https://doi.org/10.2352/EI.2023.35.15.COLOR-198


# Hue-preserving color enhancement under a cylindrical model without geometric deformation of the RGB color cube

*Tieling Chen and Onan Chew; University of South Carolina Aiken; Aiken, South Carolina, USA*

## Abstract

*Color image processing techniques are studied under a cylindrical model that does not produce geometric deformation of the RGB color cube. The main feature of this model is to use the distance from a color point in the RGB cube to the gray diagonal of the cube as a component, thereby forming a cylindrical model that does not deform the color cube. This distance component is closely related to color vividness and can be used as a color processing channel by itself. In addition to this component, the cylindrical model has an intensity component that can be represented by planes, which is also a channel for color enhancement. The space of this cylindrical model is larger than the RGB color cube, so the gamut problem may occur when color processing is not well controlled. This paper introduces a method to solve the gamut problem by compressing color overflow near the boundary of the color cube. This allows free application of conventional processing methods of redistributing the values of a component. Under this cylindrical model, with the proposed compressing method, not only the usual techniques of adjusting the distribution can be used in the intensity component without worrying about the gamut problem, but a similar strategy can be applied to the distance component to adjust color vividness. The combination processing through these two channels provides an effect that is difficult to achieve by other cylindrical models that geometrically deform the RGB color cube.*

## Introduction

The RGB color model and the color models directly based on it have a wide range of effective applications in color image processing. One reason is the widespread use of technology for displaying colors using the three primary colors, making RGB a convenient model for displaying color images on electronic devices. Another important reason is the unique analytic geometric property of its color representation, that is, colors are directly represented by the mathematical coordinates in the three-dimensional Euclidean space, so that color transformations and even the color space transformations can be achieved by mathematical equations with clear geometric meaning. Although the RGB model is a theoretical model rather than a model for representing absolute colors, due to the computational and display conveniences it brings, as well as the widely accepted industry standards for it, this model and its closely related models still have a wide range of applications.

## Pseudo-cylindrical color models

Some color models that describe the visual features of a color are based on the RGB color model. Among these color models the models HSV [5] and HSL [3] are very successful and widely used. These two models mesh the space of the RGB model through three channels, without gaining or losing colors based on what the RGB color cube contains. Color models such as HSV and HSL describe colors in terms of color visual features, such as hue, saturation, and an achromatic measure that is independent of chromaticity, representing brightness or lightness. Usually, they have the same hue channel, which meshes the RGB color cube into triangular planes. The saturation and the achromatic component are different in HSV and HSL models.

Let us call a surface with equal saturation an equi-saturation surface and call a surface with equal brightness or lightness or intensity an equi-brightness surface or an equi-lightness surface or an equi-intensity surface, respectively. In HSV, an equi-saturation surface is a hexagonal cone centered on the gray diagonal of the color cube, and an equi-brightness surface is not a planar surface. The situation is similar in the HSL model, where an equi-saturation surface is a double hexagonal cone, and an equi-lightness surface is also not a planar surface.

For the convenience of geometric representation of the color components and color selection, both HSV and HSL have cylindrical variations. Each cylindrical model is a geometric deformation of the RGB color cube, including flattening non-planar equi-brightness and equi-lightness surfaces, stretching surfaces of different sizes to the same size, and transforming hexagonal cones into cylinders. These cylinders are only variant representations of their actual color model spaces, and they are deformations of the RGB color cube. We call them pseudo-cylindrical models.

There is also a widely used theoretical model HSI [2] that is closely related to the RGB model and has similar concepts to HSV and HSL. This model has a hue component that is the same as in HSV and HSL, a saturation component that is conceptually like that of HSV or HSL, but its achromatic component intensity is not bounded by the RGB color cube. The model HSI also has a cylindrical representation that geometrically deforms the RGB color cube, so it is also a pseudo-cylindrical model. Different from HSV and HSL, the model HSI does not limit its space within the RGB color cube but circumscribes the color cube with its space. This means the geometric space of HSI is larger than the color cube, so when processing an image in the HSI model, a processing transformation might move some colors out of the RGB color cube, causing the so-called gamut problem.

