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Ultrasound Elasticity reconstruction with inaccurate forward model using integrated data-driven correction of data-fidelity gradient

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Abstract

Ultrasound elasticity images which provide quantitative visualization of tissue stiffness are reconstructed based on solving an inverse problem. Classical model-based methods are usually formulated in terms of constrained optimization problems composed of a data-fidelity term and a regularization term. The data-fidelity term incorporates the physical forward model which arises from the governing equilibrium equation discretized by finite element methods (FEMs). In elastography, the physical forward model is directly governed by the measured displacement image which leads to an inaccurate forward model in the presence of an intermediate level of noise. To tackle this issue, in the first step, we utilize a statistical representation of the physical forward model which incorporates the noise statistics with a signal-dependent correlated noise model. Next, for compensating the inaccurate forward operator error, we introduce an explicit data-driven approach for correcting the data-fidelity gradient, which can be integrated with any regularization term. The constrained optimization problem is solved using the fixed-point gradient descent where the analytical gradient of the data-fidelity term is corrected using the nonlinear mapping of a deep neural network (DNN). Finally, the proposed approach is integrated with a data-driven regularizer based on REgularization by Denoising (RED) for incorporating the prior information about the underlying elasticity patterns. Our simulation and experimental results demonstrate the improved performance of the proposed approach in various scenarios.

1. Introduction

Ultrasound elasticity imaging has achieved popularity by generating quantitative images of tissue stiffness as the most prominent indicator for characterizing bio-mechanical tissue properties [1]. In quasi-static ultrasound elastography (USE) problems [2], the general principle is, first, perturbing the tissue; second, measuring the internal tissue displacement; and finally, inferring the tissue bio-mechanical properties based on measured mechanical responses. In more detail, quasi-static loading (about 1-2% of the quasi-static axial dimension) is applied on the exterior surface of the medium resulting in some small deformation fields inside the tissue. The second step in quasi-static USE requires estimating the axial component of the internal tissue deformation by speckle tracking method which computes the correlation of B-mode ultrasound images, captured before and after applying quasi-static force loading on the surface. Moreover, for improving the accuracy of the deformation image, accumulated, averaged, or compounded multiple small deformation fields can be used.

FEMs are typically used for discretizing the medium and describing the equilibrium equations as the spatial distribution of the underlying physical law [4]. In USE, the physical model represents the relationship between the spatial elasticity distribution, the displacement spatial distribution, and applied force. The underlying equilibrium equations for linear elastic tissues can be reduced to a global stiffness equation as the governing forward model; therefore, the elasticity image can be reconstructed by solving the inverse problem of the linear global stiffness equation. Many classical model-based methods for elasticity reconstruction rely on Gaussian-Newton approaches with unstable and inaccurate performance in presence of noisy displacement measurements. Moreover, the forward operator is directly governed by the noisy displacement measurement resulting in an inaccurate forward operator which has to be taken into account to prevent degradation in the reconstruction quality when solving the ill-posed inverse problem.

In this regard, for obtaining a more accurate elasticity image, it is essential to compensate for the error due to noisy measurement and the resulting imperfect forward operator. In the first step for improving the performance of the existing model-based methods for ultrasound elastography using noisy measurements, we introduce a statistical representation of the forward problem by combining the elasticity forward model and displacement realization model, which leads to a signal-dependent colored noise model [5]. To address the inaccurate forward operator problem, some authors [6] use an application-specific model-based method based on explicitly correcting the gradient, while [7] suggests using the data-driven approaches to correct the approximate operator using a neural network. The major drawback of such data-driven techniques when solving an inverse problem is that correcting the forward operator using a parameterized model in the measurement manifold is not sufficient for generating gradients close to the gradients that would be produced by the exact forward operator [8]. With a different perspective, [9] suggests correcting the gradient implicitly in terms of a learning-based unfolding approach. Further, [8] performs correcting the forward and adjoint operators using two explicit networks. In this work, we aim to perform explicit correction of the data-fidelity gradient in the image domain using a neural network that can be integrated with

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data-driven regularizers such as RED. In this case, if the gradient correction network trains sufficiently, convergence to the neighborhood of the true solution can be guaranteed during solving the inverse problem.

