

# Similarity between two color areas

Tieling Chen, University of South Carolina Aiken, Aiken, SC, USA

## Abstract

*The paper introduces a method to compare two areas of colors, with each area presenting a perceptual single major color such as on a piece of cloth or a piece of tile. Each color area contains a cloud of colors in similar hues, and all colors present a scattered distribution in the RGB color cube. Existing color difference formulas comparing two single colors do not work well in this case. The new method presented in the paper includes two aspects, a new color model that better describes color distributions and the technique of comparing two areas of colors by their color distributions under the new color model. The new model uses cylindrical coordinates to describe the color features, and an area of colors shows a distribution pattern on each color feature. For two areas of colors that are perceptually similar, their distributions on each component of the new color model are compared for similarity, and a combination of these sub-similarities gives an overall similarity between the two areas of colors. The proposed method can be applied to many industrial processes where color similarity comparison is a main concern, such as color fastness test of fabrics and tile classification by color.*

## Introduction

The comparison between two single colors is determined by the positions of the two colors in some color model [5][6]. In the RGB color model the position of a color in the color cube represents its color, so a color difference formula consists of the differences in the red, green, and blue components [1]. The CIELab color mode is commonly used to measure color differences with formulas working on the difference in each component of the model between two color points [8]. Color difference depends on the subjective perception of the human visual system, and those difference formulas just take two color points as input. In real applications, color difference comparison is much more complicated. For example, color fastness tests of textiles require judgments on how fast the colors of textiles are after a certain treatment process to determine the quality of dyeing. This color comparison is not a simple comparison of two single colors. For a piece of cloth that looks like in one color in the visual sense, its digital image actually contains a cloud of colors with similar chromaticity. Each pixel in the color area of the digital image shows a color, and the colors of all the pixels are not the same. Although a color area appears in a perceptual single major color, it presents a scattered distribution of color points in the RGB color cube. If the distribution of the color clouds is concentrated, the color area may be represented by a single color that is the average of the color cloud. However, if the color distribution is relatively wide, a color average should not be a good representative of the color characteristics of the entire color area, and therefore the color difference formulas for two single colors would not work properly to compare two color areas in this case.

In many real applications, determining that how perceptually similar two color areas are to each other is more meaningful than how different they are. Many applications in the industry need to

measure the similarity between two color distributions but not two single colors. For example, the washing process control of denim clothing. Denim clothing was originally made of fabrics with a deep color, and then is washed and polished by machine to achieve a predetermined color. The comparisons in many factories are mainly conducted by experienced technicians, so the results are subjective. In digital image, the color of the washed denim clothing is in a relatively scattered distribution, although it presents a single major color to the visual system. The scattered color distribution makes it difficult to apply some difference formula for single colors to measure the similarity using computer. Similar examples include the color comparison of tiles. The same batch of tiles are not always in the same major color. It is necessary to compare their colors with the sample color, to classify the tiles according to the perceptual difference in color. The comparisons are also conducted by human beings in many factories because it is difficult to apply difference formulas for single colors since the colors of the digital image of a tile are in a scattered distribution.

The motivation of this paper is to find a way to compare the similarity between two color areas in digital images under a proper color model. The general idea is to compare two 3D color distributions through the comparisons of distributions in the three components of the color model. We need a color model that contains a hue component to express the color information. The color model is based on the RGB color model for the following reasons: because the location of a color point in the RGB color cube is identical to its color, mathematical expressions are more intuitive; the model is extensively used in commercial display devices; and there are industry-recognized color definitions for RGB models to maintain color consistency.

However, the commonly used RGB based color models bearing a hue component such as HSV [7] and HSL [4] are not optimal candidates because the measures in both saturation components and the V and L components are geometrically inconsistent. The paper proposes a cylindrical color model that better catches the distribution patterns of a cloud of colors with similar hues. In each component of the new color model, the distribution of the corresponding feature of the color cloud is computed. When comparing the similarity between two color areas with a perceptual single major color for each area, their distributions in each component of the color model are compared first, and then a formula consisting of these comparisons is used to give an overall measure of the similarity.