## Hue preserving processing and color vividness

In general applications, color image processing follows the requirement of hue preserving. Under a color model with a hue component, such as HSV, HSL, or HSI, this requirement is automatically satisfied by keeping the hue component unchanged. A class of processing techniques that are widely used in color enhancement and tone corrections is to process and transform through the achromatic component, and then combine the new achromatic component with the other two unchanged components, hue and saturation, to obtain a new image. The processing method on the achromatic component is basically to adjust the distribution of values to obtain a new distribution that is conducive to improving the visual effect of the overall image or satisfying a specific application requirement.

In the models of HSV, HSL, and HSI, after processing through the achromatic component, the vividness of the original image will change significantly. Color vividness is closely related to the distance from a color point to the gray diagonal of the RGB color cube. Modifying the vividness can significantly change the visual appearance of a color image, and therefore it is a significant measure for color image processing.

However, in the pseudo cylindrical representations of HSV, HSL, and HSI models, equi-saturation represented by cylinders are actually hexagonal cones in the RGB color cube, so the saturation component of these pseudo cylindrical color models does not correctly measure the color vividness. In [6] and [7], processing is performed in RGB model and the distance from a color point to the gray diagonal is named saturation. Such definition of saturation should not be confused with the saturation component in HSV, HSL, and HSI models. The vividness of a color cannot be represented by only one component in the color models including RGB, HSV, HSL, and HSI. It would be a good idea to manipulate the overall vividness of a color image directly as a single component of a color model.

### The gamut problem

The gamut problem occurs if a processing transformation moves colors outside the RGB color cube. In HSV and HSL models, the gamut problem is easy to prevent, because their actual model space is consistent with the RGB cube, so it is sufficient to ensure that the value of the component being processed is within the value range of this component. However, this method does not eliminate the gamut problem in the HSI model. This is because the HSI model space is much larger than the RGB color cube. A value processed by a transformation may still be in the space of the HSI, but it is beyond the bounds of the color cube.

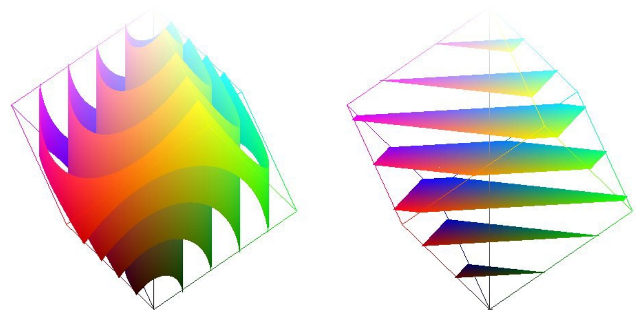
The gamut problem is also a problem to watch out for when using the RGB model directly to process images. One way to do this is to have subsequent control over the processing function to ensure that the color points are not moved outside the RGB color cube. In [4], the processing function is first applied to the intensity of a color point, where the definition of the intensity is consistent with the intensity in the HSI model, and then the transformation result of the color point is determined according to the ratio of the transformed intensity to the original intensity, to avoid the gamut problem. This method of subsequent control is improved in [6] and [7] to avoid the gamut problem, and also protect and enhance an important feature of color.

## Hue-preserving color enhancement under a cylindrical model

This paper adopts a true cylindrical color model that does not deform the RGB color cube and introduce a method that processes a color image through two components, a vividness component and an intensity component, and avoid the gamut problem brought by the cylindrical color model.

### A true cylindrical model over the RGB color cube

A cylindrical color model named HDI that is based on RGB color model is proposed in [1]. This cylindrical model does not geometrically deform the RGB color cube, but directly put on a mathematical cylindrical coordinate system to the color cube. The space of the model is a cylinder centered with the gray diagonal of the color cube, circumscribing the color cube without deforming it. The model HDI has a hue component H that is the same as that of the models of HSV, HSL, and HSI. Its distinctive feature is the D component, representing the distance from a color point to the center axis, which is the radius of the corresponding cylinder. Some cylinders with different radii restricted within the RGB color cube are displayed in Figure 1 (a). This distance is an actual value, not a value obtained by geometrically deforming the color cube. Its intensity component I is similar to that of HSI, which can be represented by planes perpendicular to the central axis. Some equi-intensity planes restricted within the RGB color cube are displayed in Figure 1 (b).



