For super-resolution, an upsampling scale "F" is used to create a denser grid of coordinates, thus generating a higher-resolution patch.

The TPB optimization process is conducted on normalized coordinates with the associated output values represented as  $I_{\text{TPB\_fit}}$ . The fitting process is given by  $f_{\text{TPB}} = f_{\text{TPB\_fitting}}(I_{\text{TPB\_fit}}, \{(\hat{x}, \hat{y})\})$ .

The TPB basis functions are constructed by multiplying basis functions along each axis, with the prediction performed using:

$$\hat{S}_{\text{TPB}}(\hat{x}, \hat{y}) = \sum_{i^h=0}^{D_h-1} \sum_{i^v=0}^{D_v-1} m_{i^h, i^v}^{\text{TPB}} \cdot B_{i^h, i^v}^{\text{TPB}}(\hat{x}, \hat{y}) \quad (3)$$

where  $m^{TPB}$  are the TPB coefficients.

The optimal solution for the TPB coefficients is obtained via the least squared solution:

$$m^{TPB,opt} = ((S_{TPB}^T S_{TPB})^{-1} (S_{TPB}^T I_{TPB,fit})) \quad (4)$$

This optimizes the coefficients for predicting the output image.

In the ensuing discourse, we will meticulously dissect the components of our methodology, shedding light on each segment's role within the overarching scheme for super-resolution and denoising. This detailed exposition is aimed at providing a granular understanding of the sophisticated interplay between the various modules that collectively contribute to the refinement and clarity of the processed images.

### Image Super-Resolution

The process of image super-resolution (SR) in our methodology occurs within the RGB domain. The goal is to produce an output image with a resolution that is  $F$  times greater than the input image of dimensions  $W \times H$ . This is achieved by extracting overlapping patches of size  $W^p \times H^p$ . Adjacent patches overlap by regions measuring  $16 \times 32$  or  $32 \times 16$  pixels to maintain continuity and consistency.

### Multilayer TPB Optimization SR

This method incorporates a multi-layer architecture with each layer consisting of a single-layer TPB optimization.

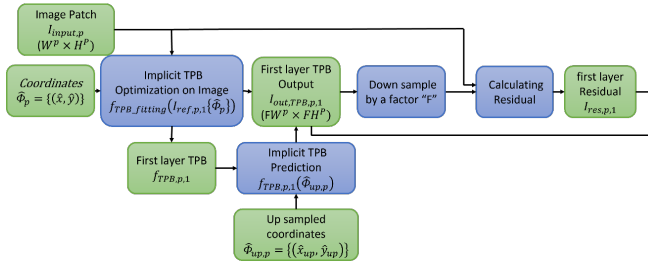


Figure 4: First Layer Structure

The first layer approximates the original image using TPB models and calculates residuals, as detailed in Figure 8.

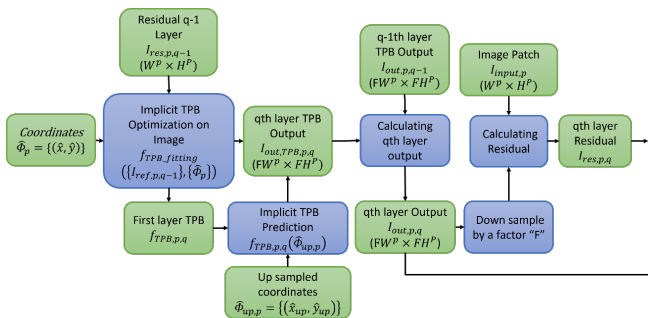


Figure 5: Even Number Layer Structure

Even-numbered layers aim to enhance the SR method by increasing PSNR and reducing error, as shown in Figure 5.

Odd-numbered layers eliminate wave artifacts and improve PSNR, as depicted in Figure 6.

