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Cartography as Spatial Representation: A new assessment of the competing advantages and drawbacks across fields of science

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Abstract

The history of cartography has been marked by the endless search for the perfect form for the representation of the information on a spherical surface manifold into the flat planar format of the printed page or computer screen. Dozens of cartographic formats have been proposed over the centuries from ancient Greek times to the present. This is an issue not just for the mapping of the globe, but in all fields of science where spherical entities are found. The perceptual and representational advantages and drawbacks of many of these formats are considered, particularly in the tension between a unified representation, which is always distorted in some dimension, and a minimally distorted representation, which can only be obtained by segmentation into sectorial patches. The use of these same formats for the mapping of spherical manifolds are evaluated, from quantum physics through the mapping of the brain to the large-scale representation of the cosmos.

Cartographic Solutions from the Roman Era

Since the time of the early Greek philosophers, the history of cartography has been marked by the endless search for the perfect form for the representation of the information on a spherical surface manifold into the flat planar format of the printed page or, more recently, the computer screen. This is a primary issue not just for the mapping of the globe, but in all fields of science where spherical structures are found. The spherical surface cannot be mapped onto the flat page without some form of relatively severe distortion.

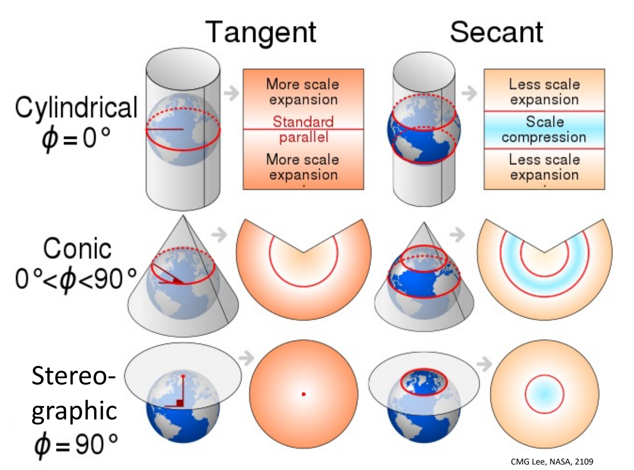

Figure 1. The 3 spatial mappings from Ptolemy's Geographia *of C2nd Graeco-Roman Alexandria (from NASA / CMG Lee).*

Claudius Ptolemy (C1st AD) proposed six formats for mapping the Roman empire portion of the sphere of the earth (Fig. 1): the three basic forms were a cylindrical mapping tangent to the earth at the equator, later known as the Mercator projection; a conic section with its apex above the north pole, thus wrapping the Earth most closely at the Mediterranean; and a stereographic projection of the sphere of the earth to a flat plane, most conveniently located at the North Pole to capture the extent of the known world in the Northern Hemisphere (from Ptolemy's viewpoint).

Each of these mappings came in two flavors. One was a literal tangent to the sphere of the Earth for each type (left column in Fig. 1), which had minimal distortion along the tangent line (or point in the case of the stereographic projection), and progressively increasing distortion away from it. The other flavor was a secant version where the projection surface was lowered below the surface of the sphere, so as to increase the extent of the undistorted circle and reduce the scale expansion away from it, at the expense of increasing the distortions in the form of relative scale compression in the original tangent regions (Fig. 1, right column). Thus the secant approach generates a more uniform degree of distortion than the tangent approach. Flattened versions of the wrapping surfaces are shown for each form of projection. It is evident that, 2000 years ago, Ptolemy was analyzing the ultimately insoluble problem of spherical mapping with a fair degree of geometrical insight.

Renaissance Ptolemaic Mapping Firsts

However, no actual maps or diagrams have survived from the Roman Ptolemaic era. The first extant map of the known world is from the Codex Urbana, a Greek translation of Ptolemy's *Geographia* from Constantinople dating to about 1300. This is a mapping of about one third of the Northern Hemisphere spanning from the Atlantic Ocean to the Malay peninsula. It therefore flattens into a roughly 180° sector rather than the 90° sector conic projection in Fig. 1.

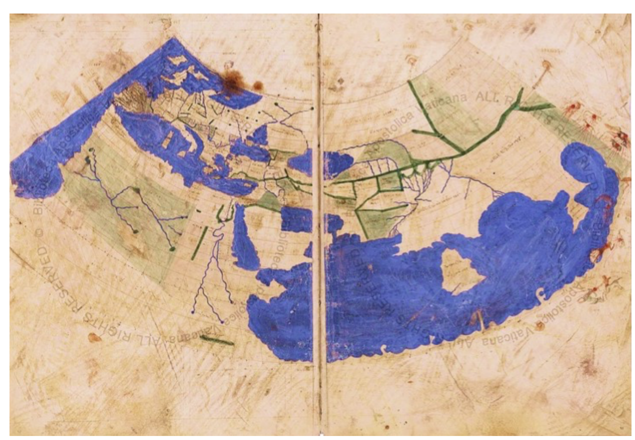

Figure 2. The world map attributed to Maximus Panudes, from the Ptolemy's Geographia *in the Codex Urbana (~1300. Vatican Library).*

A similar map from the same era is the Codex Seragliensis, a damaged Greek map of the Ptolemaiac world according to the secant form of conical projection (Fig. 3). This gives an appropriately globe-like impression due to the curved meridian lines, although far more challenging to construct with a rigorous geometry. This map is the first known implementation of the secant form of Ptolemy's conical projection (middle row, Fig. 1).