(a) (b)

***Figure 1. (a)** Cylindrical surfaces of the color model HDI within the RGB color cube. (b) Planes representing the intensities of HDI within the RGB color cube.*

Suppose $(h,d,i)$ represents a color point in HDI and $(r,g,b)$ represents a color point in RGB, the conversions between these two models are given by [1]

$$\begin{cases} h = \begin{cases} \arccos\dfrac{2r-g-b}{2\sqrt{r^2+g^2+b^2-rg-rb-gb}}, & \text{when } g \ge b \\ 2\pi - \arccos\dfrac{2r-g-b}{2\sqrt{r^2+g^2+b^2-rg-rb-gb}}, & \text{when } g < b \end{cases} \\ d = \dfrac{\sqrt{6}}{3}\sqrt{r^2+g^2+b^2-rg-rb-gb}, \\ i = \dfrac{\sqrt{3}}{3}(r+g+b). \end{cases}$$

and

$$\begin{cases} r = \dfrac{\sqrt{6}}{3} d\cos(h) + \dfrac{\sqrt{3}}{3} i, \\ g = -\dfrac{\sqrt{6}}{6} d\cos(h) + \dfrac{\sqrt{2}}{2} d\sin(h) + \dfrac{\sqrt{3}}{3} i, \\ b = -\dfrac{\sqrt{6}}{6} d\cos(h) - \dfrac{\sqrt{2}}{2} d\sin(h) + \dfrac{\sqrt{3}}{3} i. \end{cases}$$

The model HDI is a true cylindrical color model without deformation of the color cube, and it provides a valuable processing option through the distance channel in addition to the conventional processing method through the achromatic channel, the intensity channel. In other words, it has two degrees of freedom for color image processing. The main concern of using HDI to process color images is the gamut problem because the space of the model is bigger than the color cube. The dimension of the cylinder space of HDI is: $0 \le h < 2\pi$ , $0 \le i \le \sqrt{3}$ , and $0 \le i \le \sqrt{3}$ , which is bigger than the RGB color cube. The gamut problem could occur through both the distance channel and the intensity channel.

## A solution to the gamut problem of HDI

We introduce a method that does not deal with specific transformations but processes those color points that are about to be transformed out of the color cube near the border, so that they can eventually stay in the color cube in a proper way. Such a method enables conventional color image processing techniques to be used on the I and D components of HDI without gamut concerns.

This method is different from the simple but ineffective solution that replaces the color removed out of the color cube with the color on the edge of the cube, and it is also different from the method that compresses the color range that the processing function can reach along a linear direction to the range allowed by the RGB color cube in this direction. The method keeps the original intended effect of a processing function for the colors that do not cause the gamut problem, and only adjust the colors gracefully that are affected by the gamut problem.

We only consider the type of non-negative processing functions that keep the order of the processed values, that is, the processing functions are increasing non-negative functions. Assuming that along a certain linear direction, the line segment within the RGB color cube is represented by $[a,b]$, and a processing function $p$ converts the colors on this line segment to the color segment $[p(a), p(b)]$ in the HDI space. If $p(a) < a$, or $p(b) > b$, or both occur, then the gamut problem occurs in this direction either on one end or both ends. We do not consider the rarely happening situation that either $p(a) > b$ or $p(b) < a$.

The proposed method is to compress only at each end of the color segment. If $p(b)$ is outside $[a,b]$, then find a point $b'$ near $b$ inside $[a,b]$, and then use a smooth function to compress the color segment $[b', p(b)]$ to the color segment $[b',b]$.

Similarly, if $p(a)$ is at the other end of the color segment $[a,b]$, then find a point $a'$ near $a$ inside $[a,b]$, and then compress the color segment $[p(a),a']$ to the color segment $[a,a']$ smoothly. The color segment $[a,b]$ will retain a large uncompressed part, which can fully reflect the effect of the transformation function $p$. At one or both ends that cause the gamut problem, the compression of color is a smooth transition, avoiding abrupt changes in color.

## Compression functions

We use the cubic function

$$f(x) = \frac{1}{3}x^3 - x^2 + x \tag{1}$$

to compress the interval $[0,1]$ to the interval $[0,1/3]$ in a smooth way. Now we apply this function to solve the gamut problem following the ideas of the above solution.