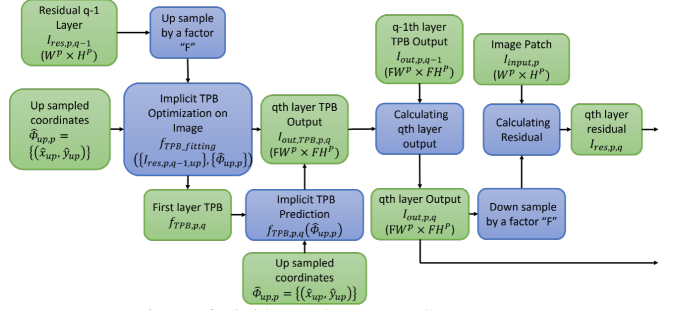


Figure 6: Odd Number Layer Structure

### Stopping Criterion of Layers

The stopping criterion is based on the stability of output PSNR values, calculated as follows:

$$v_{PSNR,p,q} = PSNR(\text{downsample}(I_{out,p,q}), I_{input,p}) \quad (5)$$

$$v_{PSNR,average,p,q} = \frac{\sum_{i=q-10}^{q-1} v_{PSNR,p,i}}{10} \quad (6)$$

### RGB Image Construction

In the process of formulating a super-resolution image, the integration of overlapping patches is critical. Our methodology utilizes patches of dimensions  $32 \times 32$  pixels, with neighboring patches sharing overlapping regions of either  $16 \times 32$  or  $32 \times 16$  pixels. This configuration is key during the fusion phase of super-resolution image generation.

The output image's corner pixels are derived from a single patch, edge pixels from two adjoining patches, and central pixels from four intersecting patches. To ensure a smooth transition across patch boundaries, we propose using variable weighting factors for each pixel. The final fused pixel value for pixels covered by multiple overlapping patches is the weighted sum of pixel values from all overlapping patches, normalized by the sum of the weight factors.

The design of the weighting factor is optimized by assigning weights based on the patch shape. We aim for equal weighting within the rectangular contour for a more accurate representation. The weighting matrix for each color channel  $W^p \times H^p$  is prepared using the following equations, where  $d_O^C$  is the dimension of the patch for color channel  $C$ :

$$\sigma_O^C = 0.2 \cdot d_O^C \quad (7)$$

$$\tau^C(x_{up}, y_{up}) = \max \left( \left| 2 \left( \frac{x_{up}}{d_O^C} - 0.5 \right) \right|, \left| 2 \left( \frac{y_{up}}{d_O^C} - 0.5 \right) \right| \right) \quad (8)$$

$$w^C(x_{up}, y_{up}) = \frac{1}{e} e^{-\left( \frac{\tau^C(x_{up}, y_{up})^2}{\sigma_O^C} \right)} \quad (9)$$

For boundary patches that do not have the full patch size, the same equations are applied by aligning the top-left corner.

Two new memory spaces are prepared with the same dimensions as the image  $W \times H$ , denoted as  $I^A$  and  $I^W$ . The channels of  $I^A$  and  $I^W$  are initialized to zero. For the patches, we define the following notation:

$$I_p^{C,O} = \text{Super-resolved patch for color channel } C \quad (10)$$

$$I_{\text{out,end},p}^C = \text{Final output for patch } p \text{ in color channel } C \quad (11)$$

The mapped values are multiplied by the weighting matrix and accumulated:

$$I^{C,A}(x_{\text{up}}, y_{\text{up}}) = I^{C,A}(x_{\text{up}}, y_{\text{up}}) + I_p^{C,O}(x_{\text{up}}, y_{\text{up}}) \cdot w^C(x_{\text{up}}, y_{\text{up}}) \quad (12)$$

$$I^{C,W}(x_{\text{up}}, y_{\text{up}}) = I^{C,W}(x_{\text{up}}, y_{\text{up}}) + w^C(x_{\text{up}}, y_{\text{up}}) \quad (13)$$

After processing all patches, the accumulated weighted pixel values are divided by the accumulated weights to obtain the final image.

$$S^C(x_{\text{up}}, y_{\text{up}}) = \frac{I^{C,A}(x_{\text{up}}, y_{\text{up}})}{I^{C,W}(x_{\text{up}}, y_{\text{up}})} \quad (14)$$

The use of overlapping patches significantly improves the quality of the super-resolution output image, providing a smooth and cohesive visual result without blocking artifacts.

### Image Denoising

Our proposed framework for image denoising operates within the YCbCr color domain and employs an implicit function, the implicit TPB, to model each color channel. We adopt a patch-based approach for processing.

The "Multilayer TPB Optimization I" method is visualized in Figure 7. This process is divided into N iterative layers.

