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Fast Prediction of Contrast Detection Probability

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Abstract

Contrast detection probability (CDP) is proposed as an IEEE P2020 metric to predict camera performance intended for computer vision tasks for autonomous vehicles. Its calculation involves comparing combinations of pixel values between imaged patches. Computation of CDP for all meaningful combinations of m patches involves approximately 3/2(m^2-m).n^4 operations, where n is the length of one side of the patch in pixels. This work presents a method to estimate Weber contrast based CDP based on individual patch statistics and thus reduces to computation to approximately 4n^2m calculations. For 180 patches of 10×10 pixels this is a reduction of approximately 6500 times and for 180 25×25 pixel patches, approximately 41000. The absolute error in the estimated CDP is less than 0.04 or 5% where the noise is well described by Gaussian statistics.

Results are compared for simulated patches between the full calculation and the fast estimate. Basing the estimate of CDP on individual patch statistics, rather than by a pixel-to-pixel comparison facilitates the prediction of CDP values from a physical model of exposure and camera conditions. This allows Weber CDP behavior to be investigated for a wide variety of conditions and leads to the discovery that, for the case where contrast is increased by decreasing the tone value of one patch and therefore increasing noise as contrast increases, there exists a maxima which yields identical Weber CDP values for patches of different nominal contrast. This means Weber CDP is predicting the same detection performance for patches of different contrast.

Introduction

It is reasonable to expect that cameras will continue to be the dominant sensor in autonomous vehicle solutions for the foreseeable future. The design of cameras and their subsequent optimization is complex, requiring balancing of numerous headline parameters while simultaneously fulfilling requirements of the operational design domain (ODD) for the vehicle. As such performance metrics are required that can predict camera capabilities to aid with the design process. Ideally, these performance metrics should work over a wide range of conditions and, in the case of autonomous vehicles, correlate with detection performance. Automotive imaging is one of the first large scale deployments of consumer imaging to the general public where safety is critical.

The IEEE P2020 Image Quality for Automotive Vehicles Working Group was set up to adapt existing or create metrics to assess image quality for automotive use cases [1]. Contrast detection probability (CDP) is an empirical metric proposed by Geese et al. [2] as an IEEE P2020 metric to predict computer vision performance for autonomous vehicles. It is based on the premise that it is the ability of an imaging system to record contrast between a target and background and its interaction with noise that predominantly determines the ability to detect objects. By examining a distribution of contrasts, CDP estimates the spread of contrast due to noise in the system and calculates the probability that measured contrasts will fall within given bounds [2]. It is suggested by Geese et al. that the bounds may be set according to the application and desired level of visibility [2]. Further details on CDP are given in the following sections.

Calculation of CDP is computationally expensive requiring comparison of large numbers of pixel values between imaged swatches to reduce error in the result. For m swatches, of n×n pixels, approximately 3/2(m^2-m).n^4 operations are needed for all meaningful combinations. Note the n^4 multiplied by the m^2 in the result. The author develops a method for fast estimation of Weber contrast CDP based on the statistics of individual swatches and is able to reduce the approximate number of operations to 4n^2m. This n^2.m order of magnitude represents a speed up of between 6500× to 41000× for swatches of 10 to 20 pixels on a side. This fast estimation facilitates more thorough exploration of the behavior of CDP across a wide range of conditions, both mathematically and physically.

Through simulation, errors in the estimation are examined and seen to be reasonable, less than 0.04 CDP, unless narrow bounds or low numbers, approximately 7 or less, of signal quanta are used.

Contrast Detection Probability

Geese *et al.* define CDP as [2]:

$$CDP_{K_{IN}} = P\big(K_{IN}(1-\varepsilon) \leq K_M \leq K_{IN}(1+\varepsilon)\big) \tag{1}$$

where, K_{IN}, is input contrast, K_M, measured contrast, ε, contrast bounds and *P()* probability. CDP is the probability that the contrast of two randomly selected pixels will fall between given bounds. Weber, Michelson, or a simple difference may be used to calculate contrast [2]. Geese *et al.* suggest the use of Weber contrast, K_W, to perform the calculation, defined below:

$$K_W = \frac{E_{MAX}}{E_{MIN}} - 1 \tag{2}$$

where E_{MAX} and E_{MIN} represent the maximum and minimum signal respectively. Weber contrast is used throughout this paper.

