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Did Tim Paint a Vermeer?

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Abstract. *Tim's Vermeer is a recent documentary feature film following engineer and self-described non-artist Tim Jenison's extensive efforts to "paint a Vermeer" by means of a novel optical telescope and mirror-comparator procedure. His efforts were inspired by the controversial claim that some Western painters as early as 1420 secretly built optical devices and traced passages in projected images during the execution of some of their works, thereby achieving a novel and compelling "optical look." The authors examine the proposed telescope optics in historical perspective, the particular visual evidence adduced in support of the comparator hypothesis, and the difficulty and efficacy of the mirror-comparator procedure as revealed by an independent artist/copyist's attempts to replicate the procedure. Specifically, the authors find that the luminance gradient along the rear wall in the duplicate painting is far from being rare, difficult, or even "impossible" to achieve as proponents claimed; in fact, such gradients appear in numerous Old Master paintings that show no ancillary evidence of having been executed with optics. There is indeed a slight bowing of a single contour in the Vermeer original, which one would normally expect to be straight; however, the optical explanation for this bowing implies that numerous other lines would be similarly bowed, but in fact all are straight. The proposed method does not explain some of the most compelling "optical" evidence in Vermeer's works such as the small disk-shaped highlights, which appear like the blur spots that arise in an out-of-focus projected image. Likewise, the comparator-based explanations for the presence of pinprick holes at central vanishing points and the presence of underdrawings and pentimenti in several of Vermeer's works have more plausible non-optical explanations. Finally, an independent experimental attempt to replicate the procedure fails overall to provide support for the telescope claim. In light of these considerations and evidence, the authors conclude that it is extremely unlikely that Vermeer used the proposed mirror-comparator procedure. © 2020 Society for Imaging Science and Technology.*
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1. INTRODUCTION AND BACKGROUND

Tim's Vermeer is a recent documentary feature film following electrical engineer and self-described non-artist Tim Jenison's effort to "paint a Vermeer"—*Lady at the Virginals with a Gentleman* (1662–65), also called *The Music Lesson*—by means of his proposed novel telescope and mirror-comparator procedure [1]. His effort had its roots in other proposals for the possible role of optics in Old Master praxis, for instance as early as the 1420s, advocated most prominently by contemporary artist David Hockney [2], and in Vermeer's oeuvre in particular as argued by academic architect Philip Steadman [3].

Thorough analysis by dozens of scholars from optics, computer vision, history of optics, art history, and related fields has led to a unanimous independent rejection of the optical tracing theory, at least for the early Renaissance [4–10]. The optical case for Vermeer, however, is still unresolved. The requisite optics, such as the camera obscura, was well known in the Dutch Golden Age [11]; in fact, the 17th-century Dutch scientist Constantijn Huygens advocated the use of the camera obscura in the making of art [12, 13]. It is certainly plausible that Vermeer was aware of basic optics, perhaps (though unlikely) learned through an acquaintance with fellow Delft resident Antonie van Leeuwenhoek, executor of Vermeer's estate and creator of a simple single-lens microscope. However, van Leeuwenhoek was not scientifically trained and was rather secretive. Documents show his earliest interest in optics came several years *after* he would have had to instruct Vermeer. Other possible sources of optical knowledge are engineer practitioners such as van der Wijcke [14]. However, we do not have evidence that Vermeer had ever met such optics experts, let alone discussed optics [15]. Furthermore, we have no independent evidence that Vermeer even owned such imaging optical devices—for example, his financial records include no entries related to optics; no such devices (not even a camera obscura) appeared in the detailed inventory of his estate compiled immediately after his death [16].

A key technical weakness of proposals based on the use of a camera obscura centers on the fact that such a device does not (directly) aid in the rendering of subtleties in color and shading, widely considered hallmarks of Vermeer's style [16]. Although an image projected by a camera obscura might facilitate the tracing of contours and thus ensure the

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accuracy of geometric perspective, it does not directly help in rendering color because the appearance of the colors of the pigments is disrupted by the colored light of the projection itself [17]. Indeed the superimposed projection makes the rendering of color even more difficult. It is conceivable that the artist's memory of the mere *sight* of a projected image might influence his ability to judge colors applied elsewhere, but this alternate and weaker hypothesis has not been adequately supported. There are, moreover, numerous alternative explanations for improvements in the fidelity of color rendering of that time, including novel pigment mixing (such as the introduction of lapis lazuli to Vermeer's white pigments to add luster) and complex glazing methods.

Much of our analysis of Jenison's proposal is based on evidence gleaned from his documentary itself, as well as from his notes and diagrams, web postings, and our correspondence with an independent artist who has attempted to reenact the proposed method, as we shall see [18]. As such, we must proceed with a modicum of caution because to date there have been no rigorous peer-reviewed presentations of details of the relevant optics, procedure, or supporting evidence. We very much hope to see a clear, complete, and scholarly presentation of the proposed methods soon. Regardless, we believe our representation of the proposal is sufficiently complete and accurate, and our analyses sufficiently robust, that some conclusions can be made. Throughout we try to be as clear as possible about our assumptions, which of course might have to be modified if and when further details of the proposal become available.

We begin in Section 2 with a description of the optics of the proposed telescope and painting procedure, with special consideration of its relation to camera lucida, camera obscura, optical telescopes, and the state of optics in the Dutch Golden Age. We then turn in Section 3 to the visual evidence adduced in support of the mirror-comparator hypothesis. This particular evidence includes the rendering of the perspective and geometry of the painting, the fidelity of tones (notably the luminance gradient across the rear wall in the studio and the "tile" floors), and a slight curve to a single contour on the case of the virginals in the original painting. In Section 4, we review attempts by an independent professional artist to replicate the proposed mirror-comparator procedure. In Section 5, we mention briefly a number of other considerations, including the relation of the mirror-comparator proposal to other optical hypotheses for Vermeer. We summarize our conclusions in Section 6.