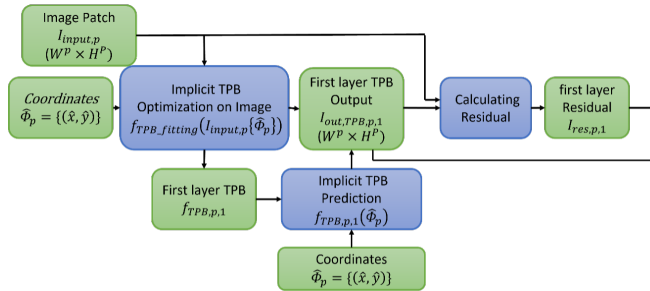


Figure 7: Multilayer TPB Optimization I process

The first layer is crucial in approximating the original image, which is depicted in Figure 8.

The mathematical representation of the first layer is as follows:

$$f_{\text{TPB},p,1} = f_{\text{TPB\_fitting}}(I_{\text{input},p}, \Phi_p) \quad (15)$$

$$I_{\text{out,TPB},p,1} = f_{\text{TPB},p,1}(\Phi_p) \quad (16)$$

$$I_{\text{out},p,1} = I_{\text{out,TPB},p,1} \quad (17)$$

$$I_{\text{res},p,q} = I_{\text{input},p} - I_{\text{out},p,q} \quad (18)$$

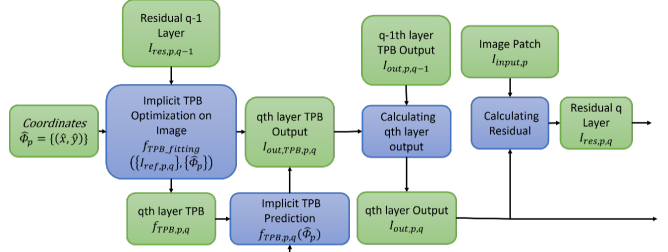


Figure 8: First layer of the Multilayer TPB Optimization I process

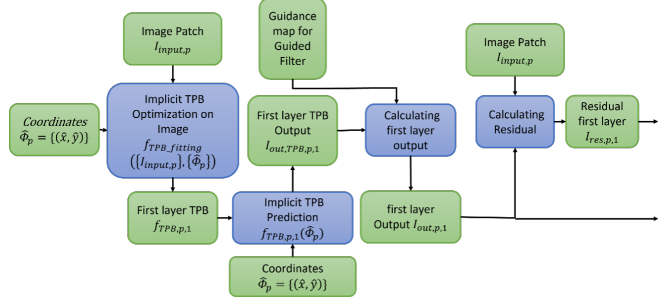


Figure 9: Subsequent layers in the Multilayer TPB Optimization I process

Subsequent layers are designed to further enhance the PSNR. The process for these layers is detailed in Figure 9.

The equations for the subsequent layers are given by:

$$f_{\text{TPB},p,q} = f_{\text{TPB\_fitting}}(I_{\text{res},p,q-1}, \Phi_p) \quad (19)$$

$$I_{\text{out,TPB},p,q} = f_{\text{TPB},p,q}(\Phi_p) \quad (20)$$

$$I_{\text{out},p,q} = I_{\text{out,TPB},p,q} + I_{\text{out},p,q-1} \quad (21)$$

$$I_{\text{res},p,q} = I_{\text{input},p} - I_{\text{out},p,q} \quad (22)$$

"Multilayer TPB Optimization II" is detailed in the figure below and incorporates a guided filter with a guidance map derived from the Y channel of the "Multilayer TPB Optimization I" output.

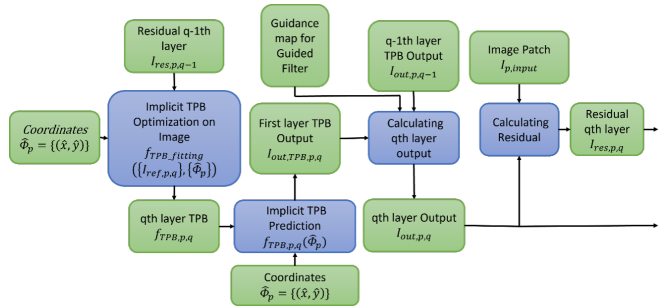


Figure 10: Multilayer TPB Optimization II process

The guided filter operation is defined by:

$$I_{\text{out\_gf}} = \text{GF\_filter}(I_{\text{in\_gf}}, I_{\text{GF\_map}}) \quad (23)$$

The process follows a similar structure to the "Multilayer TPB Optimization I," with adaptations to include the guided filter. The first layer and subsequent layers are processed similarly with the addition of the guided filter.