The purpose of this paper is not to establish an absolute formula to compare two color areas, but to raise such a problem and try to find a solution. The method in this paper can be implemented in other suitable color models.

## The difficulties on comparing two color areas

Many objects such as cloth pieces and tiles present a perceptual single major color to the visual system. Their digital images present

single major colors too. In a digital image of a color area, the single major color perceived by the visual system does not reflect all the colors of the area. It is more like some color made by the visual system to try to capture the main feature of the color area. Each color area presents a scattered distribution in the space of its color model. For example, in figure 1, there are three computer program generated color areas in the top row, and each one presents a perceptual single major color. They all started with a same single color in the RGB color model but are randomly processed with normal distributions with different deviations. The perceptual major color of each area shares the same hue information, but the three areas are apparently different from each other. The left area appears in a single color but actually its 3D distribution in the RGB color model is a small cloud of colors, shown in the left image at the bottom row of figure 1. The middle area has a more scattered 3D distribution, and the right area has an even more scattered distribution, shown in the middle column and the right column of figure 1, respectively.

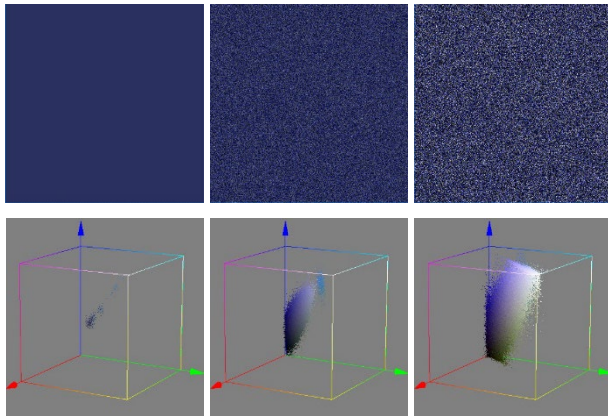


Figure 1. Top row displays three computer program generated color areas. Bottom row displays the corresponding 3D color distributions in the RGB color model.

The major difficulty of comparing their similarities or difference is that no proper single color can be used to represent the color cloud of each area, especially for a color cloud that scatters widely. For the images in figure 1, comparing the average color of each color cloud does not work because the three perceptually distinct areas have the same average value. This implies the straightforward method of using difference formulas for comparing single colors to compare color areas does not work.

Other difficulties include the selection of a proper color model. The RGB color model is an improper one because it does not provide the information of chromaticity directly. A color model with a hue component would be a better choice. However, the commonly used color models containing a hue component such as HSV and HSL still do not provide satisfactory options because they use geometrically inconsistent measures to fit the shape of the color cube, which can be seen in figure 2 [2]. The top row displays the shapes of the components of the HSV color model, Hue, Saturation, and Brightness, respectively from left to right. The polar angles of the color points on the same hexagonal cone representing the Saturation are not the same. Also distances to the zero plane from the color points on the same shell representing the Brightness are not the same. The shapes of these two components are geometrically

altered so their measures are not consistent. The situation is similar in the HSL color model, shown in the bottom row of figure 2.

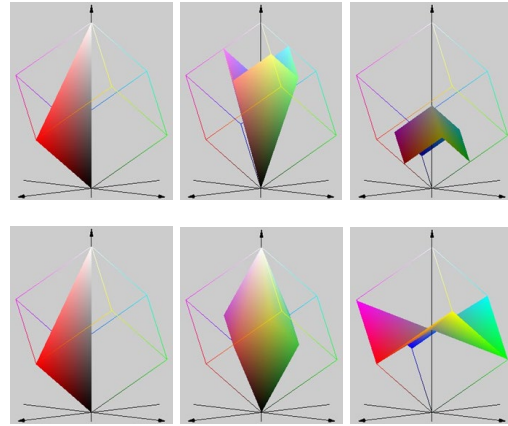


Figure 2. Top row displays the shapes of Hue, Saturation, and Brightness components of the color model HSV. Bottom row displays the shapes of Hue, Saturation, and Lightness in the color model HSL.