The rest of this paper is organized as follows. We overview the classical approaches for solving the inverse problem for elasticity reconstruction in Section 2. The statistical approach for improving the elasticity reconstruction is presented in Section 3. The learning-based method for correcting the data-fidelity gradient is elaborated in Section 4. The simulation and experimental results for evaluating the elasticity reconstruction performance are presented in Section 5, and finally, concluding remarks are outlined in Section 6.

2. Classical approaches for elasticity reconstruction

As depicted in Fig. 1, elasticity distribution of the tissue cross-section, discretized over the nodes of a mesh, is known for the forward elasticity problem. Using the known force as boundary conditions (BCs) and applying this information to the global stiffness equilibrium equation as the governing physical model, the lateral and axial displacement image can be obtained. This equation is a linear function of $\mathbf{u}$ and non-linear function of $\mathbf{E}$. For the inverse problem, the noisy version of the displacement

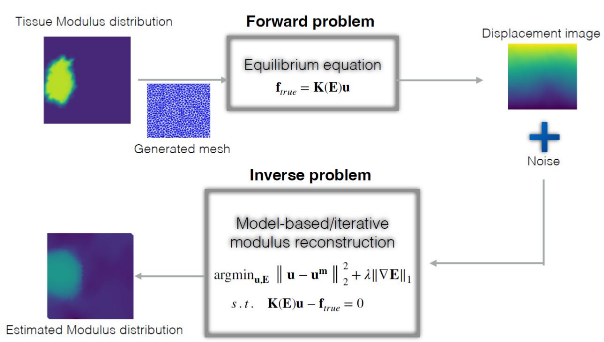

Figure 1. *Elasticity image reconstruction using a classical iterative approach.*

image $\mathbf{u^m}$ is available and the main goal is to estimate the elasticity modulus distribution $\mathbf{E}$. Such existing methods are based on iterative reconstruction of the modulus, which try to estimate the clean displacement $\mathbf{u}$ as well as the latent elasticity image $\mathbf{E}$ by minimizing an objective function consisting of the displacement data-fidelity term and a total variation (TV) or ℓ_2 norm regularizer, subject to the constraint that these estimations satisfy the equilibrium equation [10]. The solution to this optimization problem [11] is obtained with an iterative scheme that requires the Jacobian computation of the global stiffness as a function of unknown elasticity parameters leading to significant computation time for a large number of nodes. Moreover, for updating the elasticity modulus, the Jacobian matrix is multiplied with the noisy displacement measurements; this causes instability and inaccuracy for elasticity reconstruction even in the presence of a small level of noise as illustrated in Fig. 1 in comparison with the ground-truth elasticity modulus image.

3. Statistical Formulation for Elasticity Reconstruction

In the first step for addressing some of the weaknesses of classical methods for elasticity imaging, we use a new statistical framework to reconstruct the elasticity modulus by solving a regularized optimization problem. In this approach, a joint objective function is utilized by integrating the equilibrium equation as the forward model and the displacement observation model into the data-fidelity term. In practice, the equilibrium equation of elasticity imaging presented in Fig. 1 would be contaminated by some noise in the applied force which results in the following statistical representation of the forward model:

$$\mathbf{f} = \mathbf{D}(\mathbf{u})\mathbf{E} + \mathbf{w} \qquad \mathbf{w} \sim \mathcal{N}(0, \Sigma_w) \tag{1}$$

where $\mathbf{f}$ represents the applied force BCs and $\mathbf{w} \in R^{2N\times 1}$ (N is the number of nodes) denotes the axial and lateral Gaussian noise which implies an imperfection in the force measurements. Furthermore, we utilize the displacement realization model as $\mathbf{u^m} = \mathbf{u} + \mathbf{n}$ where $\mathbf{n} \sim \mathcal{N}(0, \Sigma_n)$, $\mathbf{u}$ is the clean displacement measurement and $\mathbf{u^m}$ is the corrupted displacement measurement with noise $\mathbf{n} \in R^{2N\times 1}$ with covariance Σ_n. Combining the statistical forward model in (1) with the displacement realization model yields to:

$$\begin{aligned} \mathbf{f} &= \mathbf{K}(\mathbf{E})\mathbf{u} + \mathbf{w} = \mathbf{K}(\mathbf{E})(\mathbf{u^m} - \mathbf{n}) + \mathbf{w} \\ &= \mathbf{K}(\mathbf{E})\mathbf{u^m} - \mathbf{K}(\mathbf{E})\mathbf{n} + \mathbf{w} \end{aligned} \tag{2}$$