**Evaluation**  
**Objective Results**  
**Super-resolution**

The efficacy of various super-resolution techniques was quantitatively and qualitatively assessed using the Set14 dataset, which encompasses a variety of commonly encountered image types.

**Dataset Description:** The Set14 dataset, integral to super-resolution benchmarking, consists of 14 images that represent a cross-section of real-world scenarios. These images, with resolutions spanning from  $321 \times 481$  to  $768 \times 1024$  pixels, include a spectrum of scenes and textures.

**Methods Evaluated:** Our study encompasses a suite of super-resolution methodologies, ranging from conventional coordinate Multilayer Perceptrons (MLPs) and Convolutional Neural Network (CNN)-based SinGAN Neural Knitwork frameworks to our innovative Tensor Product Basis (TPB) approach.

**Evaluation Metrics:** To gauge the quality of the super-resolved images, we computed the Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity Index Measure (SSIM), supplementing these quantitative metrics with visual inspections to assess fidelity to the high-resolution ground truth.

**Average Results:** Table 1 summarizes the performance metrics, illustrating the relative effectiveness of each method in enhancing image resolution.

Table 1: Average PSNR/SSIM results for Set14 dataset using various super-resolution methods.

Upscaling	Bicubic	MLP	SinGAN	Neural Knit-work	TPB
x2	28.43/	27.22/	14.21/	24.31/	25.62/
	0.823	0.89	0.41	0.82	0.84
x4	24.04/	22.45/	14.32/	21.72/	22.37/
	0.687	0.76	0.33	0.75	0.74

Our findings reveal that the TPB-based method exhibits competitive performance with existing super-resolution techniques, positioning it as a viable alternative for enhancing image quality.

**Denoising**

We compared various denoising approaches by analyzing their performance across different noise levels, with results summarized in Table 2.

Table 2: Quantitative comparison of denoising methods under various noise levels.

Image	TPB	DIP	N2V	N2S
Kodak01	30.96	29.83	27.39	29.02
Kodak02	32.86	33.38	32.84	32.92
Kodak03	33.70	34.75	33.06	34.21
Kodak12	32.96	35.27	33.84	33.89



Figure 11: Example of upscaled image (x4) using our proposed method.



Figure 12: Example of denoised using our proposed method.

You can see example of our TPB based output for both super-resolution and denoising task on Figure 11 and Figure 12.

The TPB-based denoising method performs comparably to other established denoising techniques, reinforcing its practicality for image processing applications.

**Future Work**

The promising results obtained from the application of the Tensor Product Basis (TPB) approach to image super-resolution and denoising pave the way for several exciting avenues of future research. The scalability and efficiency of the TPB framework make it an attractive candidate for extension to other image processing tasks such as image inpainting, segmentation, and texture synthesis. Future work will explore the integration of TPB with deep learning architectures to create hybrid models that can leverage the strengths of both traditional spline-based approaches and modern neural networks. Another area of interest lies in the application of the TPB framework to video processing tasks, where temporal coherence can be modeled using higher-dimensional TPBs.

Moreover, the adaptability of the TPB method to different image modalities suggests its potential utility in medical imaging, where it can be used to enhance the resolution and reduce the noise of MRI and CT scans, potentially aiding in more accurate diagnosis and treatment planning. The extensibility of the

TPB framework will also be examined in the context of real-time processing applications, such as in autonomous vehicles and mobile photography, where computational efficiency is paramount. Adapting the TPB method to work on edge devices with limited processing capabilities remains a challenging but worthwhile goal.

## Conclusion

In conclusion, this study presented a novel image super-resolution and denoising approach using the implicit Tensor Product Basis (TPB). Our methodology, characterized by its non-iterative nature and computational efficiency, demonstrated competitive performance against established methods across standard benchmark datasets. Through rigorous experimentation, the TPB-based approach not only met the quality standards set by traditional methods but also showed potential for significant computational savings. The use of TPB allows for precise control over the modeling of image features, which is crucial in preserving image details and reducing noise artifacts.