To capture the color characteristics, we need a color model with a hue component, but require that the remaining two components keep geometrically consistent. The following cylindrical color model satisfies the requirements.

## The cylindrical color model HDI

To establish the cylindrical color model, we first rotate the RGB color cube such that its grey diagonal is vertical. Then we impose the mathematical cylindrical model over the color cube such that the black color point sits on the zero plane and the grey diagonal is the center of the cylinders.

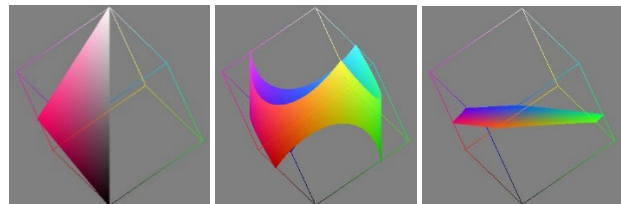


Figure 3. The geometrical shapes of the components of the color model HDI. Left. A triangle for a fixed  $H$ . Middle. A cylinder for a fixed  $D$ . Right. A plane for a fixed  $I$ .

The three components are shown in figure 3. The left image shows the shape of the Hue component. The triangle is the part of the half plane passing through the center line that is within the color cube, which is exactly the same as the Hue component in HSV and HSL. The domain of the  $H$  is  $[0, 2\pi)$ , and the hue plane with degree 0 passes through the red color.

The middle image of figure 3 displays a cylinder restricted inside the color cube. On the cylinder every color point has the same distance to the center grey diagonal. This component, denoted by  $D$ , replaces the Saturation components in HSV and HSL and keeps the measure geometrically consistent for all the color points on the same cylinder. The  $D$  component is closely related to the color vividness, and its domain is  $[0, \sqrt{6}/3]$ .

The right image of figure 3 displays a restricted plane perpendicular to the grey diagonal. The level of the plane is the distance from the zero plane. This component is conceptually similar to the Brightness in HSV and the Lightness in HSL but keeps a geometrically consistent measure. We call this component Intensity and use  $I$  to denote it. The  $I$  component is the same as the Intensity component in the theoretical color model HSI [3]. The domain of  $I$  is  $[0, \sqrt{3}]$ .

Another significant argument that the  $D$  component is more proper than the Saturation components in HSV and HSL comes from the fact that color distributions of many real objects such as cloth pieces and tiles display a pattern that the color cloud is oriented parallel to the grey diagonal, as demonstrated in the following figure 4 and figure 5.

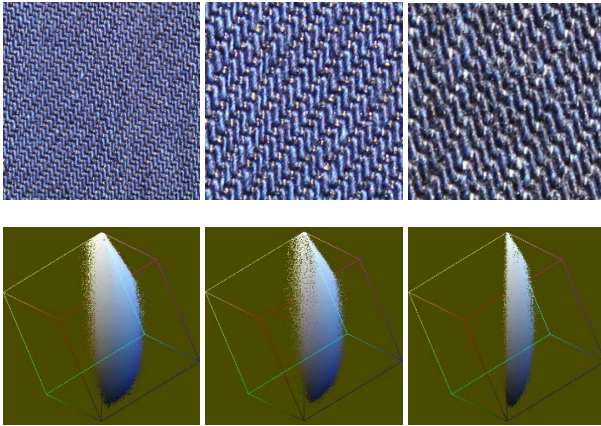


Figure 4. Top row displays three pieces of jean cloth. Bottom row displays the corresponding 3D color distributions in the color model HDI.

Figure 4 displays three pieces of jean cloth and their corresponding 3D color distributions in the HDI color model. The clouds of color points show an orientation parallel with the center grey diagonal of the color cube. The same phenomenon is also clearly shown in figure 5, displaying three tiles and their color distributions.