Figure 3. The Ptolemaic world map from the Codex Seragliensis (GI 57, Topkapi Palace, ~1300).

The next projective advance to be drawn from the Ptolemaic canon was the complete conical projection by Florentine cartographers Giovanni Contarini and inscribed by Francesco Rosselli, dating to about 1506 (Fig. 4). Information about the Americas was beginning to filter in to cartographers, so they completed the global wraparound in this wing-shaped realization of the globe south toward the Antarctic Circle, despite having only scant information about the northern coastline of South America and other southern lands.

Figure 4. The first full conical projection of the globe, by Giovanni Contarini and Francesco Rosselli (~1506, British Library, London).

The next form of global Ptolemaic projection known to have been implemented was Ptolemy's cylindrical projection, generally associated with Gerhard Mercator from his 1569 version of it but actually implemented before he was born in a little-known version by the same Francesco Rosselli from as early as 1508. This is the first complete cylindrical planisphere mapping, with a precise connection of the left and right sides of the map. (Like all such projections, it cannot represent the polar regions without severe distortion, but Rosselli avoids the problem by cropping the projection at about the Arctic and Antarctic Circles. This map explicitly adheres to the cylindrical projection property, an important property for sea navigation, that straight lines are straight trajectories on the globe (even if the distances are distorted). Thus, a navigator can use thus projection to set a heading for a defined target location. Note that this was very early in the exploration of the Americas and Rosselli's very incomplete view of the Americas has only parts of the coastline of South and North America, although he spreads them out to extend across the Pacific adjacent to China. This map is the first formalized exemplar of Ptolemy's cylindrical projection (top row, Fig. 1).

Figure 5. The first complete cylindrical planisphere projection, by Francesco Rosselli (1508, Greenwich Museum, London), truncated at the Artic and Antarctic circles. This map anticipates the familiar Mercator cylindrical projection by more than half a century.

Ptolemy's final form of global projection, the stereographic, was drawn soon afterwards by Vesconte Maggiolo. Although the initial impression is that it appears quite similar to the Contarini-Rosselli conical projection, the polar regions continue around the full circle, and there is no connection between the lands depicted at upper left (vestigial America) and upper right (Central Asia). These features indicate the implicit continuity of the projection around the complete circle, as required for the stereographic projection. This Renaissance map is therefore the first extant implementation of Ptolemy's azimuthal stereographic projection (lower row, Fig. 1).

Figure 6. The first stereographic projection of the globe, by Vesconte Maggiolo (1511, John Carter Brown Library, Providence, RI).

Renaissance Mapping Innovations

The next figure to enter the story of global cartography innovations is Leonardo da Vinci. Although he has an extensive reputation as a cartographer, drawing maps for a wide range of hydro-engineering projects for the rulers of Florence, Milan, Arezzo and the Vatican, amongst others, and all of his acknowledged maps are of local regions where he was working in Italy. In this connection, it is somewhat surprising to encounter a painting associating da Vinci with a map of the globe (Fig. 7). This is a painting attributed to his friend Donato Bramante (the architect of St Peter's Basilica in Rome), showing portraits of the two friends as the Greek philosophers Democritus and Heraclitus on either side of a depiction of the globe. As far as can be determined, this is the first cartographic depiction of the globe in world history (except for some crude Roman era mosaics of the globe in connection with groups of philosophers). As befits a picture, the globe is painted in an optical projection. In this sense, it approximates what would not be called a perspective projection, in which the globe is viewed from a near point in space somewhere above Egypt. However, the scale is not realistic, since the globe surface is overfilled with a Eurocentric range of the Ptolemaic global features. In fact, it quite closely resembles the corresponding inflation in the Behaim Globe of 1492 (Fig. 7, left; the first extant model of the globe of Earth), especially in the eastward distortion of southern Africa. This resemblance suggested that the two authors had access to some depiction of Behaim's Erdapfel globe, 400 miles north in Nuremberg. Following this lead, many artists in the early 16th century used cartographic depictions of the globe in their works.

Figure 7. 'Heraclitus (the Stoic) and Democritus (the Epicurean)' by Bramante (~1490, Pinoteca di Brera, Milan). The globe is in an expanded orthographic, or perspective, projection, with cartography reminiscent of the Erdapfel Globe (right panel).

Despite these cartographic connections, Leonardo da Vinci is not generally acknowledged as authoring a world map spanning the globe, which was the domain of a few specialized cartographers of the era. Nevertheless, there is a world map among his papers in the Royal Library, Windsor (Fig. 8), which is one of the very first maps in history to use the name the AMERICA. As far as can be determined, this map is not mentioned in any of the hundreds of books devoted to the works of Leonardo da Vinci, despite its presence in one of the best-authenticated collections of his works. The Windsor collection. Based on considerations of the known extent of the continents depicted in those years, and the description of the voyages of Amerigo Vespucci disseminated widely in Florence where da Vinci was resident, this world map has been dated to about the year 1505 [1-4].

Figure 8. Da Vinci world map projected onto Reuleaux (spherical) triangles (from the da Vinci papers in the Royal Collection, Windsor). Antarctica and the Artic Ocean are at the centers of the two rosettes.