Practical calculation of CDP has been investigated by Ebbert [3]. Jenkin [4] and Artmann *et al.*[5] have written on the calculation of CDP and interpretation of results. Two uniform tone patches, representing the brightest and darkest components of a desired contrast level, are recorded in chosen illumination conditions by the imaging system under analysis. The patches should be large enough that a reasonable statistical sampling of the noise processes of the imaging system are captured. Typically, 10×10 pixels in the final image for each is sufficient. After transformation of the patch data into linear input units via the system tone curve, calculation proceeds by evaluating the contrast of every pixel combination between the two patches to estimate a distribution of contrasts, Figure 1. CDP for the contrast, illumination and system parameters used is then yielded by calculating the proportion of the distribution within the given limits. This procedure may then be repeated to calculate CDP values for different illumination and contrast combinations. Geese *et al.* suggest that a bounds of 50% on the nominal input contrast is a good

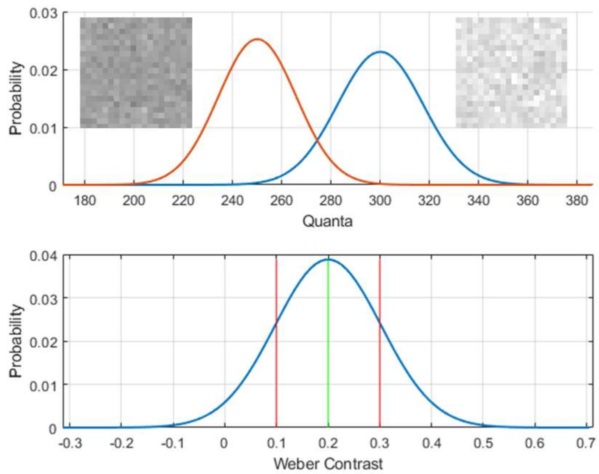

Figure 1, Distribution of Weber contrasts as calculated from recording bright and dark image patches and comparing each combination of pixels.

indicator for a threshold of visibility and detectability [2], though this has not been established with psychovisual calibration and may well change with display and illumination level. As noise increases, the probability that two pixels will yield a contrast level within the desired bounds decreases and conversely, as noise improves, so the probability of correctly recording the contrast also rises. Geese *et al.* suggest that the output from CDP may be correlated to the performance of specific imaging tasks [2] and Jenkin has previously compared CDP to results from detectivity [4].

Number of Calculations

Calculation of CDP involves the comparison of every pixel combination between two recorded swatches to yield a single measurement. For all meaningful combinations of m patches to yield a CDP surface as detailed by Artmann [5], the number of calculations may be estimated as follows. For a square patch of n×n there are n^2 pixels and to compare every pixel combination between two patches there are n^4 combinations. Weber contrast, as per Equation 2 is calculated for every combination. This involves one divide and one subtraction and yields $2n^4$ calculations. Additionally, n^4 additions are required to sum the results to yield the cumulative distribution function (CDF) from which the CDP value may be yielded to give a total of approximately $3n^4$ operations for a single pair of swatches.

To compare m swatches, duplicate and self-combinations are eliminated. Swatch A compared to B is the same as B compared to A for CDP calculation. Also comparing swatch A or B etc. to itself is meaningless. Thus, this yields $(m^2\text{-}m)/2$ meaningful swatch combinations. Multiplying the number of operations for a single swatch pair by the number of meaningful combinations, the total computational load, O, for m, n×n pixel patches is:

$$O = \frac{3}{2}(m^2 - m).n^4 \tag{3}$$

The n^4 and m^2 terms in Equation 3 cause the number of operations to grow quickly. For 180 swatches of 10×10 pixels, which are reasonable parameters to yield a CDP surface, approximately 483 million operations are needed. For 180, 25×25 pixel swatches, this grows to 18.9 billion. Given these results, it may be seen that meaningful investigation of performance across a wide range of illumination and camera parameters involves a large number of operations and provides the motivation to seek an efficient estimation method.