Consider the first case mentioned in the above. We want to compress the interval $[b', p(b)]$ to the interval $[b',b]$, where $b' < b < p(b)$. The point $b'$ is selected in the way that the length of $[b', p(b)]$ is three times longer than the length of $[b',b]$, which is $p(b) - b' = 3(b - b')$. Then we get

$$b' = 1.5b - 0.5p(b) = b - 0.5(p(b) - b). \tag{2}$$

With a standard transformation of the function in (1), we obtain the following function

$$F(x) = b' + (p(b) - b')f\left(\frac{x - b'}{p(b) - b'}\right), \tag{3}$$

which meets the requirement of compressing the interval $[b', p(b)]$ to the interval $[b',b]$ smoothly.

For the gamut problem on the other end, we first determine $a'$ by the condition that the length of $[p(a),a']$ is three times longer than the length of $[a,a']$, which is $a' - p(a) = 3(a' - a)$, and get

$$a' = 1.5a - 0.5p(a) = a + 0.5(a - p(a)). \tag{4}$$

By a similar analysis, we find the function

$$G(x) = a' - (a' - p(a))f\left(\frac{a' - x}{a' - p(a)}\right) \tag{5}$$

meets the requirement of compressing the interval $[p(a),a']$ to the interval $[a,a']$ smoothly.

## Solving the gamut problem in the intensity channel

Suppose $p$ is a processing function on the I component of the HDI model. The effect of the processing is to move colors along the direction parallel to the gray diagonal of the color cube. For fixed H and D values, the range of I is restricted by a lower face and an upper face of the color cube, illustrated in Figure 2 (a). The intensities at the two ends can be calculated, so we get the interval $[a,b]$ for the intensities in this segment. Then for the intensity $i \in [a,b]$, we have $p(i) \in [p(a), p(b)]$.

If $p(b) > b$, then gamut problem occurs on top. We use equation (2) to find the value of $b'$ and use equation (3) to deal with the gamut problem. The function is a composition of $p$ and $F$ on the original intensity, which is $F(p(i))$ for $i \in [b',b]$.

Similarly, if $p(a) < a$, then gamut problem occurs at the bottom. We use equation (4) to find the value of $a'$ and use equation (5) to deal with the gamut problem. The function is a composition of $p$ and $G$ on the original intensity, which is $G(p(i))$ for $i \in [a,a']$.

Then the main task is to find the interval $[a,b]$. For this we need consider the six different sectors of H respectively because the projection of the color cube onto the base plane is a hexagon. For every fixed $h$ and $d$ in a sector, the minimum intensity and the maximum intensity can be obtained from the equations of the lower face and the upper face of the color cube over the sector. For example, in the sector of $0 \le h < \pi/3$, following the suggested calculations we can find

$$a = \frac{\sqrt{2}}{2}d\cos(h) + \frac{\sqrt{6}}{2}d\sin(h), \text{ and } b = \sqrt{3} - \sqrt{2}d\cos(h).$$

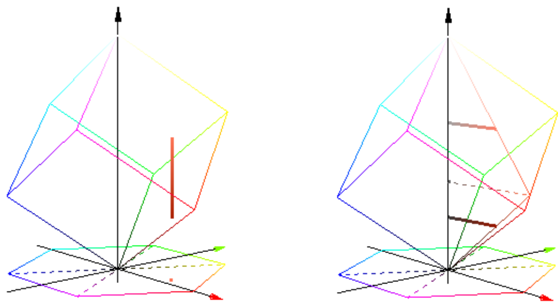



***Figure 2****. (a) The thickened vertical line segment depicts the range of intensity I for fixed values of H and D. (b) The dashed line is at the intensity level of $i_0$. The thickened horizontal line segments depict the range of D for fixed values of H and I. The upper one is for a bigger I value and the lower one is for a smaller I value.*

## Solving the gamut problem in the distance channel

Suppose $q$ is a processing function on the D component of the HDI model. The effect of the processing is to move colors along the direction perpendicular to the gray diagonal of the color cube. For fixed H and I values, the range of D is restricted by either a lower face or an upper face of the color cube, depending on the height of the intensity, illustrated in Figure 2 (b). If we can find the out end of the distance component $b$, then the interval for the distances is $[0,b]$. For any distance $d \in [0,b]$, we have $q(d) \in [q(0), q(b)]$.