2. PROPOSED MIRROR COMPARATOR AND PROCEDURE

We first consider Jenison's proposed mirror-comparator instrument and its relation to optical instruments, such as the camera lucida and camera obscura, and its purported place in the history of optics and telescopes. We then describe the proposed copying/transcription procedure.

2.1 *Proposed Optical Telescope and Mirror Comparator*

It is important to distinguish Jenison's mirror-comparator telescope from other optical devices that have been proposed for the creation of visual artworks because his instrument provides novel functionality, new technical difficulties and challenges, and new classes of visual evidence in the artworks produced through its use. Perhaps, the simplest such optical device is the *camera lucida* (Latin, "light room"), which has several embodiments. The most primitive embodiment is a small pane of glass held at 45° to the artist's gaze toward the tableau. The artist can look *through* the glass to see the tableau and at the same time see *in reflection* from the glass the support (paper, canvas, and so on) and his marks (pencil lines, brush strokes, and so on). (Alternatively, the artist can look down through a glass pane onto the support and see the tableau in reflection.) Thus the artist sees the tableau and support "directly" and overlapping and can thereby trace visual contours accurately. Note that a camera lucida forms no traditional real image as do all other optical devices we consider. (Technically, in this embodiment, no true image of the tableau is formed, and a virtual image is formed of the support.) Only one viewer can see through such a camera lucida at a time.

There are several problems with using this simple camera lucida in the creation of art. The first is that because of the reflection from the glass, the marks on the support are inverted compared to the scene. That is, when the artist is marking the *head* of a portrait subject, his marks lie at the *bottom* of the support; conversely when the artist is marking the *feet* of the subject, his marks lie at the *top* of the support. Some artists find this geometric inversion a modest cognitive challenge.

A second impediment concerns the relative brightnesses of the view of the tableau and that of the support. The intensity of the light reflected from the pane is far less than that transmitted. Therefore, the artist may find it difficult to see the dim marks on the support—especially if the overall illumination of the tableau is low [19].

These impediments do not arise in the prism-based camera lucida invented by William Hyde Wollaston in 1806 [20]. The multiple reflections in the prism flip a view so that the orientations of the view of the tableau match that of the support (e.g., paper). Moreover, these multiple reflections of the tableau lead to a slightly improved balance of the brightnesses of the tableau and support.

These improvements led some artists and scientists in the 19th century to use the device, primarily for quickly sketching portraits or transcribing scientific images as in microscopes. We have secure evidence that some artists and scientists used Wollaston's camera lucida in the 19th century. John W. Audubon used a camera lucida to copy the drawings of his father, ornithologist/artist John James Audubon, in *Birds of America*. There has been speculation that certain 19th-century artists used the device, most notably Jean-Auguste-Dominique Ingres [2]. However, there is none of the expected documentary or physical evidence to

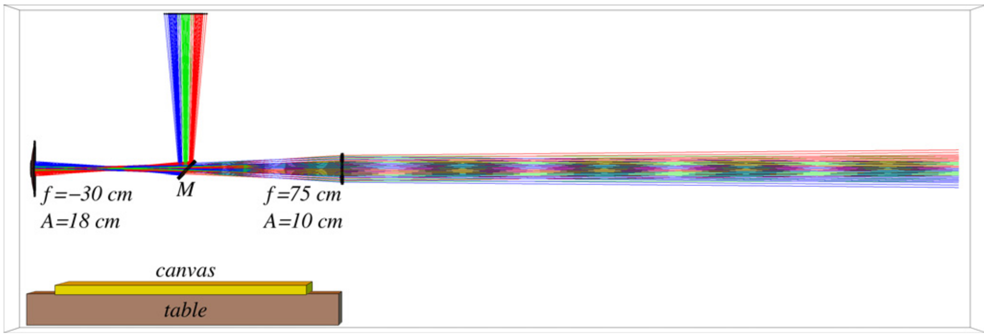

Figure 1. Ray-tracing diagram of Jenison's optical telescope and mirror comparator used for copying/transcribing the studio reconstruction of Vermeer's *Lady at the Virginals with a Gentleman* [32, 34:01]. The studio is off to the right and light passes from it through a converging lens (focal length $f = 75$ cm and aperture diameter $A = 10$ cm), then to a concave mirror ($f = -30$ cm and $A = 18$ cm) back to a small plane "secondary mirror," M, and then up to the artist's eye. The focal lengths, aperture diameters, and rough separations of components were taken from a diagram from Jenison's notebooks. The colored rays correspond to distance points on the optical axis and at $\pm 0.5°$. The artist/copyist could look down at the plane secondary mirror, M, to see the real image of the tableau and then shift slightly to the side to see the corresponding position on the canvas where he had applied paint. The artist/copyist could adjust the applied paint until the two images matched precisely.

support this speculation, and there are plausible non-optical explanations for the visual evidence in his artworks.

The second optical device implicated in art praxis is the *camera obscura* (Latin, "dark room"). In its simplest form, the device is a room or box with a small hole in the middle of one side to admit light, thereby creating an inverted image on the opposite wall [19]. Such images are much dimmer than the external scene, and this inherent brightness limitation was overcome by enlarging the hole and placing a convex lens over this aperture to project a real, inverted image onto the rear wall. There are a number of variations of the device, including the introduction of a ground-glass screen, so that the inverted image can be seen from outside the box, or of a single plane mirror at 45° so that the image appears conveniently on the top of the camera. Several people can view such a projected image simultaneously just as many people can simultaneously watch the same movie projection.

The proposals for the use of a camera obscura in art center on tracing the contours of an image projected onto a piece of paper acting as a screen. Thereafter, the artist would transfer these contours to a canvas, for instance, by means of *pouncing*, and then apply paint within the contours [3]. No serious artist or scholar suggests that an artist would paint *directly* under such an optical projection because of the extraordinary difficulty in judging colors as the (colored) light of the projection itself alters the appearance of the applied paint. Paradoxically, if the artist wants to paint under an optical projection to make the image appear in proper color, he should paint the entire canvas white—like a movie screen. A number of artists are known to have used a camera obscura for tracing a projected image or just observing a scene in two dimensions for "blocking" (setting composition), most notably Canaletto in the 18th century [21].