The qualitative and quantitative results underscore the versatility of the TPB method, affirming its applicability to a broad range of image processing tasks. By providing a balance between performance and efficiency, the TPB framework stands as a testament to the potential of spline-based methods in the era of neural networks. As we look to the future, the TPB approach is poised to become a cornerstone in the development of advanced image processing techniques. Its ability to be integrated with other computational models opens up a realm of possibilities for innovation and application across diverse fields. This research not only contributes to the academic discourse but also holds significant implications for practical applications in industry and beyond.

## References

- [1] Francis Williams, Zan Gojic, Sameh Khamis, Denis Zorin, Joan Bruna, Sanja Fidler, Or Litany, Neural Fields as Learnable Kernels for 3D Reconstruction, Presented at IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), 2022
- [2] Yuan-Chen Guo, NeRFReN: Neural Radiance Fields with Reflections, Presented at IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), 2022.
- [3] Nilesh Kulkarni, Justin Johnson, David F. Fouhey, Scene-DRDF, Presented at the European Conference on Computer Vision (ECCV), University of Michigan, Ann Arbor, 2022.
- [4] Yinbo Chen, Xiaolong Wang, Transformers as Meta-Learners for Implicit Neural Representations, European Conference on Computer Vision, 2022.
- [5] Yiheng Xie, Towaki Takikawa, Shunsuke Saito, Or Litany, Shiqin Yan, Numair Khan, Federico Tombari, James Tompkin, Vincent Sitzmann, Srinath Sridhar, Neural Fields in Visual Computing and Beyond, arXiv:2111.11426 [cs.CV]
- [6] Wentao Shangguan, Yu Sun, Weijie Gan, Ulugbek S. Kamilov, Learning Cross-Video Neural Representations for High-Quality Frame Interpolation, arXiv:2203.00137 [eess.IV], <https://doi.org/10.48550/arXiv.2203.00137>, 2022.
- [7] Wentao Shangguan, Yu Sun, Weijie Gan, Ulugbek S. Kamilov, Learning Cross-Video Neural Representations for High-Quality Frame Interpolation, arXiv:2203.00137 [eess.IV], <https://doi.org/10.48550/arXiv.2203.00137>, 2022.
- [8] Trevor Hastie, Robert Tibshirani, Jerome Friedman, The Elements of Statistical Learning: Data Mining, Inference, and Prediction, Published
- [9] Tamar Rott Shaham, Tali Dekel, Tomer Michaeli, SinGAN: Learning a Generative Model from a Single Natural Image, Presented at Arxiv, 2019
- [10] Mikolaj Czerkawski, Javier Cardona, Robert Atkinson, Craig Michie, Ivan Andonovic, Carmine Clemente, Christos Tachtatzis, Neural Knitworks: Patched Neural Implicit Representation Networks, Presented at Arxiv, 2022
- [11] Dmitry Ulyanov, Andrea Vedaldi, Victor Lempitsky, Deep Image Prior, Presented at Arxiv, 2017
- [12] Alexander Krull, Tim-Oliver Buchholz, Florian Jug, Noise2Void - Learning Denoising from Single Noisy Images, Presented at Arxiv, 2018
- [13] Joshua Batson, Loic Royer, Noise2Self: Blind Denoising by Self-Supervision, Presented at Arxiv, 2019
- [14] Frank-Uwe Weller, B-Spline Surfaces with Knot Segments, Published in 1994, Mathematics, Computer Science.
- [15] Taro Kawasaki, Pradeep Kumar Jayaraman, Kentaro Shida, Jianmin Zheng, Takashi Maekawa, An Image Processing Approach To Feature-preserving B-spline Surface Fairing, Presented at Computer-Aided Design, February 2018.
- [16] Benjamin W. L. Margolis, Kenneth R. Lyons, ndsplines: A Python Library for Tensor-Product B-Splines of Arbitrary Dimension, Journal of Open Source Software, vol. 4, no. 42, p. 1745, 2019, DOI: 10.21105/joss.01745.

## Author Biography

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