The gamut problem may occur on the right end when $q(b) > b$, but it does not occur on the left end because $q(0) \in [0,b]$. When the gamut problem occurs, we use equation (2) to find the value of $b'$ and use equation (3) to deal with the gamut problem. The function is a composition of $q$ and $F$ on the original intensity, which is $F(q(d))$ for $d \in [b', b]$.

Then the job focuses on finding the value of $b$. For a fixed hue value $h$, the value of $b$ is determined by the value of $i$. As illustrated in Figure 5 (b), suppose $i_0$ be the intensity of the outer vertex of the equi-hue triangle for the hue value $h$, if $0 \le i \le i_0$ then $b$ is determined by the lower side of the triangle, and if $i_0 < i \le \sqrt{3}$ then $b$ is determined by the upper side of the triangle.

To find $i_0$ and $b$, we also need to consider six different sectors of the H component respectively. Still take the sector of $0 \le h < \pi/3$ as an example. Mathematical calculations find that

$$i_0 = \frac{3\sin(h) + \sqrt{3}\cos(h)}{\sqrt{3}\sin(h) + 3\cos(h)},$$

and if $0 \le i \le i_0$ then

$$b = \frac{\sqrt{2}i}{\cos(h) + \sqrt{3}\sin(h)},$$

and if $i_0 < i \le \sqrt{3}$ then

$$b = \frac{\sqrt{3} - i}{\sqrt{2}\cos(h)}.$$

For other sectors of $h$, the value of $i_0$ and the two values of $b$ can be found similarly.

## Hue preserving processing under HDI without gamut problem

Under the HDI model, color images can be processed with hue preserving from two channels, I and D. In addition to processing through the achromatic channel I as in the HSV, HSL, and HSI models, color images can also be processed in the distance channel D to adjust the overall vividness of the image. The processing method on each channel is to use a transformation function to adjust the distribution pattern of the values on that channel. According to the distribution pattern of the tones of the image, color images are roughly divided into categories that are visually too dark, too bright, and too flat [2], then the corresponding processing functions can be designed to redistribute the tones. These methods can be used on the I component of HDI.

Similar processing techniques can also be applied on the D component. If an image is over vivid, too dull, or lacks contrast in vividness, a processing function can be accordingly designed to adjust the distribution of values on the distance channel.

There is a concern about the order of processing through the two channels D and I. In fact, both order work and they do not make a significant difference. Since the processing functions on these two components are independent, in the space where no gamut problem occurs, the two different orders give the same processing result. Where there may be a gamut problem near the border of the color cube, the different processing orders may produce subtle differences, but they will basically not affect the processing effect.

## Experimental results

For convenience, in our experiments a processing function $g$ is defined in the interval $[0,1]$ and has a range not bigger than the interval $[0,1]$. Also, $g$ is a continuous increasing function. The function is accordingly stretched to fit the domains of $i$ in the intensity channel and $d$ in the distance channel. This says, when using $g$ in the intensity channel, the processing function is $p(i) = \sqrt{3}g\left(\frac{i}{\sqrt{3}}\right)$, and when using $g$ in the distance channel, the processing function is $q(d) = \frac{\sqrt{6}}{3}g\left(\frac{3d}{\sqrt{6}}\right)$. In the following examples, a given image is processed through the intensity channel first and then through the distance channel. Except the original image and the resultant image, we also display the 3-dimensional distributions of colors of both images. The corresponding processing function for the intensity channel $g_i$ and the processing function for the distance channel $g_d$ are provided too.

Figure 3 (a) displays an image with low visual quality of a farm field with a driveway. Its 3D color distribution shown in Figure 3 (c) indicates that the image is too strong in high intensities and is very weak on the overall vividness. With the proposed method of processing the image through the I and D channels under the color model HDI, two simple power functions, $g_i(x) = x^p$ with $p = 2$ and $g_d(x) = x^p$ with $p = 1/2$, improve the visual quality of the image apparently. Any potential gamut problems are handled by this method. The resultant image is displayed in Figure 3 (b), which is more vivid and makes the details more noticeable in the bright part.