Figure 1 shows the optical elements and design of Jenison's proposed optical device, replotted from a hand-drawn diagram in his notebooks, which also includes documentation of focal lengths, apertures, and other optical parameters. It is a *catadioptric* telescope, that is, one where the focusing is due to both a lens and a concave mirror. The ray-tracing diagram here, created in *Optica*, shows in color the passage of rays from three distance points: one on the axis and the others at $\pm 0.5°$. The rays pass through a converging lens and to a concave mirror and then form a real image within the telescope. The rays then pass to a small front-surface plane mirror angled at 45° upward and then to the artist's eye above. The concave mirror helps form the real image as well as increase its brightness. Moreover, the added reflection from the plane mirror ensures that the parity (left–right orientation) of the final image matches that of the scene—a property violated in a traditional camera obscura.

This instrument magnifies a small field of view (approximately 4° in diameter depending on the configuration) and is technically a telescope because it forms a real image within the space within the optical system, which is then viewed by the artist. The moderately large concave mirror both collects light and magnifies the image. The small flat mirror at 45° acts very much as an injection mirror or "secondary mirror" used in many telescope designs [22]. The image is brighter than in a camera obscura because the artist is looking at the real image *directly*. By contrast, the light forming the real image in a camera obscura strikes a screen and is scattered throughout the full 2π steradians, making the image dim by comparison. This is precisely why one can look through a telescope to see very dim stars but cannot see such stars when they are projected by similar optics onto a screen. This improved brightness means that Jenison's telescope does not need a baffle or light-tight box—the "camera" of a camera obscura.

2.2 *The Place of the Proposed Telescope in the History of Optics*

Table I compares the camera lucida, camera obscura, and the proposed optical telescope with respect to a number of optical and other properties.

Table I. Comparison of properties of three optical devices proposed as tools in art praxis. The fields of view are approximate and depend on details of the particular designs.

	Camera lucida	Camera obscura	Mirror-comparator telescope
Number of core optical elements	1	1 or 2	3
Real image projected	None	Real (on screen)	Real (in space)
Number of simultaneous observers	1	Several	1
Field of view	≈30°	≈40°	≈3°
Sensitivity to alignment errors	Low	Low	Very high
Documented circa 1663	None definitive*	Yes	No

* The Dutch engraver Hendrik Hondius (1573–1650) described artists looking through a tilted glass frame, which might duplicate the optics of a simple camera lucida.

The proposed telescope as demonstrated by Jenison—had it existed in 1663—would have been one of the most complicated optical systems of its time. Its optics is nearly as complex as the Keplerian telescope (c. 1611), which modified for terrestrial use had three lenses (objective, erecting, and eyepiece) and the Wiesel and Divini telescopes (c. 1654), which had five lenses (objective and four-lens eyepiece) [23, 24]. No system documented from that time employed all the proposed different *types* of optical elements: converging lens, concave mirror, and front-surface plane mirror. Specifically, this device is more complex than the important two-mirror telescope invented by Laurent Cassegrain, first documented nearly a decade *after* the date of Vermeer's painting. It is also more complex than the famous reflecting telescope (with a 45° secondary mirror) invented in England by Isaac Newton in 1668 (but published in 1672), a decade and a half after the date of Vermeer's painting.

Most importantly, the proposed telescope is far more sensitive to alignment than any optical system appearing decades after Vermeer. Most telescopes, then and now, ensure axis alignment and centrations of components by means of tubes. This approach cannot be applied to the mirror-comparator telescope. This is because any tube would not only block light between the support and the artist but would also prevent the direct readjustment of positions and tip angles required for imaging throughout the full visual scene. It is not only the alignment but also the *separations* of the optical elements and support (canvas) that must be proper for the device to be used in comfort for extended periods. Specifically, the optical distance from the artist's eye to the real image and the artist's eye to the support must be nearly equal. If these distances differ significantly, then the artist must accommodate (focus his eyes) in order to see and thus accurately compare the two images one after the other. In Section 4, we shall see the practical ramifications of these technical drawbacks.

The German polymath Samuel Hartlib wrote around 1655 (roughly the time of Vermeer's painting) to document that a form of projection of a bright outdoors onto a table was available in The Hague, a mere 10 km from Delft [14, as quoted in]:

> At [The] Haage now to bee performed by one paire of glasse in the window to represent and conveigh all the objects without upon the Streets upon the table in de middle of the roome. The inventor, as I take it, is Van der Wijcke, the Belgick Reeves at Delfe, who makes all manner of Tubes and Microscopes excelling those of Braband. The Tubes hee fits to the sight of every ones age. [He is] a most rare Workeman.

A related case is that of Gerrit Dou, a celebrated realist painter and contemporary of Vermeer. There is firm reporting that Dou would use as many as three pairs of eyeglasses when executing some of his works. Moreover, there is suggestive testimonial evidence that he may have used a single concave mirror as a primitive telescope to magnify and thus better see details in the still-life tableaus he painted [25]. There is no evidence that his mirror was as complicated as Jenison's telescope and no evidence that he employed the mirror-comparator procedure described in Section 2.3.

This is not the venue for a full study of how Vermeer might have learned optics or invented this device. Delft and environs was a center for optics, where important inventions and improvements in early telescopes were made. Antonie van Leeuwenhoek, Vermeer's contemporary in Delft, developed a single-lens microscope, which was basically a tiny magnifying glass mounted in the hole of a brass plate. A telescope was demonstrated in nearby The Hague in 1608 and spyglasses were available to students in Leiden University in 1610. Moreover, much optical knowledge was acquired and spread by lens grinders and optics artisans such as Evert Harmansz Steenwijck, van der Wijcke, and others [14]. It must be stressed, however, that despite the manifest opportunities for Vermeer to learn optics, we have no textual evidence that he even met with any optics experts, much less learned optics, or became such an expert so as to invent and use the sophisticated mirror comparator. Furthermore, the inventory of his estate documents no such needed optical devices. His executor, van Leeuwenhoek, would almost surely have been expert enough to at least recognize and document such devices had they existed.