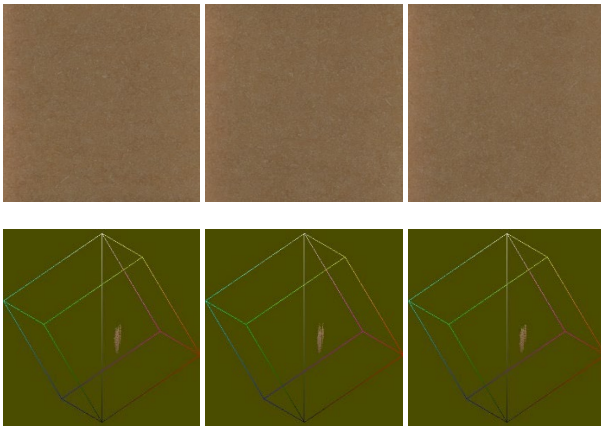


Figure 5. Top row displays three tiles. Bottom row displays the corresponding 3D color distributions in the color model HDI.

The conversions between the RGB color model and the HDI color model can be obtained with standard mathematical transformations.

Equation (1) gives the formula converting from RGB to HDI. The domains of the  $R$ ,  $G$ , and  $B$  components are  $[0, 1]$ . Equation (2) is the formula converting from HDI to RGB.

$$\begin{cases} H = \begin{cases} \arccos \frac{2R - G - B}{2\sqrt{R^2 + G^2 + B^2 - RG - RB - GB}}, & \text{when } G \geq B \\ 2\pi - \arccos \frac{2R - G - B}{2\sqrt{R^2 + G^2 + B^2 - RG - RB - GB}}, & \text{when } G < B \end{cases} \\ D = \frac{\sqrt{6}}{3} \sqrt{R^2 + G^2 + B^2 - RG - RB - GB} \\ I = \frac{\sqrt{3}}{3} (R + G + B) \end{cases} \quad (1)$$

$$\begin{cases} R = \frac{\sqrt{6}}{3} D \cos H + \frac{\sqrt{3}}{3} I \\ G = \frac{-\sqrt{6}}{6} D \cos H + \frac{\sqrt{2}}{2} D \sin H + \frac{\sqrt{3}}{3} I \\ B = \frac{-\sqrt{6}}{6} D \cos H - \frac{\sqrt{2}}{2} D \sin H + \frac{\sqrt{3}}{3} I \end{cases} \quad (2)$$

The conversion formulas provide convenience for displaying images with RGB and comparing color distributions with HDI. Notice that the cylinder  $[0, 2\pi) \times [0, \sqrt{6}/3] \times [0, \sqrt{3}]$  of the HDI space is bigger than the color cube. This may cause the gamut problem if colors are modified in the HDI space after they are converted from the RGB and then converted back to the RGB for display. However, the HDI model is only used for color distribution comparison, so the colors are not modified in the HDI space. During the comparison, the gamut error never occurs in the process.

Another thing to note is that RGB is a theoretical model, and the HDI model is related to it, so HDI is also a theoretical model and cannot be used to express absolute colors. This paper only proposes a theoretical method of color distribution comparison. In practical comparison, the data and conclusions all depend on the actual colors that the RGB model can express. The experimental data in this paper are obtained in the sRGB model.

## Color distribution comparison

To compare the distributions between two color areas, a reasonable approach is to compare their distributions in the three components  $H$ ,  $D$ , and  $I$ , and then combine the comparisons to get an overall measure.

Given the image of a color area with perceptual single major color, we use equation (1) to convert the RGB data to HDI, and then collect the data in the three components of  $H$ ,  $D$ , and  $I$ . For consistency, we normalize and discretize the domain of  $H$  to the interval  $[0, n)$  and the domains of  $H$  and  $I$  to the interval  $[0, n]$  with a preselected number  $n$ .