Development of Estimation Method

To develop a fast estimation method for Weber based CDP, we first note that calculating Weber contrast for every pixel combination, Equation 2, to yield the contrast distribution, we are essentially calculating the ratio distribution for a pair of swatches. E_{MAX} and E_{MIN} are replaced by the distributions of the brighter and darker swatches respectively in Equation 2. Assuming the noise of individual swatches may be approximated by Gaussian rather than Poisson statistics for sufficiently large means, estimating the variance of the ratio distribution will yield a fast method to estimate the contrast distribution, and further the cumulative distribution function (CDF) of the contrast.

The ratio distribution for independent Gaussian variables with zero means is a Cauchy distribution [6]. Our distributions, however, have non-zero means, which significantly complicates the result. Hinkley formulated a general result for the ratio of two correlated normal variables, $f(x)$, [7], below:

$$f(x) = \frac{b(x)d(x)}{\sqrt{2\pi}\sigma_1\sigma_2 a^3(x)}\left[\Phi\left\{\frac{b(x)}{\sqrt{1-\rho^2}a(x)}\right\} - \Phi\left\{-\frac{b(x)}{\sqrt{1-\rho^2}a(x)}\right\}\right] + \frac{\sqrt{1-\rho^2}}{\pi\sigma_1\sigma_2 a^2(x)} exp\left\{\frac{c}{2(1-\rho^2)}\right\} \tag{4}$$

where,

$$a(x) = \left(\frac{x^2}{\sigma_1^2} - \frac{2\rho x}{\sigma_1\sigma_2} + \frac{1}{\sigma_2^2}\right)^{\frac{1}{2}}$$

$$b(x) = \frac{\theta_1 x}{\sigma_1^2} - \frac{\rho(\theta_1 + \theta_2 x)}{\sigma_1\sigma_2} + \frac{\theta_2}{\sigma_2^2}$$

$$c = \frac{\theta_1^2}{\sigma_1^2} - \frac{2\rho\theta_1\theta_2}{\sigma_1\sigma_2} + \frac{\theta_1^2}{\theta_2^2}$$

$$d(x) = exp\left\{\frac{b^2(x) - ca^2(x)}{2(1-\rho^2)a^2(x)}\right\}$$

and,

$$\Phi(y) = \int_{-\infty}^{y} \phi(u)du$$

where,

$$\phi(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}.$$

The standard deviations of the variables are σ_1 and σ_2, the means Θ_1 and Θ_2 and the covariance, ρ. The result is not readily accessible, relies on a covariance term and is generalized for correlated normal distributions. By setting the variance of each distribution equal to the mean and the covariance term to zero, such that $\Theta_1=\sigma_1^2=\lambda$, $\Theta_2=\sigma_2^2=\mu$ and $\rho=0$ Griffin used the above to yield an approximation for the mean and variance of the ratio distribution of independent normal random variables[8], E_R and V_R respectively:

$$E_R = \frac{\lambda}{\gamma} \tag{5}$$

$$V_R = \frac{\lambda}{\gamma^2} + \frac{\lambda^2}{\gamma^3} \tag{6}$$

where,

$$\gamma = \frac{\mu}{1-e^{-\mu}}.$$

In summary, providing noise behavior is close to Poissonian and given sufficiently large signal means, the mean and variance of the ratio distribution may be estimated from the variance of the individual swatches using Equation 6. The mean of the ratio distribution is corrected to reflect the Weber contrast in Equation 2 by subtracting a value of one from the result:

$$E_R' = \frac{\lambda}{\gamma} - 1 \tag{7}$$

To calculate CDP, the CDF of the contrast distribution must be calculated, and the difference taken between the bounds. The Gaussian or normal distribution, Equation 8, has no analytical solution for its integral [9] as it is based on an error function.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{8}$$