Note especially that van Leeuwenhoek was a draper (cloth salesman) and showed no evidence of interest in microscopes before 1668 when he visited England—six years after Vermeer's painting. Suggestions that he had optical influences before that time are based on mere speculation. We mention in passing that van Leeuwenhoek's microscope was in essence a single spherical lens mounted in a brass plate and much simpler than the proposed telescope. Yet even with wide dissemination of the design and its manifest value to the budding science of biology, scientists found van Leeuwenhoek's microscopes very difficult to use and generally did not adopt them [26]. In short, it seems rather implausible that this draper would be giving sophisticated optical instructions to Vermeer in 1663.

2.3 *Painting/Copying Procedure*

Jenison's hypothesis under consideration is that Vermeer used the mirror-comparator telescope of Fig. 1 in the following way. The artist looks at the secondary mirror from above and sees in reflection the real image of the tableau; the artist also sees the support and his brush strokes adjacent to the mirror edge. He rocks his head position slightly, back and forth, so as to alternately view the projected image and then the corresponding strip of canvas in the same direction, thereby comparing them. He then applies paint so as to match the artwork to the viewed image.

This procedure exploits the human visual system's exquisite sensitivity to shape and misalignment of lines ("Vernier acuity") corresponding to arcminutes of visual angle [27–29]. Likewise, it exploits the visual system's extraordinary sensitivity to differences in color; under optimal conditions, humans can distinguish as many as 40 million colors [30]. Of course, it is important that the color and lightness in the scene be expressible in paints available to the artist. The typical range of reflectivities (albedos) of artists' pigments is approximately a factor of 100:1 (or in photography terminology, $\log_2 100 \approx 7$ stops). For this reason, the limited luminance range in a Dutch interior might find full match in paint, whereas that of a sunlit landscape with deep shadows would not.

The angle of view in such a fixed telescope setup is fairly small, and for that reason, the telescope must be adjusted to copy regions throughout the full tableau. Repositioning and refocusing of the full optics for a fixed canvas would be extraordinarily difficult as the angles, separations, focus, and so forth would have to be adjusted extremely carefully, and there is little guarantee that adjacent passages would align. Moreover, the perspective would become discernibly inconsistent as the effective center of projection was changed. Instead, Jenison's table (and canvas) was on wheels, and the copyist slides the table into position and then adjusts the lens and mirrors accordingly, for instance, by pointing higher or lower in the visual field. (As we shall see in Section 4, despite this aid, this overall readjustment remains extremely difficult.) This procedure minimizes disruptions to the location of the center of projection and attendant inconsistencies in the global perspective.

Because the mirror-comparator process, including refocusing, is so slow and arduous, copying human figures presented novel challenges. Such figures of course cannot hold their poses fixed unaided during the weeks it would take to copy them. For this reason, Jenison used a mechanical brace or retort stand to hold his models' heads in place during the copying. We have no independent textual corroboration that Vermeer's models (nor any models of the time) were constrained in this unusual and uncomfortable manner.

Despite the alleged benefits of this mirror-comparator method, Jenison did not adhere to this method exclusively. For certain passages, such as the marble tiled floor, he used a rule to ensure long lines were straight. Such a procedure can be aided by the addition of a pin placed at the central vanishing point to either hold a chalked string snapped against the canvas or to guide the placement of a rigid straightedge. We find pinprick holes at the central vanishing point in several of Vermeer's paintings, most notably *The Art of Painting* executed c. 1666–68.

3. ANALYSIS OF VISUAL EVIDENCE

We now turn to the three primary classes of evidence used in support of the mirror-comparator hypothesis: geometric accuracy, lightness and color accuracy, and an anomalous bowed contour.

3.1 *Geometric Accuracy*

The ideal way to test the geometric accuracy of the resulting painting is of course to compare it with a photograph of the tableau taken from the center of projection of Jenison's telescope. We do not have access to such a photograph [2]. One can check the internal consistency of this painting by a number of geometric means, for instance, confirming that the vanishing point defined by orthogonals, horizon line, and related distance points is geometrically consistent [31]. Note, though, that because some of these lines—particularly on the floor of the studio—were drawn with straightedges, such tests reveal little about the spatial fidelity achievable with the mirror comparator used alone.

A more challenging issue is the perspective structure of scene elements at oblique angles to the main central vanishing point (such as the chair behind the table in this painting). Its correct perspective could most likely be achieved by an expert use of the optical projection method coordinated with the horizon level of the central vanishing point or by an advanced knowledge of multiple vanishing point perspective construction (which was not known to such advanced exponents of perspective as Pozzo and Canaletto, let alone Vermeer's contemporaries in Holland). Only a few of the lines in the chair in this work are discriminative (there must be physical parallels visible in the painting), but in other paintings of such chairs, they are perfectly consistent as far as can be assessed. The requisite rules are that the orthogonal sets of parallels must converge at two points on the same horizon level as the central vanishing point and that the angle between them (from the center of projection defined by the distance points of the tile floor)