Defining $\tilde{\mathbf{w}} = -\mathbf{K}(\mathbf{E})\mathbf{n} + \mathbf{w}$ and utilizing $\mathbf{D}(\mathbf{u^m})\mathbf{E} = \mathbf{K}(\mathbf{E})\mathbf{u^m}$ and employing these in (1) leads to the following unified statistical forward model:

$$\mathbf{f} = \mathbf{D}(\mathbf{u^m})\mathbf{E} + \tilde{\mathbf{w}} \qquad \tilde{\mathbf{w}} \sim \mathcal{N}(0, \Gamma) \tag{3}$$

where Γ is defined as:

$$\Gamma = \Sigma_w + \mathbf{K}(\mathbf{E})\Sigma_n\mathbf{K}(\mathbf{E})^T \tag{4}$$

This joint forward model introduced in (3) describes the underlying noise with a signal-dependent colored noise model. For reconstructing the elasticity image $\mathbf{E}$, it is required to solve the inverse problem using $\mathbf{f}$ and $\mathbf{u^m}$ measurements. To this end, we use the following regularized objective function to estimate of the elasticity image:

$$\begin{aligned} \hat{\mathbf{E}} = \text{argmin}_{\mathbf{E}} \quad & \tfrac{1}{2}\|\mathbf{f} - \mathbf{D}(\mathbf{u^m})\mathbf{E}\|^2_{\Gamma^{-1}} + \tfrac{N}{2}\log|\Gamma| + \lambda R(\mathbf{E}) \\ s.t. \quad & \mathbf{E} > 0 \end{aligned} \tag{5}$$

where $\|\mathbf{A}\|^2_{\mathbf{B}} := (\mathbf{A}^T\mathbf{B}\mathbf{A})$ and λ is the regularization parameter. The first term in (5) ($g(\mathbf{E}) = \frac{1}{2}\|\mathbf{f} - \mathbf{D}(\mathbf{u^m})\mathbf{E}\|$) denotes the data-fidelity term, $R(\mathbf{E})$ describes the regularization term, and $\mathbf{E} > 0$ is the positivity constraint. For solving (5), we utilize the fixed-point gradient descent method [12] by fixing Γ while estimating $\mathbf{E}$, and plugging the estimated $\mathbf{E}$ into (4) to update Γ. For estimating $\mathbf{E}$ in each iterate of the fixed-point method, we use the gradient descent update rule as follows:

$$\mathbf{E}_{n+1} = [\mathbf{E}_n - \gamma(\nabla g(\mathbf{E}_n) + \nabla R(\mathbf{E}_n))]_+ \tag{6}$$

where $[]_+$ indicates the positivity constraint on the estimated elasticity modulus and γ is the step size. It is worth mentioning that, the physical forward operator $\mathbf{D}(\mathbf{u^m})$ for elasticity imaging problem is directly governed by the measured displacement images. In scenarios with an intermediate level of noise, leading to low signal to noise ratio (SNR), we denote the noisy displacement images as $\mathbf{u^n}$ (compared to the less noisy displacement image $\mathbf{u^m}$) which leads to inaccurate forward model $\mathbf{D}(\mathbf{u^n})$. For such low SNR scenarios, the introduced statistical formulation performs poorly for elasticity reconstruction. Thus, it is essential to compensate for the error induced by such an imperfect forward operator for reconstructing more accurate stiffness images. The good news is that we can benefit from separate updates of the data-fidelity term and regularization term; hence, we will be able to compensate for the error of data-fidelity gradient and integrate it with any regularization term. In this regard, we propose a learning-based gradient correction method for elasticity imaging in the next section.