Vazquez-Leal et al. provide a simple analytical approximation of the CDF of the standard normal distribution, *C(x),* [9]:

$$C(x) \approx \left[exp\left(-\frac{358x}{23} + 111\tan^{-1}\left(\frac{37x}{294}\right)\right) + 1\right]^{-1} \tag{9}$$

for $-\infty \leq x \leq \infty$.

The value of x in Equation 9 must be corrected to reflect the actual mean and standard deviation of the CDF of Weber contrast distribution, below:

$$x' = \frac{x - E_R'}{\sqrt{V_R}} \tag{10}$$

The value of CDP may then be estimated by calculating C(x') at the upper and lower contrast bounds as given in Equation 1. As the fast estimation technique only relies on knowing the mean and variance of each input swatch, it is relatively simple to predict CDP by using a mathematical model to estimate photons at the imaging plane for different lighting, lens and sensor geometries.

Number of Calculations for Prediction

As previously, the number of operations for the prediction may be estimated. Standard deviation, S, for a swatch is calculated as:

$$S = \sqrt{\frac{\sum|\bar{p}-p|^2}{N}} \tag{11}$$

where $\bar{p}$ is the mean value of the swatch, p, individual pixel values and N, the number of pixels. The calculation of the mean for a swatch is n^2 operations, the difference between the mean and each pixel, another n^2 operations, as is the square and the summation to yield a total of $4n^2$ operations per swatch. For m swatches this becomes $4n^2m$. This is the vast bulk of the computational operations. There are perhaps a few dozen operations necessary to yield the CDP result for each meaningful combination, but this component becomes insignificant at the size of swatches that are needed to reduce the error in the result to a reasonable amount. For 180, 10×10 pixel swatches, the approximate number of operations is now 72k or a 6500x decrease, and for 180, 25×25 pixels, 450k or a 41000x decrease.

Simulation and Results

Swatches of 64×64 pixels were created and Poisson noise added to represent 100% Weber contrast at different linear full wells. CDP is then calculated as per Geese *et al.* and the proposed method for varying bounds. Figures 2(a), (b) and (c) illustrate the result comparing 500 and 250 quanta swatches with CDP bounds from 1 to 200%. As may be seen, the error in the result is low and remains below 2% until contrast bounds are reduced to approximately 20% of nominal input, Figure 2(c). Some of the difference should be attributed to the error in the estimation of the result using the original CDP calculation method as this itself is subject to estimation error due to the finite numbers of pixels used to calculate the result.

Reducing the patch quanta to 50 and 25 respectively, Figures 3(a), (b) and (c), the percentage error increases to under 5% for a large proportion of the range and a slight positive bias is introduced at wider bounds. This turns towards negative at lower bounds. Given the increase in speed of the calculation, the trade off in accuracy is modest. Again, some of the difference should be attributed to the calculation of CDP itself, rather than the fast prediction.

At patch levels of 5 and 2.5 quanta, Figures 4(a), (b) and (c) the difference between the fast prediction and the CDP result becomes 20% for majority of the range. This could be attributed to a number of things. As the Poisson distribution is left-clipped, there is an increasingly deviation of the actual distribution from the Gaussian which has support from -∞ to ∞. There is additionally increased quantization of results due to the low number of quanta. Again, positive bias is seen at higher bounds which turns negative as bounds become tighter.

To illustrate investigation of performance bounds using the fast prediction, results were calculated using a contrast sweep, Figure 5. A dark swatch is held at 7 quanta and a bright patch swept from 7 to 350 quanta. Weber contrast and CDP were estimated for each combination for both 25% and 50% bounds. As may be seen, the results track closely and there is a slight positive bias in the fast-estimated result because of the dark patch that is held a 7 quanta.