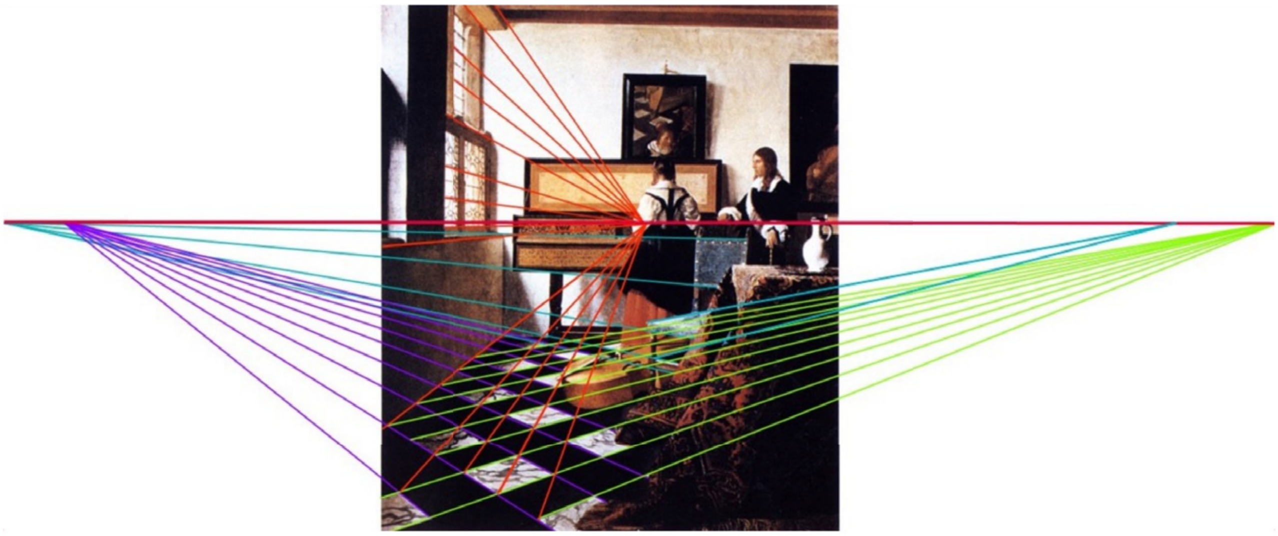

Figure 2. Perspective scheme in Vermeer's *Lady at the Virginals with a Gentleman* (*The Music Lesson*). The red lines mark the central vanishing point; the green and purple lines define the distance points. © C. W. Tyler 2000, with permission.

must be 90°. Few artists of that era, or any other, were fully conversant with these rules, but Vermeer may have been one of the few.

The remarkable characteristic of *Lady at the Virginals with a Gentleman* is that its perspective is indeed photographically realistic not only within the tile floor but also in the convergence from the windows at left to the same central vanishing point, and of the obliquely aligned chair, as just described. Other than by a diligent application of the rules of perspective, this degree of alignment of the three-dimensional structure could possibly be obtained by the kind of whole-field camera obscura proposed by Steadman [3]. Such coherence does not arise in the kind of piecemeal approach proposed by Hockney [2]. Since implementing the perspective construction shown in Figure 2 would require implausible extensions of a solid surface structure on either side of the painting to support the straightedge while making the distance-point construction, these observations seem to better support Steadman's proposal.

3.2 *Accuracy in Rendering of Lightness, Color, and Tone*

As mentioned in Section 2.3, the mirror-comparator telescope greatly facilitates the copying and matching of colors in the scene. Steadman, a proponent of the mirror-comparator hypothesis, states, "The most powerful evidence [for the mirror-comparator hypothesis] has to do with tonality . . ." He claims that the falloff in lightness along the rear wall in Vermeer's painting "cannot be seen"; "Not even Vermeer could have seen [the difference between luminance at the right and left]. You just cannot see a shadow gradient in tone like that" [32, 38:53]. Likewise, we hear in the documentary that the luminance gradient ". . . is something Vermeer has reproduced *that he could not have seen*" (emphasis added).

In fact, this claim is patently false because we can clearly see the gradient of lightness on walls such as depicted in Vermeer's painting (Fig. 2). The only reasonable interpretation of this repeated statement is that we cannot see the gradient veridically—that it appears as a much shallower gradient of illumination than is physically present and hence that the painter cannot capture the strength of this gradient due to the perceptual flattening of its physical steepness. However, this logic is flawed in the same way as the "El Greco fallacy" [33]. The painter's goal is to capture the perceptual experience of the physical gradient and so is in the position to match the physical gradient by adjusting the paint mixtures to provide a perceptual match between the scene gradient and the painted gradient. This match is possible as long as some gradient is perceivable; it only becomes impossible if, indeed, the gradient is literally invisible (which is manifestly not the case).

Here, knowledge of basic visual psychophysics and comparable paintings is essential for judging this evidence central to the proponents' argument. The phenomenon of color constancy (including lightness constancy) has been understood for well over a century [34]. In fact, Chevreul's studies of this and other perceptual phenomena, in particular simultaneous contrast, were explicitly applied by Impressionists such as Georges Seurat [35]. The peripheral human visual system evolved to respond primarily to *relative* brightness of regions throughout a scene as this allows us to recognize, for example, a berry as blue despite wide variations in the intensity and chromaticity of illumination. However, this cannot be the sole brightness-related channel in human vision; the human visual system is *also* able to distinguish subtle variations in overall light level [36–41]. After all, without this channel, we could not distinguish a brightly lit noonday scene from the same scene at night lit by a full moon. Despite adaptation and color constancy, humans *can* sense overall brightness in regions of a scene.

Although the explicit claim that Vermeer "could not have seen" the luminance gradient in his studio (yet rendered

Physics based computer graphics model of *Lady at the virginals with a gentleman*?

Tim Jenison's after *Lady at the virginals with a gentleman* (74.6 × 64.1 *cm*), oil on canvas (2012)

Johannes Vermeer's *Lady at the virginals with a gentleman* (76.4 × 64.1 *cm*), oil on canvas (1662–65)

Pieter de Hooch's *Young woman drinking* (69 × 60 *cm*), oil on canvas (1658)

Frans Hals and Pieter Codde's *The Meager Company* (209 × 429 *cm*), oil on canvas (1633–37)

Anthonie Palamedesz's *A happy company* (56.5 × 70 *cm*), oil painting (1673)

Edward Hopper's *Girl at a sewing machine* (48.3 × 46 *cm*), oil on canvas (1921)