Specifically, for the  $H$  component, the original domain  $[0, 2\pi)$  is normalized and discretized to  $[0, n)$  with all integer values.

Similarly, the original domain  $[0, \sqrt{6}/3]$  of D and the original domain  $[0, \sqrt{3}]$  of I are normalized and discretized to  $[0, n]$  with all integer values.

We need obtain the histograms in H, D, and I first. Each one is stored in a buffer. For H, once a value is obtained by equation (1), find its corresponding nearest integer between 0 and  $n$ , then use this integer as the index to the buffer to increment its count. The buffer holds all the counts of the integers, and it gives a histogram for the H component. With similar procedures, we can get histograms for the D and I components.

Then normalize the histograms to obtain the corresponding normalized distributions. Still use H as an example. With its histogram buffer, by dividing the count of each integer by the total count we get the normalized distribution. We can save the distribution to another buffer and call it the distribution buffer. The sum of all the values, which are between 0 and 1 now, is 1. Obtain the distribution buffers for D and I in the same way. Each distribution buffer is a discrete function over the integers from 0 to  $n$ . The value  $n$  is excluded for H because both  $n$  and 0 are corresponding to the red color so the weights are all put on 0.

To compare two discrete distribution functions in the H component, we exploit the differences at all the integers between 0 and  $n - 1$ . Suppose  $Dist_{H1}$  and  $Dist_{H2}$  are two discrete distribution functions. Denote  $S(H)$  the similarity between the two functions. We use the following formula to find the similarity in H.

$$S(H) = 1 - \sum_{i=0}^{n-1} w_H(i) |Dist_{H1}(i) - Dist_{H2}(i)| \quad (3)$$

where  $w_H$  is a discrete weight function adjusted by actual applications. If the two distributions are the same, then  $S(H)$  is 1. The closer the two distributions are, the closer to 1  $S(H)$  is. If the two distributions are different, then  $S(H)$  is smaller than 1, the greater the difference the smaller the value.

The similarity formulas  $S(D)$  for D and  $S(I)$  are in a similar format. They are given by equation (4) and equation (5), respectively,

$$S(D) = 1 - \sum_{i=0}^n w_D(i) |Dist_{D1}(i) - Dist_{D2}(i)|, \quad (4)$$

$$S(I) = 1 - \sum_{i=0}^n w_I(i) |Dist_{I1}(i) - Dist_{I2}(i)|. \quad (5)$$

Here  $Dist_{D1}$  and  $Dist_{D2}$  are two distributions in the D component,  $Dist_{I1}$  and  $Dist_{I2}$  are two distributions in the I component,  $w_D$  and  $w_I$  are the weight functions depending on actual applications.

For the overall measurement, we consider the following formulas.

$$similarity = S(H)^\alpha S(D)^\beta S(I)^\gamma, \quad (6)$$

$$similarity = \alpha S(H) + \beta S(D) + \gamma S(I), \quad (7)$$

$$similarity = \min(S(H), S(D), S(I)). \quad (8)$$

where  $\alpha, \beta, \gamma$  are weights selected in actual applications, and they are selected in such a way that the value of *similarity* is between 0 and 1. The value 1 means the two color areas under comparison are identical. The closer the value is to 1 the more similar the two color areas are.

Note that these formulas for the overall similarity are introduced for demonstration only. Proper formulas could be obtained through psychophysical studies.

## Experimental results

The optimal weights in the comparison formulas should be obtained in real applications through experiments. To demonstrate the effectiveness of the comparison method we simply set

$$w_H(i) = 1/2, \quad w_D(i) = 1/2, \quad \text{and} \quad w_I(i) = 1/2,$$

in the equations (3), (4), and (5), because when two distributions are disjoint, the sum of their differences is 2. For the overall similarity formulas, we set  $\alpha = \beta = \gamma = 1$  in equation (6) and  $\alpha = \beta = \gamma = 1/3$  in equation (7) and call them *similarity by product* and *similarity by average*, respectively. Also, call the formula in equation (8) *similarity by minimum*.