4. Learning-based Gradient Correction Method:

In this part, we introduce a solution for elasticity reconstruction $\mathbf{E}$ with the imperfect forward operator $\mathbf{D}(\mathbf{u^n})$ (due to intermediate level of noise in measured displacement image $\mathbf{u^n}$). In this regard, we acquire two observations of the displacement fields in terms of $\mathbf{u^m}$ with a low level of noise (high SNR) and $\mathbf{u^n}$ with an intermediate level of noise (low SNR), to compute the accurate forward operator and inaccurate forward operator respectively. As mentioned earlier, the data-fidelity term $g(\mathbf{E})$ of the optimization problem using inaccurate forward operator $\mathbf{D}(\mathbf{u^n})$ can be computed as:

$$g(\mathbf{E}) = \frac{1}{2} \| \mathbf{f} - \mathbf{D}(\mathbf{u^n})\mathbf{E} \|^2_{\Gamma^{-1}} \quad (7)$$

$$= \frac{1}{2}(\mathbf{f} - \mathbf{D}(\mathbf{u^n})\mathbf{E})^T \Gamma^{-1} (\mathbf{f} - \mathbf{D}(\mathbf{u^n})\mathbf{E}) \quad (8)$$

and the corresponding gradient of the data-fidelity term would be:

$$\nabla g(\mathbf{E}) = (\mathbf{D}(\mathbf{u^n}))^T \Gamma^{-1} (\mathbf{f} - \mathbf{D}(\mathbf{u^n})\mathbf{E}) = (\mathbf{D}(\mathbf{u^n}))^{\mathbf{T}} \tilde{\mathbf{r}} \quad (9)$$

where $\tilde{\mathbf{r}}$ is the weighted residual term. It is worth noting that although it might seem enough to correct the inaccurate forward operator $\mathbf{D}(\mathbf{u^n})$ by a trained network in the displacement data manifold using $\mathbf{u^m}$ and $\mathbf{u^n}$, that would not guarantee that the modified forward operator leads to accurate elasticity reconstruction since the gradient computation is directly governed by the adjoint of the forward operator $(\mathbf{D}(\mathbf{u^n}))^{\mathbf{T}}$. In this regard, we use a neural network G_θ as a non-linear mapping operator to adjust the gradient operator rather than explicitly correcting the forward operator. The schematic of the training procedure for correction of the data-fidelity gradient is illustrated in Fig. 2 which can be described as: we have two different acquisitions of displacement where $\mathbf{u^n}$ is noisier compared to $\mathbf{u^m}$. We compute the forward operator using high SNR acquired displacement image $\mathbf{u^m}$ leading to a more accurate forward operator $\mathbf{D}(\mathbf{u^m})$ and compute the forward operator using low SNR displacement image $\mathbf{u^n}$ which generates an inaccurate forward operator $\mathbf{D}(\mathbf{u^n})$. We plug the forward operators and force measurements to the data-fidelity term, compute the gradients and train the network G_θ to learn the mapping from the inaccurate gradient to the accurate gradient. We also use the back-projection estimate of elasticity as the initial elasticity $\mathbf{E}$. Regarding the network loss function, we use gradient consistency of the data-fidelity terms based on (9) as the penalty term during the training as follows:

$$Loss = \sum_i \| G_\theta \left((\mathbf{D}(\mathbf{u^n}))^{\mathbf{T}} \tilde{\mathbf{r}} \right) - (\mathbf{D}(\mathbf{u^m}))^{\mathbf{T}} \mathbf{r} \| \quad (10)$$