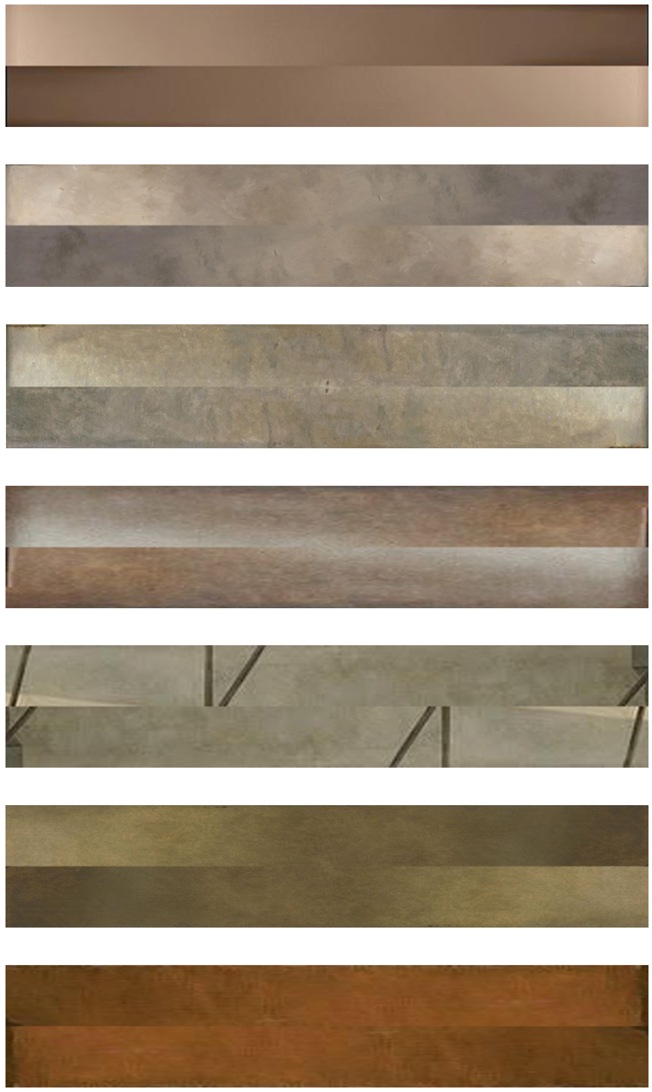

Figure 3. Details from five paintings showing luminance gradients on rear walls, with counterposed (reversed) copies to reveal the contrast gradients. The top panel shows a physics-based computer graphics rendering in lieu of a photograph of Vermeer's actual studio as an indication of the gradient that would likely have been present. The following counterposed gradients show that despite color (or lightness) constancy, at least some Old Master painters and a 20th-century painter can see and paint luminance gradients of the sort Steadman claims "could not be seen."

it in his painting) was voiced in the documentary by perceptual psychologist Colin Blakemore [1], and several times by both Jenison and Steadman in different venues, and was the central reason Steadman found the telescope hypothesis persuasive, that claim is false. Experiments in perceptual psychology demonstrate this fact as do the paintings by a wide range of independent artists with no supporting evidence that they used optics. Nevertheless, we can consider a weaker and presumably less persuasive view that an artist could not perceive and render such a gradient *accurately* without the use of the proposed mirror-comparator telescope. This weaker view also suffers from a number of technical problems. Note that given such a gradient would have been visible *at all*, the artist could (and presumably would) apply paint to his canvas so as to produce a similar lightness perception. In fact, an experimental demonstration described in Section 4 proves that a realist artist can indeed see such gradients and render them in paint quite accurately in an absolute sense.

Perhaps the most direct and relevant way to test the claim that Vermeer "could not have seen" the luminance gradient is from the innumerable realist paintings in which gradual gradients are in fact rendered. Figure 3 shows details of such gradients taken from several realist paintings from a range of artists and periods. Note especially that throughout this wide range of art periods and media, the kind of luminance gradient proponents claim Vermeer "could not have seen" is captured directly by artists for whom there is no suggestion that they used a mirror comparator. The physics-based computer graphics model suggests that the actual gradient may have been slightly more visible than those rendered by the other artists [42]. Furthermore, as we will see in Section 4, an independent realist artist proves that

Figure 4. (L) Johannes Vermeer's *Lady at the Virginals with a Gentleman* (74.6 × 64.1 cm), oil on canvas (1662–65), Royal Collection Trust/© Her Majesty Queen Elizabeth II 2019. (C) A detail of the painting, horizontally compressed to reveal the slight upward bow in the horizontal line along the case of the virginals. Note that every adjacent line, such as the horizontal line beneath the bowed contour, and indeed every nominally straight line in the scene (e.g., furniture) is in fact straight. (R) Detail of Gabriël Metsu's *Lady at the Virginals* (1660–67), scaled to reveal a slight bowing in a single contour on the virginals.

one does not need the mirror comparator—or optical aids of any sort—to accurately render such luminance gradients.

One final rebuttal to the mirror-comparator theory concerns the rendering of lightness on the floor tiles. Such black and white floors were called "Italian floors" at the time due to the source of white marble; the less expensive black marble came from Belgium. The "tile" floor patterns differ among several of Vermeer's works presumably executed in his studio such as *The Music Lesson*, *The Art of Painting*, and *Lady Standing at the Virginals*, proving that at the very least, *some* of the floor designs are "fictive." The proponents are impressed that the rendering of the lightness of the tiles in Vermeer's studio apparently matches the intrinsic reflectance properties of an actual tile floor. However, some historians argue that tiled floors were rare in Dutch interiors of the time and extremely rare in all but the homes of the wealthy and that large tiles such as those depicted in Vermeer's paintings were especially expensive. Such floors appeared primarily in entranceways and reception rooms [43–48]. Tile floors appear in many paintings by Pieter de Hooch for wealthy patrons, such as *A Woman Seated by a Window with a Child in the Doorway*, *Card Players in a Sunlit Room*, *Man Holding a Letter to a Woman in the Entrance Hall of a House*, *Card Players in an Opulent Interior*, *A Lady Surprised by her Lover*, *Portrait of a Family Playing Music*, and *At the Linen Closet*, but on the ground floor (as far as the visual and other evidence shows) as is evident by views of street life outside the window or by the massive formal fireplaces or very high ceilings that appeared on ground floors only. Such floors would not have appeared in any homes but of the wealthy; further, tiles would never appear in an *attic* of such a home (where Vermeer painted this and several other works). Artists' oils (as well as rust) severely damaged tiles, making such tiles vulnerable in a studio such as Vermeer's. A careful inventory after Vermeer's death did not mention a tile floor in this studio or indeed anywhere in this home though such floors were considered an immovable part of the home and hence might not be inventoried. The proponents are explicitly placing great credence in Vermeer's rendering of the reflectance properties of a marble floor—a floor for which there is very little evidence even existed.