Da Vinci's projection is the first entirely novel approach to global cartography since the Roman era, and the first to completely span the surface of the globe. Remarkably, he correctly intuits the configuration of the Artic (as an open ocean) and Antarctica (as a small continent), a feat not achieved by any subsequent cartographer about the next 300 years! Moreover, it has a unique cartographic projection onto eight spherical-geometry triangles that provide close to isometric projection throughout the globe. (This non-Euclidean form of triangle was not analyzed again until Reuleaux in 1876 [5]). It is thus a pioneering foray into the field of spherical geometry, constructing triangles whose angles sum to 270° (three right angles) on the sphere, and even more than this in the planar projection. It can be calculated that the distortion of in any part of the map is no greater than 7% in principle, although of course many parts of the map have far greater distortions than this in practice due to the sparse exploration of the Western Hemisphere by this early date.

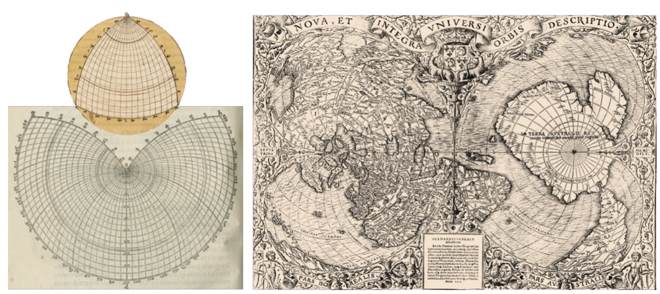

Figure 9. Left: Reuleaux spherical triangle, and it development into the cordiform cartographic metric (from Oronce Fine, 1542). Right: double-cordiform map by Oronce Fine (1531).

Although currently obscure, Leonardo da Vinci's world map evidently had a chain of influence through the centuries, as evidenced by the cartographic developments of Oronce Fine, mathematician and cartographer to the French King François 1er. The king was a devotee of Leonardo da Vinci and brought him to France as the vanguard of Italian Renaissance culture, where he died in 1519. Thus, as the king's confidant in cartographic matters, Fine would have had direct access to da Vinci's papers. Based on the geometric developments in Fig. 9, it may be inferred that Fine took da Vinci's Reuleaux triangle concept and developed it into an integral butterfly or heart-shaped (cordiform) projection, completing into the sophisticated double cordiform projection of his world map of 1531 (Fig. 9, right), perhaps inspired by the twin-circular arrangement of the octants in the da Vinci world map (Fig. 8). It does not seem much of a stretch to see Fine's map as, in turn, the inspiration for the double-hemisphere map of his compatriot, Jean Rotz published in 1542 (Fig. 10), which became the model for this popular double-hemisphere, or globular, format for world cartography for the next three centuries.

Figure 10. The first double-hemisphere stereographic (globular) projection of the known world, published by Jean Rotz in 1542. Note that Rotz is unusually careful not to go beyond the known coastlines in Asian and North American regions , but nevertheless commits to some bizarre speculative reconstructions in the continents of South America and "New Holland" (Australia).

Of course, the price of the da Vinci octant projection is that the world map is no longer projected integral form, which has been an intuitive preference for almost all subsequent cartographic projections. As further evidence of the ingenuity of Francesco Rosselli, his next cartographic innovation is his most well-known map, the oval projection of 1508 (Fig. 11). Remarkably, it is found on the obverse of the very same sheet as his cylindrical projection of Fig. 5. This oval world map is the first in history to integrate the entire surface of the globe into a seamless whole, and this approach has become a de facto standard for global and cosmic mapping since that time.

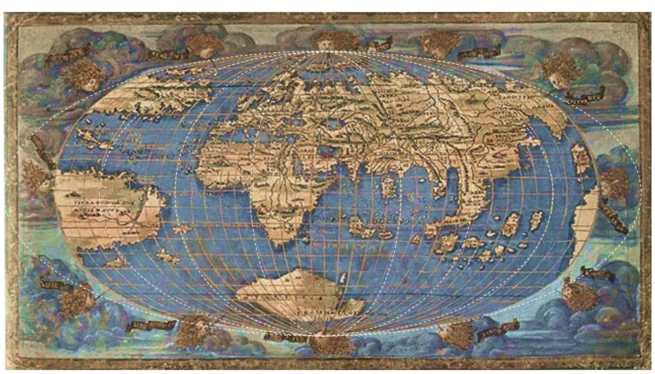

Figure 11. The Rosselli (1508) quasi-elliptical projection of the world. Dashed curves are fiducial ellipses at the 50, 100, 150 and 180° meridians.

Though hand-drawn, the meridians in Rosselli's quasi-elliptical projection are close to elliptical out to 150° from the center line, compromising to more flattened ovals beyond that. Thus, this projection anticipates to a remarkable degree the fully elliptical Mollweide projection of 1805, which is widely used for cosmological mapping of all kinds.

Interestingly, we find the Rosselli projection included as one of da Vinci's sketches of almost all early 16th century cartographic projections (Fig. 12, as extracted by Tyler [1]) on one undated page of his notes (Codex Atlanticus, 521r, Biblioteca Ambrosiana, Milan). It is fascinating to see the mind of Leonardo at work comparing the diverse projections and apparently developing several of his own in the process. His notes indicate that he was in contact with the renowned Florentine cartographer Paolo Toscanelli, so one can imagine that these images were a form of notes following a discussion of global mapping with Toscanelli himself. (These notes also mention storing a world map with his friend Giovanni de Benci before leaving for Milan in 1482.) However, Toscanelli died in that same year, while the pseudo-elliptical projection attributed to Rosselli did not appear until 1508, and the date of this sheet of drawings is unknown. Thus, either da Vinci/Toscanelli developed the concept of the quasi-elliptical projection many years prior to Rosselli, or da Vinci was in later contact with Rosselli (who had a public map store in Florence in this time period). It is hard to imagine that da Vinci, who was a prolific map-maker his whole life, would not have visited such a store and been interested in the array of forms of projection to be found there. Thus, either scenario is highly plausible, and both are compatible with the wide range of dating of the Codex Ambrosiana.