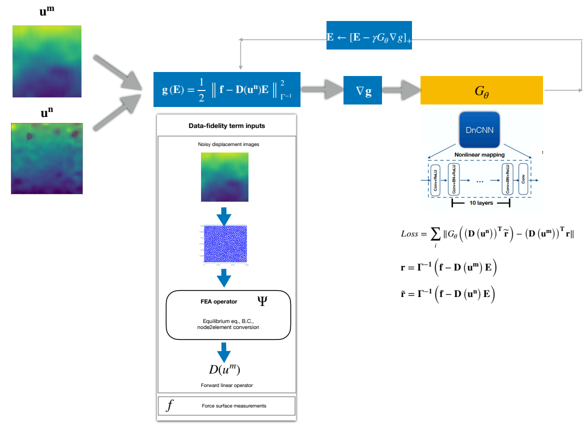

Figure 2. *Training procedure for data-fidelity gradient correction.*

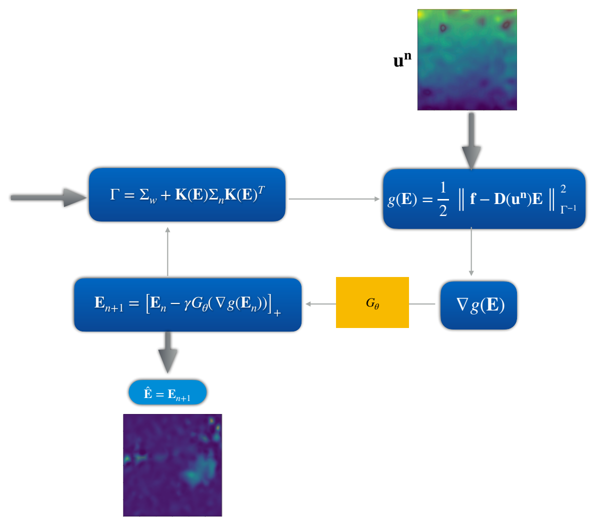

Figure 3. *Elasticity reconstruction using data-driven correction of data-fidelity gradient without any regularizer.*

$$\mathbf{r} = \Gamma^{-1} (\mathbf{f} - \mathbf{D}(\mathbf{u^m})\mathbf{E}) \quad (11)$$

$$\tilde{\mathbf{r}} = \Gamma^{-1} (\mathbf{f} - \mathbf{D}(\mathbf{u^n})\mathbf{E}) \quad (12)$$

After training G_θ during a number of epochs, we update the elasticity image $\mathbf{E}$ for computing the next gradient and we perform the training recursively.

After training the gradient correction network G_θ, in the inference phase, we use the iterative scheme of gradient descent without any regularization (explicit data-driven regularizer will be integrated later in this section). We use the low SNR displacement image $\mathbf{u^n}$ for calculating the inaccurate forward operator $\mathbf{D}(\mathbf{u^n})$, we compute the data-fidelity gradient based on this forward operator and then correct the gradient using the trained network G_θ. Next, the estimate of the elasticity image is updated and the new elasticity estimate is plugged for covariance matrix computation, and this scheme continues iteratively to update the final elasticity reconstruction $\hat{\mathbf{E}}$. This procedure is depicted in Fig. 3.

In the final reconstructed elasticity image in Fig. 3, there could still be some artifacts and it is required to improve the elasticity reconstruction. Therefore, we aim to integrate a learning-based regularizer based on the RED paradigm [13] to benefit from some

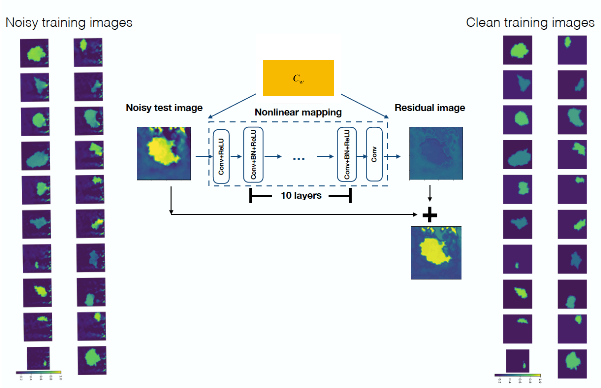

Figure 4. *Denoiser training procedure.*

prior information about the underlying elasticity structure of the tissue. In the RED approach, this data-driven prior information is learned using a DNN denoiser C_w and the residual of such denoiser is plugged into the optimization task as the gradient of the regularizer. The denoiser training procedure is depicted in Fig. 4. We utilize residual learning for capturing more details and high-frequency prior information about the underlying elasticity pattern. The network is trained using clean and poor unregularized elasticity images with a pixel-wise loss function.