3.3 *Bowing of a Single Contour*

Figure 4 shows Johannes Vermeer's *Lady at the Virginals with a Gentleman*, the work replicated by Jenison. At the right is a passage scaled horizontally by a factor of approximately .59 to better reveal a very slight bowing or warping in one contour on the wooden case of the virginals, which we presume was physically straight in the studio. One might attempt to utilize this single contour as evidence for the use of optics by claiming that it was due to an optical aberration or to geometric factors in the mirror-comparator procedure. Such an explanation for Vermeer's praxis can be summarily dismissed based on additional and more plentiful visual evidence: all other contours at every orientation, including nearby parallel ones on the virginals and indeed throughout the painting, are straight. If the bowing of this line were due to optical effects, we would certainly find similar bowing elsewhere throughout Vermeer's painting—but we do not. Indeed, such a bowed line does not appear in Jenison's painting. Likewise, a single such very slightly bowed contour appears in Vermeer's contemporary Gabriël Metsu's *Lady at the Virginals*, but here too numerous other lines in his painting are straight. Perhaps this single bowed line arose when Vermeer did not stretch a drawing guide string sufficiently taut or simply moved his hand slightly across the direction of the contour when drawing a guide line using a straightedge.

There is, moreover, another difficulty for the optical explanation for long contours. The mirror-comparator telescope has a fairly small field of view, and thus long contours would require several repositionings and refocusings of the optics. Any curvature recorded within a *single* setup or "exposure" (as proposed) would not cohere into a single long uniform bowing executed across *multiple* such exposures. Steadman's derived floor plan for *The Music Lesson* [3] suggests that the horizontal angle subtended by the bowed contour is approximately 7.2°, which would likely need two (and possibly more) "exposures" requiring readjustments. In that case, the expected contour would resemble a sequence of two or more curved arcs, much like a drooping telephone line suspended by a sequence of three telephone poles. Other contours (such as the corner of the walls, ceiling beams, and painting frames) are longer and would lead to several such drooping arcs. Of course, too, such purported bowing need not be restricted to linear contours; it would apply to, for instance, the array of decorative dolphins on the virginals. Although there may be rare cases of very slight distortion, we do not find such pervasive distortion throughout Vermeer's (or Jenison's) painting.

4. INDEPENDENT REENACTMENTS

The mirror-comparator painting procedure is extremely slow, tedious, difficult, and frustrating for both Jenison and at least one independent artist. Jenison took 130 full days of painting to complete the 74.6 × 64.1 cm painting and admitted on camera that he would have stopped out of boredom and frustration had he not been committed to finishing the documentary film. He states his procedure is "nerve-wracking," "difficult," and that he experienced "a wave of revulsion . . ." in returning to use his telescope [1]. One imagines, too, that part of his motivation to persevere was to convince others that his proposal would indeed work after nearly a year of creating the studio with accurate props, committing to a documentary film project, and so on. As he states weeks before finishing: "This certainly is not easy" and "If we weren't making a film I would definitely find something else to do."

As with all such scholarship, it is essential that experiments involve independent practitioners. To this end, the professional painter Jonathan Janson (who has a decades-long interest in Vermeer's painting techniques) attempted to use Jenison's telescope in both Jenison's studio in Texas and a replica (for Vermeer's *Woman Standing at a Virginals*) at the Museum of Old and New Art in Hobart, Tasmania. All the emphasis in Janson's report below has been added to highlight technical points under consideration [49].

The difficulty in using the device is evident throughout all his attempts, and in fact no attempt succeeded without Jenison's direct assistance in adjusting the telescope "every time":

> I [Janson] continued to practice on a very small area but was unable to paint more than a few square inches at a time. It should [be] clearly understood that the area that can be painted without re-aligning all three of the optical elements is small with respect to a scene such as those represented in Vermeer's paintings. *I was unable to align them on my own so Tim had to intervene every time I wished to move to a new area to be painted or a new motif.*

Janson reiterates the difficulty of the procedure:

> In the following days I attempted to paint a corner of Tim's mock-up of Vermeer's *Music Lesson* that was featured in his movie in order to test the device in a more realistic setup. Again, *I was unable to proceed without Tim repositioning all three optical elements.*

Janson found that he did not need to use the mirror comparator in order to paint tones and gradients accurately:

> Out of curiosity, I first executed [without optics] a monochrome underpainting of the area to be painted (the corner of the room where the furthest side window abuts against the back wall and projects a diagonal shadow downward) in order to test my sense of tonal value against the objective values that would be given by the [mirror-comparator] device. I was surprised to discover that I had been able to come very close to the true tones [without using optics] when I checked them with the comparator.

Again, Janson's experience comports with the evidence summarized in Fig. 3, showing that rendering tonal gradients accurately is not a particularly challenging task for trained artists. The following confirms the extreme sensitivity to alignment errors discussed in Section 2.1:

> Despite my most earnest attempts, *I was almost completely unable to align the elements correctly so that I could progress from one area of the scene to another,*

> *except for tiny movements to the left or right*. Even the slightest error in alignment causes a sort of blacking out of the image on the mirror or an insurmountable distortion. In fact, one must keep in mind that the correct alignment is determined not simply by aligning the three elements along a single axis . . ., but by many other factors, including but not limited to the distance between each element with respect to the others, the distance from the comparator to the canvas below, as well as the tilts of each of the elements. The tilt of the concave mirror is particularly crucial in that even the slightest movement may cause the image in the comparator to black out or wildly distort. I often spent minutes trying to recapture images I had lost this way. Moreover, even after one is able to site a new area successfully the new image may be too big, too small or distorted with respect to the previous "image" and it will no longer align correctly with the painted scene. This happened to me many times.