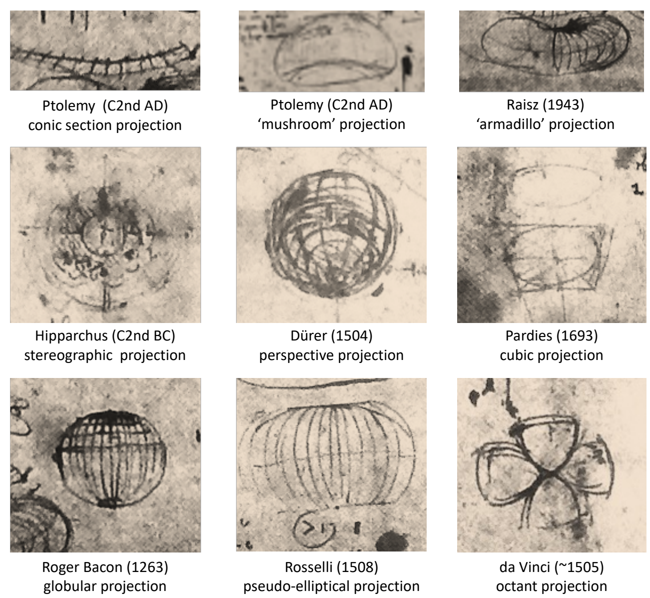

Figure 12. Da Vinci's nine global cartographic mappings on p 521r of the Codex Ambrosiana. (Modified from Tyler [1]).

Da Vinci also sketches a cubic projection that represents a simple way to capture the globe in six stereographic views. This concept was not realized in a full global projection until Pardies (1693, Rumsey Map Center, Stanford), who only used it for a mapping of the mythological sky chart. It still does not seem to have been realized for the terrestrial globe in the form envisaged by da Vinci. There are two different approaches that can be taken to a cubic projection of the globe. Da Vinci's sketch indicates an orthographic projection, in which each side would essentially show a picture of the globe projected as if viewed from its direction. The other approach is to project the globe to the surface of the cube by means of six stereographic projections, one to each face of the cube, as if stretching the globe surface over the cube as an elastic skin. This latter approach may be found among the 70 or so projections listed at ArcMap, or the 77 or so projections listed on the Wikipedia page of map projections (where it is only used for sky projections, however). An example of the cubic projection (Fig. 13) can be found at https://line.17qq.com/articles/godhlhdpv.html. The areal and anisotropic distortions are limited to the corner regions of each face. Note that, as a flat map, it does not connect simply in the elevation (vertical) direction, as each polar face would have to be replicated four times to connect to each of the equatorial faces, as in its 3D connectivity. This would bring inappropriate adjacency of the replicates to each other, however, an issue that is avoided by splitting the north and south polar faces each into four triangles.

Figure 13. Stereographic cubic projection of the globe, with the polar faces subdivided into triangles (from https://line.17qq.com/articles/godhlhdpv.html).

Mapping of Quantal Resonance Manifolds

Dating back to Pythagoras, the ancient Greek philosophers were concerned with the three-dimensional geometry of regular solids, which are known as the Platonic solids but were developed centuries before his time. This interest was resuscitated in the Renaissance by Piero della Francesca (see Tyler [6]), and became of great interest to Leonardo da Vinci, who collaborated in realizing them, and many other semi-regular solids, as both solid and skeleton-figure illustrations (Fig. 14). The important aspect is that the constraint of regularity, in that all lengths of the edges and all angles between them are identical, limits the configurations to just five polyhedrons, as opposed to the infinity variety of polyhedrons that can be generated without this constraint.

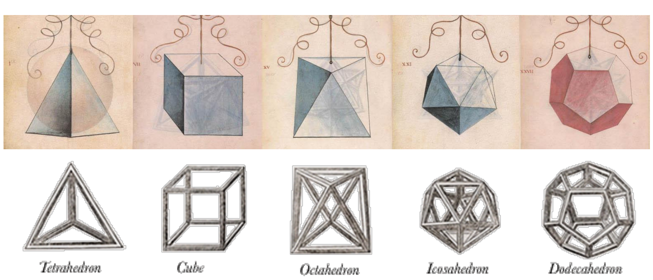

Figure 14. Leonardo's solid and 'woodframe' drawings of the five regular polyhedral solids (from https://keplersdiscovery.com/harmonicesMundi.html).

This quantization of structure illustrates the kind of idea the Pythagoras may have had in mind when enunciated that "the universe is governed by number". Here, the number is defined by the inherent symmetries of the spherical space inhabited by regular polyhedrons. While there are an infinite number of regular polygons in two dimensions, there are only five possible regular figures when the dimensionality is increased to three. The geometry thus enforces a form of whole-number quantization that anticipates the logic of Quantum Mechanics.