In Fig. 5, the overall reconstruction pipeline is depicted based on the gradient descent scheme which requires the data-fidelity gradient denoted as ∇g and regularizer gradient as ∇R. In each iteration, the data-fidelity gradient is adjusted using G_θ and the regularizer gradient is replaced by the residual of learned denoiser C_w.

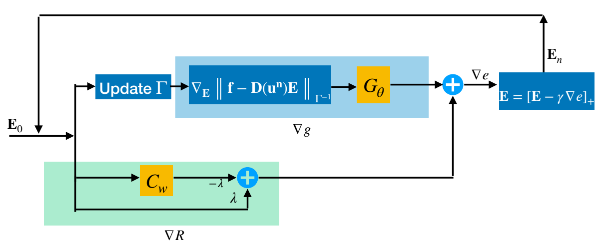

Figure 5. *Elasticity image reconstruction pipeline using learned gradient correction method and RED regularizer.*

5. Simulations and experimental results

To validate the performance of the proposed approach, we attempt to reconstruct the latent elasticity image $\mathbf{E}$, using low SNR measured displacement $\mathbf{u^n}$ and applied force $\mathbf{f}$ as Neumann BCs. The gradient correction network G_θ needs to be trained using two different acquisitions of the displacement for each phantom. To this end, we generate a dataset of 653 simulated phantoms where for each phantom, the ground truth elasticity image $\mathbf{E}$ and two acquisitions of the displacement images are produced. For generating the dataset of simulated phantoms, some mask B-mode images [14] are used as the phantom cross-section images. In particular, each mask B-mode image contains a lesion with an irregular shape embedded in the background tissue and random scalar values are assigned to the lesion and background elasticity modulus resulting in a synthetic map of the ground truth elasticity image. Moreover, the displacement images of each phantom are acquired by solving the forward problem $\mathbf{K}(\mathbf{E})\mathbf{u} - \mathbf{f} = 0$ and adding Gaussian noise with $SNR = 26dB$ for $\mathbf{u^n}$ and $SNR = 35dB$ for $\mathbf{u^m}$. We train the gradient correction network G_θ with the DnCNN architecture with 10 layers by feeding $\mathbf{u^n}$ images as the inputs and $\mathbf{u^m}$ images as the target images using the aforementioned gradient consistency loss function. Then, we solve the unregularized optimization problem for the low SNR displacement image $\mathbf{u^n}$ utilizing the trained G_θ for adjusting the gradients and reconstructing the elasticity images. Although the elasticity reconstruction performance is improved compared to not employing the gradient correction approach, we aim to reduce the remaining artifacts by training a denoiser for learning the underlying prior elasticity patterns. In this regard, we train the denoiser network $\mathbf{C}_w$ with 10 layers of DnCNN by feeding the reconstructed elasticity images from the previous step as the input and the ground truth elasticity images as the target images. Finally, we solve the optimization problem for the test images where the data-fidelity gradient is corrected by the learned gradient correction network G_θ and the regularizer gradient is replaced by the learned denoiser $\mathbf{C}_w$ residual. Figs. 6-8 demonstrate the reconstruction performances for two simulated test phantoms and one experimental phantom based on real measurements. In the first and second rows of Figs. 6-8, we use high SNR displacement image $\mathbf{u^m}$, where the results of the conventional method and our RED-based approach are illustrated without the need for operator correction. In the third row of these Figs., we use the low-SNR displacement image $\mathbf{u^n}$ where methods without gradient correction are not able to generate acceptable results while our proposed approach involving RED and operator correction indicates significant reconstruction improvement.

6. Conclusion

In this article, we proposed an explicit data-driven method for correcting the gradient when solving the inverse problem for ultrasound elastography. In this application, the forward operator is directly governed by the noisy measured displacement image which leads to an imperfect forward operator in scenarios with low SNR. The introduced statistical method is able to compensate for the error where a small amount of noise is present in the displacement image. To improve the potential of this method for compensating the error in low SNR scenarios with the consequent inaccurate forward operator, we used a data-driven network for adjusting the data-fidelity gradient. Moreover, this approach is able to be integrated with any regularization term where we used the RED paradigm for replacing the regularizer gradient with the learned denoiser residual. The simulation and experimental results verify the effectiveness and robustness of the proposed method.