Data are collected in sRGB color model and calculations are performed in the corresponding HDI color model. The results are affected by the value of  $n$ , but when  $n$  is larger enough the results become stable. The results obtained in the experiments are stable ones after big values of  $n$  are tested, and all results are rounded to the hundredth.

The above setting is used to compare the color areas in figure 4 first. The original image of the one on the left of the top row has a size of 1024 by 1024 pixels, and the original image of the middle one is a local area chopped from the original image of the left one, with a size of 512 by 512 pixels. An effective comparison should give a very high similarity to these two color areas.

The comparison in the H component gives a high value,

$$S(H) = 0.94,$$

which is demonstrated by the top chart of figure 6 because the two distributions have a very large overlap.

To clearly show how similar two distributions are, the two distributions are drawn together. In each chart of figure 6, the distribution in blue is for the middle image in figure 4, and the red one is for the left image in figure 4. For demonstration purposes, each line of the red distribution that is the background is drawn thicker, and the blue distribution that is drawn on the surface is thinner, so that the approximate shapes of both distributions can be seen. Also, for demonstration's sake, each chart of figure 6 is drawn when the value of  $n$  is set to 128, but the result of similarity in each component is obtained for large values of  $n$  when it becomes stable, in which case the blue distribution would be dense enough to cover the shape of the blue distribution.

The comparison in the D component also gives a high value,

$$S(D) = 0.94,$$

demonstrated by the middle image of figure 6. For the I component, a high value in similarity is also obtained,

$$S(I) = 0.97,$$

demonstrated by the bottom image in figure 6. The overall similarities, by three different formulas, are all in high values:

$$similarity \text{ by product: } 0.86,$$

$$similarity \text{ by average: } 0.95,$$

$$similarity \text{ by minimum: } 0.94.$$

With the current parameters setting and considering the two images under comparison are from the same cloth, it seems that similarity by average is the most optimal formula, while similarity by product is too sensitive and similarity by minimum might encapsulate the similarities of the other two components.

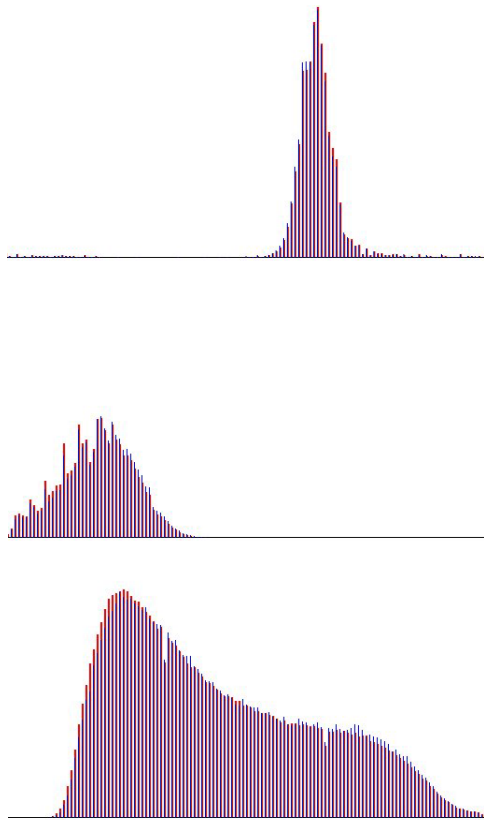


Figure 6. Distributions in  $H$ ,  $D$ , and  $I$  of two color areas. A distribution in black is for the middle image in the top row of figure 4, and a distribution in red is for the left image in the top row of figure 4. The top, middle, and bottom images display the distributions in  $H$ ,  $D$ , and  $I$  components, respectively.