The heart of quantum mechanics is the Schrödinger Wave Equation, which takes the form of spherical harmonics for the system of orbitals of the hydrogen atom, conceptualized as consisting of the combination of an electron and a proton. In terms of the wave equation, this means that the atom is equivalent to a three-dimensional standing wave resonance at some ultra-high oscillation frequency. Just as standing waves in a tube or on a string form various one-dimensional oscillation patterns, or harmonics, and two-dimensional standing waves on a drum or a plate form a variety of two-dimensional patterns, known as Chladni patterns, so do three-dimensional oscillations around the sphere form a discrete set of three-dimensional wave patterns, or harmonics, as visualized as two-dimensional projections in Fig. 15. Each solution is the product of a spherical harmonic function and a radial harmonic function. The orthogonal property of the spherical harmonics is the basis for the Pauli exclusion principle of the unique occupancy of each orbital.

Each harmonic solution is characterized by three numbers that control the pattern of the resonance in a complex manner depending on its energy level. The intensity scale in Fig. 15 is usually considered to represent the probability of finding the electron at any point in the spherical volume around the location of the atom, but it can also be considered to represent the continuous energy distribution of a given solution of the Schrödinger equation. It is the discreteness of these standing wave solutions that forms the primary source of the quantization on Quantum Mechanics. These discrete resonance structures are the atomic equivalent of the mathematical structure of the regular polyhedrons of Fig. 14.

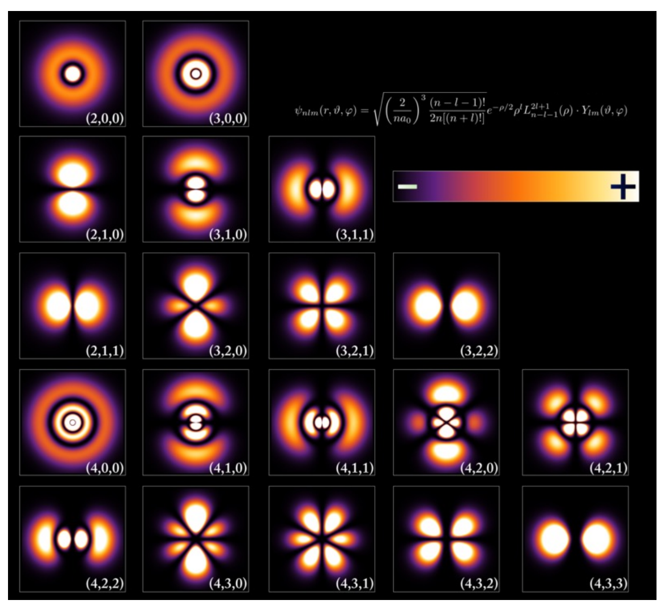

Figure 15. Wave functions of the hydrogen atom at different energy levels. The brighter areas represent a higher probability of interactions of the atom with incident energy. The solutions to the hydrogen atom Schrödinger equation are products of a spherical harmonic functions and a radial harmonic function.

Modern Uses of the Elliptical Projection

The Mollweide projection is the first widely recognized form of elliptical projection, although a number of modified ones have been developed. The modern globe in this projection is shown in Fig. 16, to give a sense of its advantages and drawbacks. The advantages are that it captures the entire globe in a unified projection, gives a sense of the sphericity of the globe, and completely avoids the extreme ***area*** distortions of the cylindrical projection towards the poles. However, despite the equal-area advantage, the elliptical projection still introduces severely anisotropic ***shape*** distortions near the poles, so the relative distances in various directions are correspondingly distorted. There is also the obvious fact that the left and right sides do not connect, and cannot even be connected by repetition of the map in adjacent positions. The only projections that have this property are cylindrical projections such as Rosselli's of Fig. 5, which would connect only in the meridional direction, however.

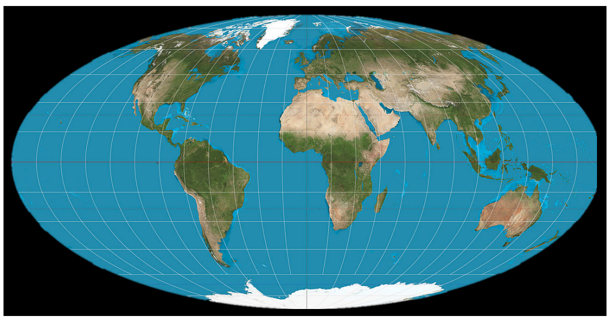

Figure 16. The globe of Earth mapped in the elliptical projection. Note the severe anisotropic distortions near the poles.

As mentioned, all global projections to the flat plane have to make substantial compromises in one form or another. In cosmological mapping, one of the preferred compromises seem to be those of the elliptical projection, as exemplified by the following cosmological examples.

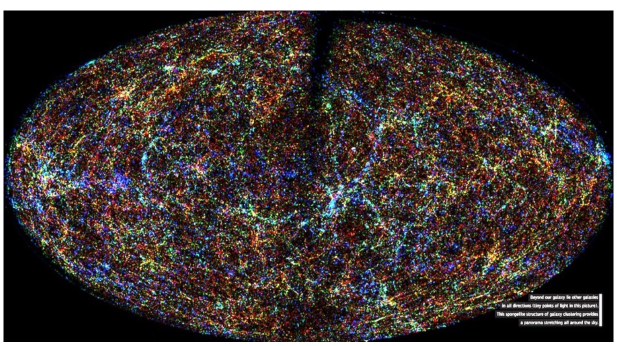

Figure 17. Whole-sky map of the galaxies from the viewpoint of Earth, illustrating the filigree threads of galaxy clustering. The dark center line is our own galaxy, the Milky Way (from NASA / T. Jarrett).