Likewise, the artist describes being "stuck" using the device:

> I traveled to Tasmania and began to paint from the mock-up MONA [Museum of Old and New Art] of Vermeer's *Lady standing at the virginals*, a scene which I had chosen because it appeared a valid but relatively simple testing ground. I soon found out that, given the environmental constraints of the installation, while I had been to make satisfactory painting when using traditional technique for the first picture, *I was almost immediately stuck when I began using the comparator to paint the scene a second time*.
>
> Tim had returned to the US [from Tasmania] and *without him, I was unable to align the elements consistently and so made little progress in painting. I attempted to solve the alignment problem day after day but was unable to come up with anything that might resemble a method*. Believing that there must exist some logical procedure by which one can move from one area of the scene to the other more or less comfortably I interrogated Tim on various occasions via live stream but he was unable to provide me with a clear set of rules but, instead, insisted that it will come with practice, as it evidently did for him. I proceeded as best I could, using the comparator to make tonal spot checks at crucial points of the painting, but unable to use the device to paint the scene piece by piece as Tim could have had done and had expected I would also be able to do.

These "reenactments" did not involve human models, which would been even more difficult, as described above.

5. ADDITIONAL CONSIDERATIONS

Perhaps the most compelling evidence put forth by Steadman in support of his particular camera obscura proposal concerns the sizes of canvases of six paintings he assumed were painted in the same studio [3]. That assumption must be seriously questioned in light of recently discovered historical evidence. Steadman argued that these sizes match the size of a "usable" projected image in a camera obscura having its screen located at the rear wall of his studio. (Why this master painter would have chosen his canvas sizes based on optical constraints rather than on artistic and compositional matters and patron preferences has never been explained.) Recently discovered compelling evidence, based on image heraldry in the depicted window and historical records of real estate, and more, shows that Vermeer executed two of those six paintings, *The Glass of Wine* and *The Lady with Two Gentlemen*, in the home of the van Nederveen family, wealth gunpowder merchants [50]. At the very least, these results seriously compromise Steadman's claims of numerical implausibility concerning canvas size for two reasons. First, at best, the number of "matching" canvas sizes is reduced from six to four, which due to basic combinatorics makes any correspondences far more likely. Second, the fact that two Vermeer paintings identified by Steadman have the matching dimensions while *not* having been executed in Vermeer's studio shows that such dimensions are likely based on esthetic—not optical—considerations.

Moreover, Steadman's explanation for the canvas sizes collapses in Jenison's hypothesis, which makes the canvas size evidence moot. Because of the freedom in telescope setup and that the relation between the object size and the angle of view ("visual pyramid") is not rigidly constrained as in the camera obscura model, if Jenison's proposal is correct, the canvas size evidence is irrelevant. In that case, then the sizes of the canvases are artistic or patron choices and their "optical" correspondences are just chance—the explanation Steadman explicitly rejected [3].

Consider the two works just mentioned almost surely painted *outside* Vermeer's studio. It seems rather unlikely that Vermeer spent months executing these paintings using a mirror-comparator method and that there would be no documentary records from the wealthy patrons that he did so. In light of this new evidence, it would seem that supporters of the mirror-comparator hypothesis would have to concede that Vermeer executed some of his paintings *without* optics, and at least one other painting *with* optics, even though all such paintings share geometric, tonal, and other properties. Such an explanation would lack scholarly parsimony.

Previous arguments for the use of optics by Vermeer were based in part on claims that optics would speed up (and ease) the rendering of paintings [2, 3]. By that criterion, the mirror-comparator hypothesis has the opposite effect: it slows down and makes painting far more complex, tedious, frustrating, and difficult. Given the verified extreme difficulty in the proposed mirror-comparator procedure, let us not forget that Vermeer fathered 15 children, 9 of whom survived to childhood, was an art dealer, and lived at a time when even simple chores took far more time and were more burdensome than today. Moreover, he died at age 42 from the crushing burden of financial ruin. (His widow, Catharina Bolnes, filed for bankruptcy shortly after his death.) It is hard to imagine Vermeer having the time (let alone expertise) to operate and keep aligned what was (had it existed) almost surely the world's most complex optical device. Then, too, there is the unanswered question of how Vermeer would have secretly *invented* such a complex device.

6. SUMMARY AND CONCLUSIONS

The proposed catadioptric telescope and mirror-comparator transcription procedure has the important property in that it allows accurate—indeed extremely accurate—copying of local contour, color, and tone and thereby overcomes a key drawback of prior proposals based on a camera obscura. However, we find that at the time of its purported use, the mirror-comparator telescope and procedure would have been among the world's most complicated optical systems, and its central optics would have been even more complicated than revolutionary and celebrated devices invented by leading scientists and optical designers years later. Moreover, the proposed telescope system requires greater precision in alignment and element separations than any optical system known for many decades thereafter, all without using the ubiquitous use of tubes to align the optical elements. Although circumstantial evidence makes it plausible that Vermeer might have been aware of optics, we have no corroborating textual evidence—such as correspondence, personal notes, and sales or estate records—that he owned or even knew of such optical devices. Even his most "scientific" painting, *The Astronomer*, depicts no optical devices.

To the extent that the geometry can be verified by means of orthogonals, distance points, and related geometric conditions, it appears that the geometry in the executed painting is accurate. The luminance gradient along the back wall (and indeed throughout Vermeer's painting) would have been easily seen by Vermeer as proven by psychophysical experiments and confirmed by numerous other painters' renderings of similar gradients. The single very slightly bowed line corresponding to an edge on the virginals appears elsewhere in a Dutch Golden Age painting. Any optical explanation for such bowing can be immediately rejected because of the numerous straight lines that would also have been bowed under the proposed optical aberrations but are not.

In light of the above evidence, we conclude that it is highly improbable that Vermeer used the proposed mirror-comparator telescope in the execution of his paintings.

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