The 3D color distribution of the resultant image is shown in Figure 3 (d), reflecting a better visual quality of the image.


***Figure 3**. (a) The original image with a low visual quality of a farm field with a driveway. (b) The resultant image with a better visual quality using the proposed method under the color model HDI. (c) The 3D color distribution of the original image. (d) The 3D color distribution of the resultant image.*

Figure 4 (a) is an image of a path in the woods, and the image is dark and dull in color, which is reflected by its 3D color distribution shown in Figure 4 (c). We also use two simple power functions, $g_i(x) = x^p$ and $g_d(x) = x^p$, both with $p = 1/2$, to demonstrate that the commonly used technique of distribution rebalancing can be freely used on the I and the D components of the color model HDI, with the gamut problem well handled.


***Figure 4**. (a) The original image with a low visual quality of a path in the woods. (b) The resultant image with the visual quality improved with the proposed method under the color model HDI. (c) The 3D color distribution of the original image. (d) The 3D color distribution of the resultant image.*

The resultant image is displayed in Figure 4 (c). The new image has a stronger overall vividness and a brighter appearance, giving a better visual quality, which is reflected by its 3D color distribution shown in Figure 4 (d).

## Conclusions

This paper introduces a technique of color enhancement in a true cylindrical color model HDI that avoids the gamut problem. This cylindrical model is established directly on the RGB color cube without geometric deformation. The space of the model is a cylinder circumscribing the color cube. The D component of this model is the radius of the cylinder, which is directly related to the color vividness. Since D is an undistorted component, processing through the D channel can consistently adjust the overall vividness of the image, achieving results that are difficult to obtain through processing on the saturation component on other models such as HSV, HSL, and HSI. Conventional processing techniques for adjusting the distribution on the achromatic component can be applied to the I component of HDI. Similar processing techniques can also be used on the D component. Since the space of the HDI model is larger than the RGB color cube, the processing on the I and D components may cause the gamut problem. The method to deal with the gamut problem in this paper is to perform subsequent processing after the processing of each component and compress the out stretching colors back into the cube. For this purpose, a cubic function is used to smoothly compress the out stretching colors back into the cube, and only affects a small part of the colors close to the out stretching colors while keeping most of the colors unaffected. Combining the processing on the two components yields a satisfactory overall result.

## References


[1] T. Chen, “Similarity between two color areas”, IS&T International Symposium on Electronic Imaging 2022, Color Imaging XXVII: Displaying, Processing, Hardcopy, and Applications, paper 364, 2022.

[2] R. C. Gonzales and R. E. Woods, Chapter 6, Digital Image Processing, 3rd ed., Prentice Hall, Upper Saddle River, New Jersey, 2008.

[3] G. H. Joblove and D. Greenberg, “Color spaces for computer graphics,” Computer Graphics, vol. 12, no. 3, pp. 20-25, 1978.

[4] S. K. Naik and C. A. Murthy,” Hue-Preserving Color Image Enhancement Without Gamut Problem,” IEEE Transactions on Image Processing, vol. 12, no. 12, pp. 1591-1598, 2003.

[5] A. R. Smith, “Color gamut transform pairs,” Computer Graphics, vol. 12, no. 3, pp. 12-19, 1978.

[6] S. Yang and B. Lee, “Hue-preserving gamut mapping with high saturation,” Electronics Letters, vol. 49, no. 19, pp. 1221–1222, 2013.

[7] H. Yu, K. Inoue, K. Har, and K. Urahama, “Saturation improvement in hue-preserving color image enhancement without gamut problem,” The Korean Institute of Communications and Information Sciences (KICS), ICT Express 4, pp. 134–137, 2018.


## Author Biography


*Tieling Chen is a Professor at University of South Carolina Aiken, USA. He received his PhD in Mathematics (2001) and his M.S. in Computer Science (2002) from University of Western Ontario, Canada. His research interests include Image processing and Color models and their applications.*

*Onan Chew is a senior undergraduate student in the Department of Computer Science, Engineering and Mathematics of University of South Carolina Aiken, USA. His major is in Applied Computer Science with a concentration on Gaming and Simulation.*