Fig. 17 is a whole-sky map of the galaxies visible from Earth, revealing the network of chains that form the large-scale structure of the Universe, as opposed to the purely random distribution that might be expected if these centers of local energy were generated by some purely random process throughout the volume of space. The distribution depicted in Fig. 18 is a greatly enhanced mapping of the microwave cosmic background radiation, the echo of the Big Bang across the Universe. The intuitive appeal of this elliptical projection is that it approximates the scope of the human visual field, which is roughly 180° in the horizontal direction and 90° on the vertical, giving the sense of the broad sweep of space around us.

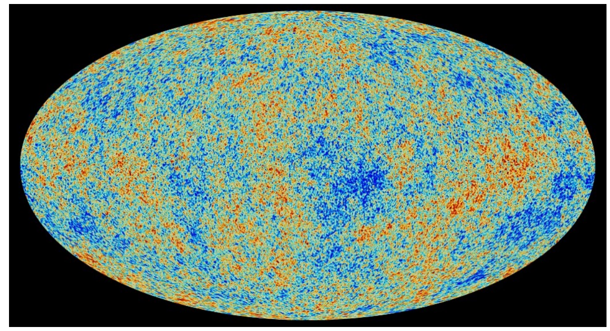

Figure 18. Whole-sky map of the cosmic microwave background radiation resounding from the Big Bang, with the density variations scaled up by a factor of 100,000 (from ESA and the Planck Collaboration).

Spherical harmonics, which are part of the specifications of the orbitals of quantum theory, may be used for the representation of any spherical mapping, such as whole-sky astrophysical surveys. One example of such surveys is the mapping of the gravitational waves predicted by Einstein's theory of General Relativity. This very recent technology is just beginning to show detectable signals. The sensitivity profile of this technique above which a signal has to appear is shown by the spherical harmonic reconstruction in Fig. 19, while a major gravitational-wave event to be detected is depicted in Fig. 20, attributed to the cataclysmic coalescence of two black holes to form one giant black hole.

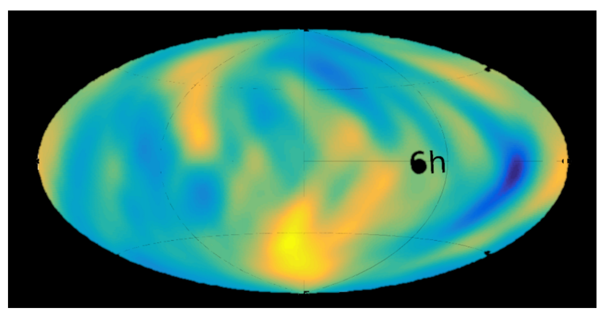

Figure 19. Spherical wave representation of the whole-sky limits of the LIGO gravitational wave sensitivity. (LIGO and Virgo Scientific Collaborations, arXiv:1612.02030, 2016.)

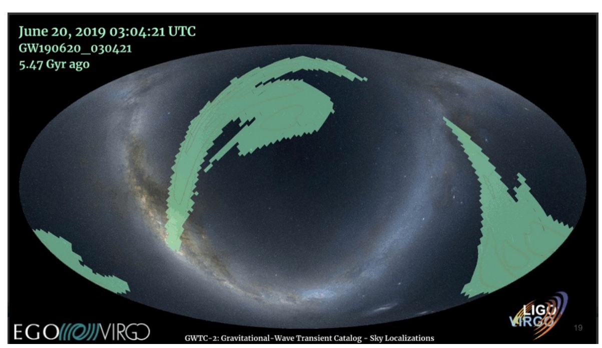

Figure 20. Elliptical projection of the whole-sky gravitational wave detection map on 20 June, 2019, attributed to the coalescence of two black holes to form one. The aqua gravitational wave energy is overlaid on a sky map featuring the crescent shape of our home galaxy at this projection angle. (LIGO and Virgo Scientific Collaborations.)

Mapping the Human Brain

Leonardo da Vinci, of course, is famous for his anatomical studies of the human body, including his cutaway drawing of the human skull, a format that he invented (Fig. 21, left). What is far less known is his drawing of the human brain. While skulls are commonplace in Christian art through the centuries, depictions of the brain are not, and this faint diagram seems to be the first depiction of a human brain in history (Fig. 21, right). Notice how it captures the characteristic folding pattern of the gyral ridges and sulcal creases.

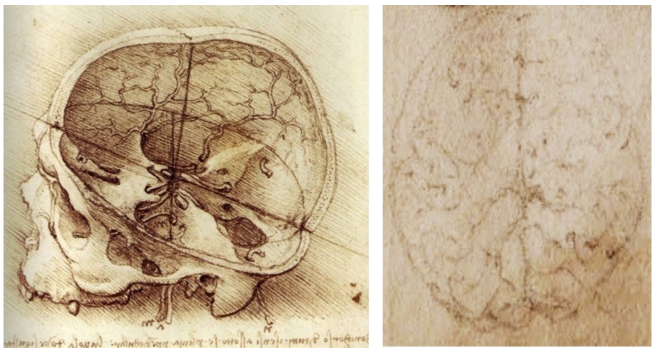

Figure 21. Drawings of the human cranium by Leonardo da Vinci Left: Human skull sectioned (~1489). Right: Dorsal view of the human brain (~1508). Royal Collection, Windsor.

In the modern era, brain mapping has progressed far beyond dissection and drawing to an array of sophisticated mapping techniques. A primary principle of brain organization is a neural form of projection in which the local activation over one 2D surface is mapped with approximate point-by-point correspondence to a 2D surface somewhere else in the nervous system. The prevalence of such 2D surface mapping in the brain is indicative of the importance of maintaining the adjacency relationships of the information entering the eyes for further processing throughout the brain.