The middle image and the right image in the top row of figure 4 present a less similarity in colors. The original image of the right one also has a size of 512 by 512 pixels. Using the same setting for the comparison formulas, we have the following comparison results, demonstrated in figure 7:

$$S(H) = 0.44, S(D) = 0.39, \text{ and } S(I) = 0.86.$$

Shown in figure 7, the distributions in blue for the middle image in figure 4 have apparent differences from the corresponding distributions in red for the right image in figure 4. The differences are properly caught by the comparison formulas. For the overall similarity, the value is given by different formula as below:

$$\begin{aligned} \text{similarity by product:} & \quad 0.15, \\ \text{similarity by average:} & \quad 0.56, \\ \text{similarity by minimum:} & \quad 0.39. \end{aligned}$$

These values are consistently smaller than the corresponding values for the comparison between the left and middle images in

figure 4. This means in figure 4, between the left image and the right image, the left image is closer to the middle image in color. It matches the visual system's perception of the color areas presented in the three images.

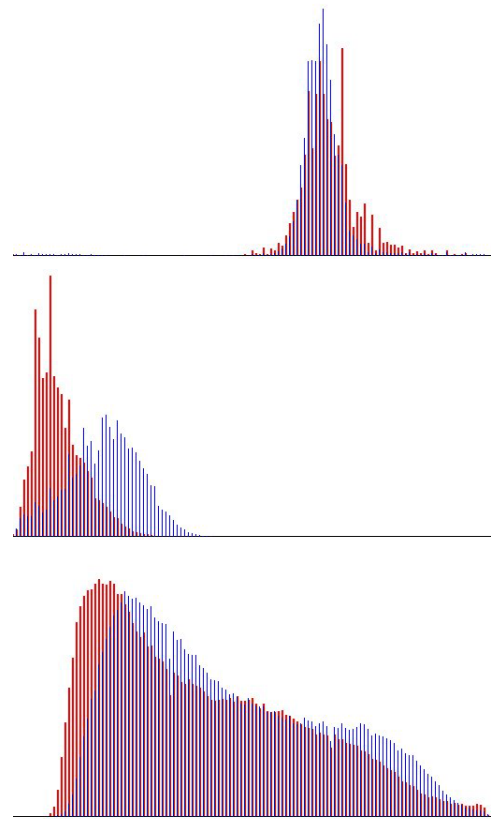


Figure 7. Distributions in  $H$ ,  $D$ , and  $I$  of two color areas. A distribution in black is for the middle image in the top row of figure 4, and a distribution in red is for the right image in the top row of figure 4. The top, middle, and bottom images display the distributions in  $H$ ,  $D$ , and  $I$  components, respectively.

The same setting of the formulas is then used to compare the similarities among the images in the top row of figure 5. Each original image is a local area of a different tile and has a size of 283 by 283 pixels. The subtle differences can be caught by the comparison method. The comparison results are summarized in the following.

Between the left image and the middle image,

$$\begin{aligned} S(H) &= 0.84, S(D) = 0.84, \text{ and } S(I) = 0.89. \\ \text{similarity by product:} & \quad 0.63, \\ \text{similarity by average:} & \quad 0.86, \\ \text{similarity by minimum:} & \quad 0.84. \end{aligned}$$

Between the right image and the middle image,

$$\begin{aligned} S(H) &= 0.92, S(D) = 0.92, \text{ and } S(I) = 0.96. \\ \text{similarity by product:} & \quad 0.81, \\ \text{similarity by average:} & \quad 0.93, \\ \text{similarity by minimum:} & \quad 0.92. \end{aligned}$$



Between the left image and the right image,

$$\begin{aligned} S(H) &= 0.79, S(D) = 0.79, \text{ and } S(I) = 0.88. \\ \text{similarity by product: } &0.54, \\ \text{similarity by average: } &0.82, \\ \text{similarity by minimum: } &0.79. \end{aligned}$$

These comparisons indicate the color areas of the middle and right images in figure 5 are the closest, which is hard to discern by naked eyes.

The computer program generated images shown in figure 1 are also compared. Each image has a size of 512 by 512 pixels. The comparison results are given below.