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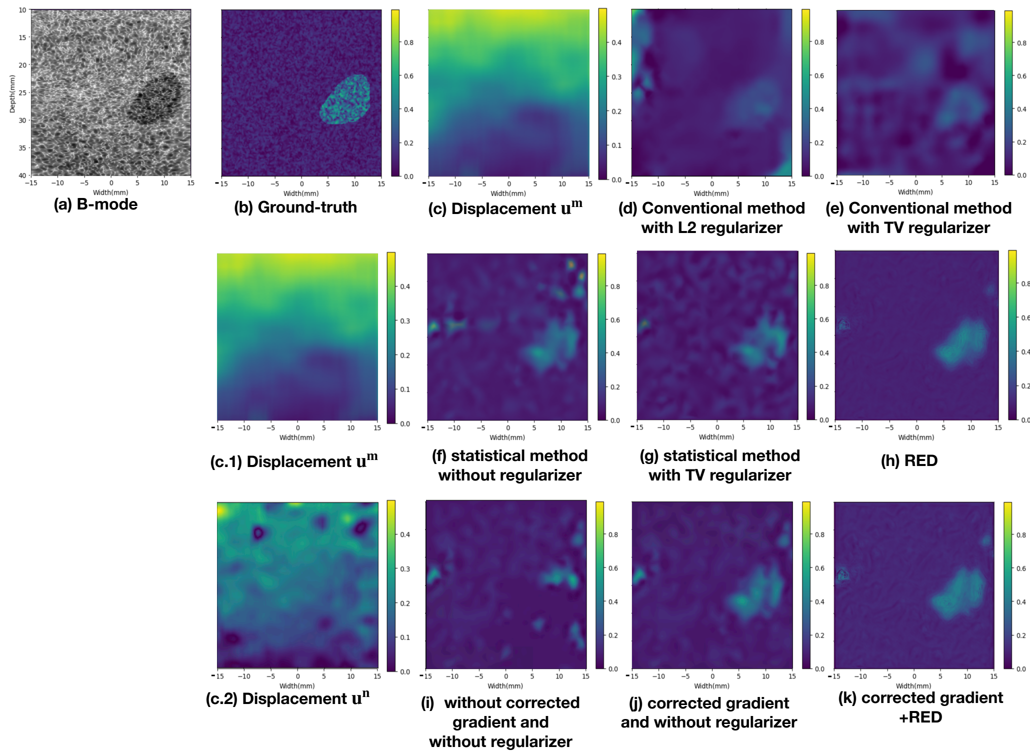

Figure 6. *Simulated test phantom. First and second row reconstructions are using the displacement image* $\mathbf{u^m}$ *and third row results are using the displacement image* $\mathbf{u^n}$. *The unit of the color bar for the elasticity image is 100 KPa.*