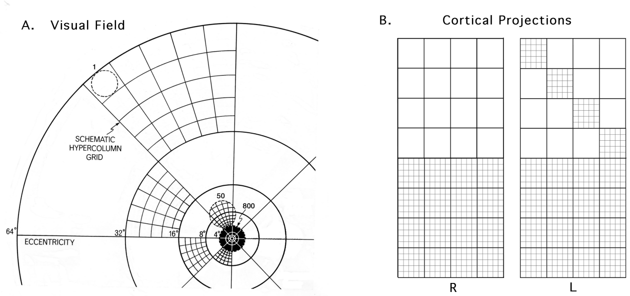

Figure 22. Diagrammatic model of the log polar mapping in human from retina to the right (R) and left (L) hemisphere maps of primary visual cortex (from [8]).

A canonical example of such mapping is the way the surface of the retina, receiving the image of the world projection optically through the pupil of the eye, is mapped to the primary visual cortex. This is not a linear mapping, however, but an approximately log polar mapping, such that the central region (the fovea) maps to a much larger area of cortex than the peripheral regions, which have proportionately compressed mapping the farther they are from the fovea [7,8], as shown in the diagrammatic encapsulation of the retinotopic mapping in Fig. 22. The small squares represent the smallest unit of cortical space that maps all 8 dimensions of local feature coding: contrast, width, length, orientation, color (2), motion, and binocular disparity, within a notional subunit of cortex known as a hypercolumn. Each hypercolumn square occupies about 4 sq mm through this cortical map.

Fig. 23 illustrates the basic methodology of retinotopic mapping by functional Magnetic Resonance Imaging (fMRI), the mapping of the locations of retinal activation to the multiple maps of the visual cortex at the back of the brain. The stimulus (center inset in Fig. 23A) is a rotating wedge of high-contrast patterns centered on a polar grid that is fixated by the person being scanned. A key issue is to have this grid present during the scan provide for optimally stable eye fixation, since the ~1 minute rotation of the wedge tends to cause eye movements away from the center of the field that distort the accuracy of the map.

The upper image in Fig. 23A is a partially-inflated computer-generated anatomical view of the two hemispheres of the cortical surface of the brain. This inflation procedure opens up the tight sulci visible in da Vinci's drawing to allow visualization of the cortex surface extending down into the sulci. Because the back of the brain has the form of a sharp ellipsoid, the next procedure is to flatten sections centered on the occipital pole (corresponding to Ptolemy's stereographic projection of the globe), to provide a minimally-distorted but fully-connected view of the visual projection areas. The boundaries of the retinotopic projection areas are then calculated from the pattern of activation induced by the rotating wedge to show that the primary visual cortex (V1) mapping is roughly triangular in each hemisphere of the brain (lower image in Fig. 23A), but the secondary and tertiary mappings are approximately rectangular, and split into lower and upper representations of the visual field above and below the V1 triangle). The best-fitting analytic function for human V1 is not a strict log polar mapping but a double-sech function developed by Schira et al. [9], as depicted in Fig. 23 B.

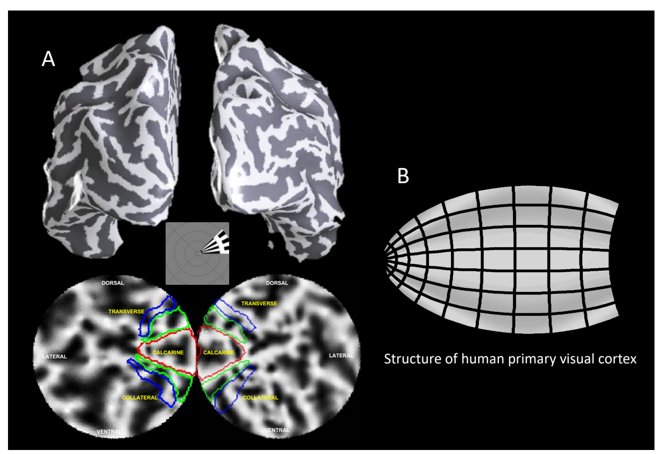

Figure 23. A. Upper portion: partially inflated cortical hemispheres of the human brain. Shading indicates gyral ridges (light) and sulcal valleys (dark) Center icon: rotating wedge stimulus used to determine the mapping from the retina. Lower portion: flattened regions of the occipital lobe showing the orientation labels (white), the names of principal sulci (yellow), and the mappings of the first three retinotopic projection areas (red: V1, green: V2; blue: V3). B. Analytic structure of each hemisphere of human V1 (from [9]).

The second dimension of the cortical mapping is the eccentricity, from fovea to periphery, which has evidently evolved to provide a smooth gradation from ultra-high resolution at the point of fixation, which would require of the order of 200 million points of information if uniform across the visual field, to a low resolution in the far periphery (see Fig. 22), providing a compression to only about 1 million fibers carrying the information up the optic nerve to the cortex. The key to employing this foveated structure is the eye movements that carry the high-resolution fovea to any point of interest detected as some point in the lower resolution regions of the periphery.