Between the left image and the middle image,

$$\begin{aligned} S(H) &= 0.05, S(D) = 0.05, \text{ and } S(I) = 0.05. \\ \text{similarity by product: } &0.00, \\ \text{similarity by average: } &0.05, \\ \text{similarity by minimum: } &0.05. \end{aligned}$$

Between the right image and the middle image,

$$\begin{aligned} S(H) &= 0.41, S(D) = 0.40, \text{ and } S(I) = 0.63. \\ \text{similarity by product: } &0.10, \\ \text{similarity by average: } &0.48, \\ \text{similarity by minimum: } &0.40. \end{aligned}$$

Between the left image and the right image,

$$\begin{aligned} S(H) &= 0.04, S(D) = 0.04, \text{ and } S(I) = 0.03. \\ \text{similarity by product: } &0.00, \\ \text{similarity by average: } &0.04, \\ \text{similarity by minimum: } &0.03. \end{aligned}$$

The experiments in this paper are mainly to demonstrate how to use the proposed method to make color comparison. In practical applications, the HDI model should be associated with a fixed color model, and the optimal parameters should be selected through many experiments to determine the comparison formula.

## Conclusions

The paper proposed a method to measure the perceptual similarity of two color areas, with each color area presenting a single major color to the eyes. The method contributes two novel ideas, one is a new color model HDI that better catches the characteristics of the distribution of a color area, and the other one is a technique to compare the color distributions of two color areas. The color model HDI is a cylindrical model with the grey diagonal of the RGB color cube as its center. The measure in each component of the model is geometrically consistent, and the D component, which is the distance of a color point to the grey diagonal, reflects well the orientation of the shape of the color cloud of a color area with a single major color. The technique of comparing two color areas under the color model HDI is to compare their color distributions in each component by normalizing and discretizing the distribution functions and then finding their weighted differences. The overall measure is a combination of the measures from each component. Three formulas for the overall measure are provided, and each one

works consistently. The proposed method provides reliable results of comparison and can find the subtle difference between two color areas that is hard to perceive with naked eyes. The method can be used in real applications where color comparison is a major concern.

## References

- [1] S. Abasi, M. A. Tehran, Mohammad, and M. D. Fairchild, "Distance metrics for very large color differences," *Color Research & Application*, vol. 45, no. 2, pp. 208-223, April 2020, doi:10.1002/col.22451.
- [2] T. Chen and J. Ma, "Comparison between HSV and HSL through tone mapping," *Proceedings of the International Conference on Computer Science & Communication Engineering*, Suzhou, China, June 2015.
- [3] R. C. Gonzales and R. E. Woods, Chapter 6, *Digital Image Processing*, 3rd ed., Prentice Hall, Upper Saddle River, New Jersey, 2008.
- [4] G. H. Joblove and D. Greenberg, "Color spaces for computer graphics," *Computer Graphics*, vol. 12, no. 3, pp. 20-25, 1978.
- [5] K. Plataniotis and A. N. Venetsanopoulos, *Color Image Processing and Applications (Digital Signal Processing)*, Springer, 2000.
- [6] S. K. Shevell, *The Science of Color*, 2nd ed. Elsevier Science & Technology, 2003.
- [7] A. R. Smith, "Color gamut transform pairs," *Computer Graphics*, vol. 12, no. 3, pp. 12-19, 1978.
- [8] D. A. Szafrir, "Modeling Color Difference for Visualization Design," in *IEEE Transactions on Visualization and Computer Graphics*, vol. 24, no. 1, pp. 392-401, Jan. 2018, doi: 10.1109/TVCG.2017.2744359.

## Acknowledgement

*This work was supported by the 2021 RISE grant of University South Carolina, USA.*

## Author Biography

*Tieling Chen is a Professor at University of South Carolina Aiken, USA. He received his PhD in Mathematics (2001) and his M.S. in Computer Science (2002) from University of Western Ontario, Canada. His research interests include Image processing and Color models and their applications.*