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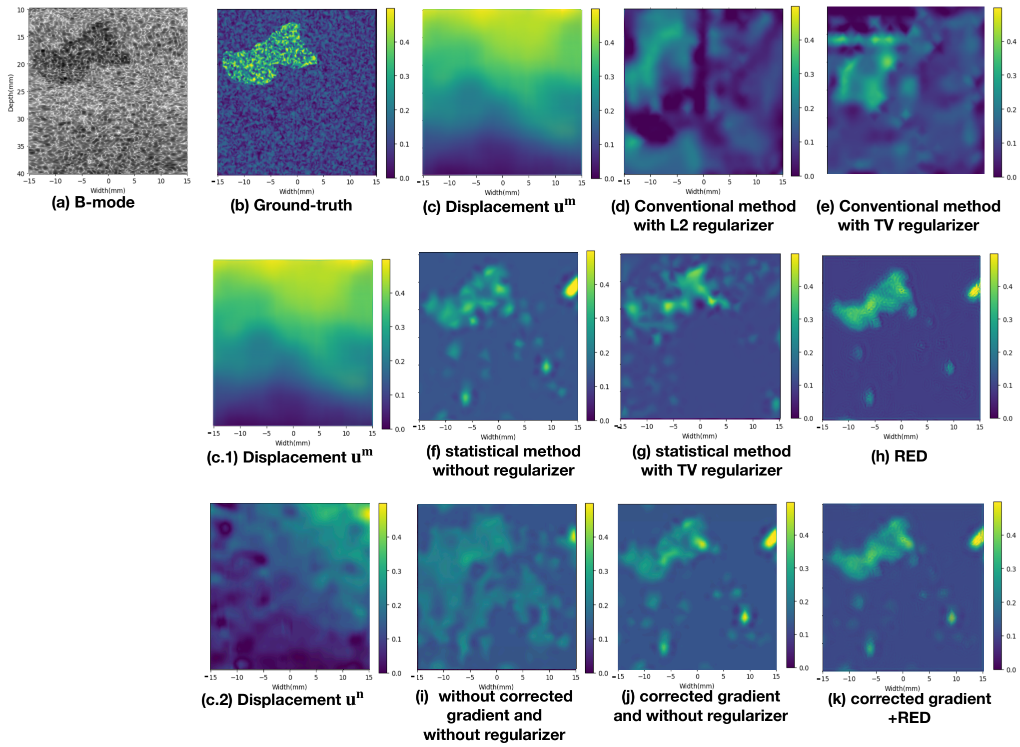

Figure 7. *Simulated test phantom. First and second row reconstructions are using the displacement image* $\mathbf{u}^{\mathbf{m}}$ *and third row results are using the displacement image* $\mathbf{u}^{\mathbf{n}}$*. The unit of the color bar for the elasticity image is 100 KPa.*

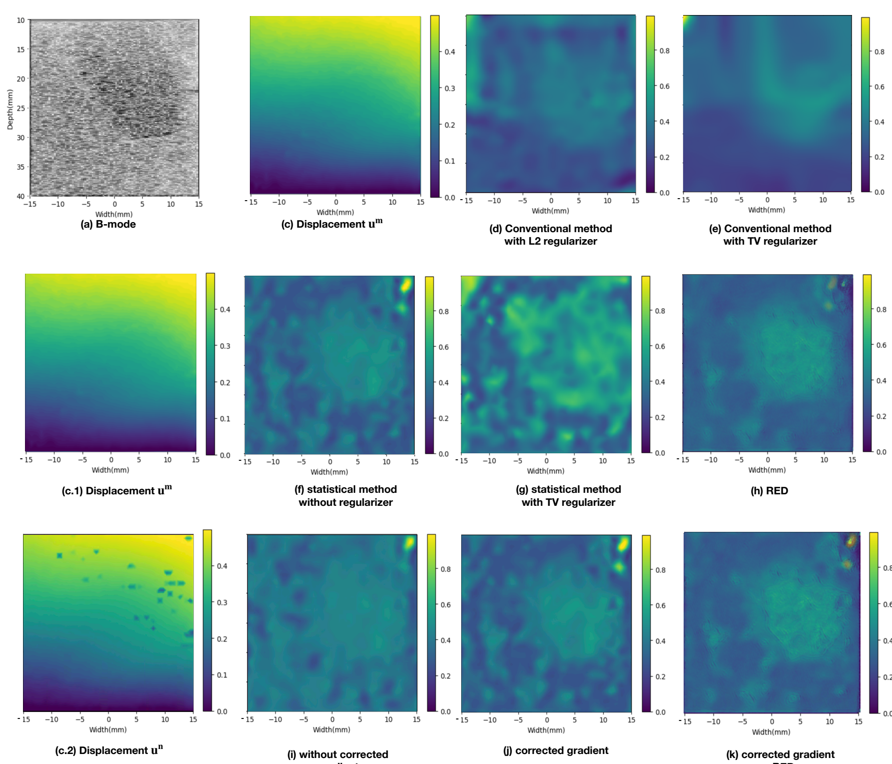

Figure 8. *Experimental test phantom. First and second row reconstructions are using displacement image* $\mathbf{u}^{\mathbf{m}}$ *and third row results are using the displacement image* $\mathbf{u}^{\mathbf{n}}$*. The unit of the color bar for the elasticity image is 100 KPa.*