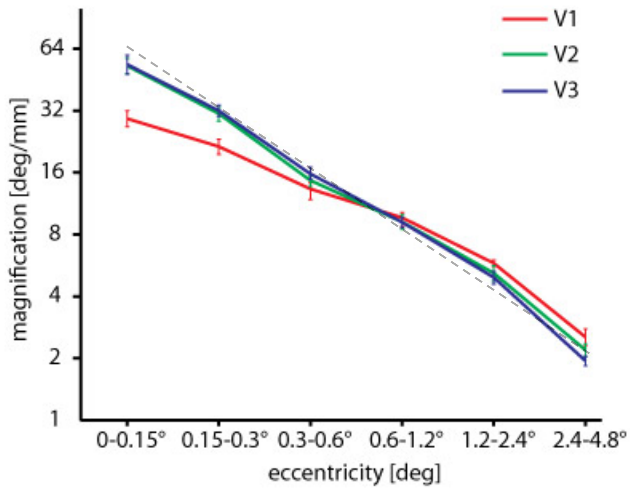

Figure 24. The magnification of the cortical mapping as a function of eccentricity in the first three retinotopic maps (from Schira et al., [10]). Dashed line is the theoretical log polar relationship.

This organization of the retina-to-cortex mapping raises the question of whether the mapping is the same or different in each cortical map. In human, this issue was first addressed by Schira et al. [10], who used the fMRI retinotopy approach to make a fine-grain comparison of the eccentricity maps in the first three retinotopic maps for the broad central region of the retina. The log polar mapping concept of Fig. 22 would imply that the cortical magnification would fall as an inverse linear function on the double-log coordinates of Fig. 24, which is well approximated by the secondary areas (V2 and V3). Contrary to expectation however, the magnification is notably shallower in the foveal region of V1, implying that less cortical area is devoted to the primary processing of the fixated region of the visual field than to its higher-level processing. (The fall-off of the V1 function from full log-polar reciprocity shown by the red curve in Fig. 24 may be attributed to the fact that the optical quality of the retinal image would not support a magnification of 64x at the foveal center; the image would be too blurred out to carry meaningful information.) The smaller magnification at the level of V1, however, can have more secondary cortex devoted to it to process the higher-order features, such as figural properties not available from V1 processing.

The retinotopic concept can be extended by similar fMRI studies to many more mappings beyond the first three. One example of such extended mapping is the canonical scheme shown in Fig. 25 from Tyler et al. [11]. The simple progression of retinotopic areas breaks up into subfields at the level of V3A/B in the dorsal direction and hV4/VOF ventral direction.

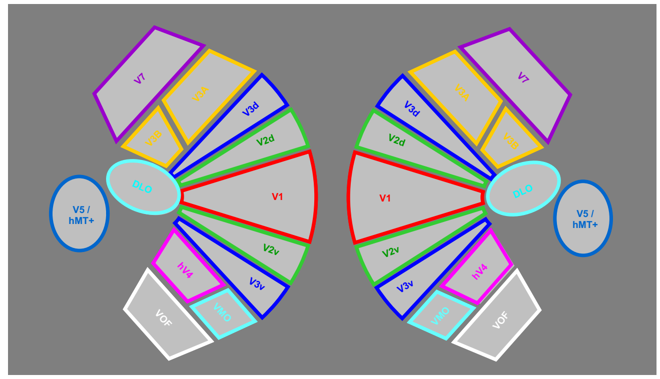

Figure 25. A canonical scheme for the retinotopic mapping in the human brain (from Tyler et al., 2005). Abbreviations are: V# - hierarchy of visual retinotopic maps; hMT+ - human middle temporal map; DLO – dorsolateral occipital map; VMO – ventromedial occipital map; VOF – Ventral occipital foveal map.

Functionally, evolutionary constraints suggest that each retinotopic mapping should play a different role in the processing of visual information arriving at the retina, but it is the subject of ongoing research to determine those distinct roles. One of the first such specializations to be identified was the processing of motion (Zeki[12]), which is the particular specialty of area V5 (whose human counterpart is known for complex reasons as hMT+). This fan of visual representation areas in Fig. 25 further emphasizes of the importance of mapping, which maintains the adjacency relationships of the information entering the eyes for further processing throughout the brain. If a more abstract principle were more relevant, the representations of the visual hierarchy would have evolved to splay out into a non-retinotopic arrangement (which they do to some extent, such as the specialized stripes in V2), rather than maintaining the retinotopic organization through so many levels.

Conclusion

In drawing parallels across the domains of global cartography, sky mapping, subatomic particle theory and the neuroscience of brain mapping, this analysis shows that many of the same principles of spherical mapping apply across these fields of science, although each has its unique aspects. This historical survey also identifies evidence for a number of little-known firsts in the history of global mapping geometry (excluding verbal descriptions):

First pictorial projection – Bramante & da Vinci (~1490), not Dürer (1515)
First accurate concept of polar regions – da Vinci (~1505)
First map of entire globe – da Vinci (~1505), not Rosselli (1508)
First minimally-distorted projection of the sphere – da Vinci (~1505)
First depiction of the human brain – da Vinci (~1508),
First cylindrical projection – Rosselli (1508), not Mercator (1569)
First full conical projection – Contarini-Rosselli (1506-8)
First stereographic projection – Maggiolo (1511), not Vespuche (1524)
First double-cordiform projection – Fine (1531)
First double-hemisphere stereographic projection – Rotz (1542)

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Christopher Tyler received his BA in psychology from the University of Leicester(1966), his Masters in applied psychology from The University of Aston (1967) and his PhD and DSc in neurocommunication from the University of Keele (1970 and 2004). Since 1970 he has pursued research in visual neuroscience at Northeastern University, University of Bristol, Bell Laboratories, Smith-Kettlewell Eye Research Institute, and City University of London, together with studies in Renaissance art history and cartography.