Holding a bright patch at 350 quanta and sweeping a dark patch from 350 to 7 quanta, Figure 6, results may be seen to more closely match until higher contrasts are reached where the dark patch is approaching the low numbers of quanta again. What may also be seen is the maxima in each of the calculated curves at approximately 200% contrast. In this instance CDP reduces as contrast increases because relative noise in darker patch is increasing more rapidly than the contrast. This yields CDP values that are identical for differing contrasts using the same bounds. The fast prediction results also display this characteristic indicating that it is not artifact of the discreet nature of the Poisson distribution. Jenkin has previously compared the performance of CDP to detection theory [4].

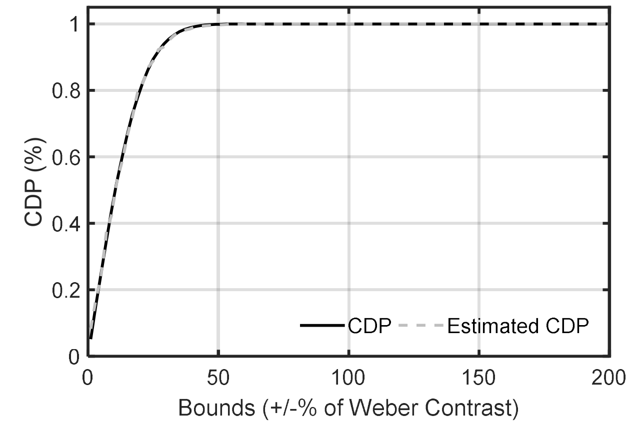

Figure 2(a) Weber CDP and fast estimation of the same for patches of 500 versus 250 quanta with 1% to 200% nominal contrast bounds.

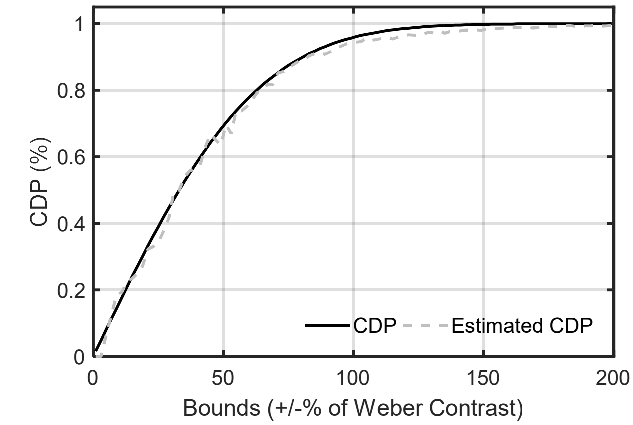

Figure 3(a) Weber CDP and fast estimation of the same for patches of 50 versus 25 quanta with 1% to 200% nominal contrast bounds.

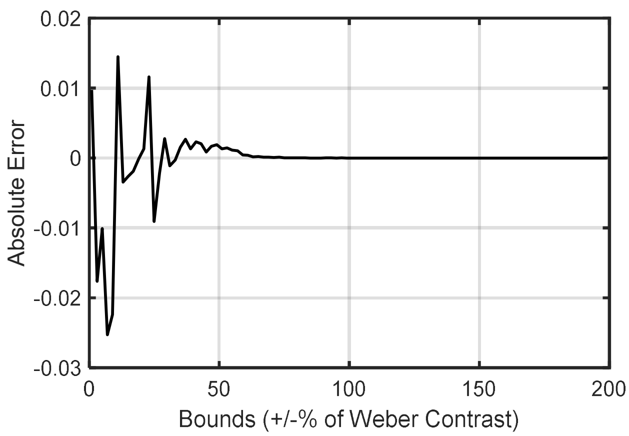

Figure 2(b) Absolute error of Weber CDP versus fast estimation of the same for patches of 500 versus 250 quanta with 1% to 200% nominal contrast bounds.

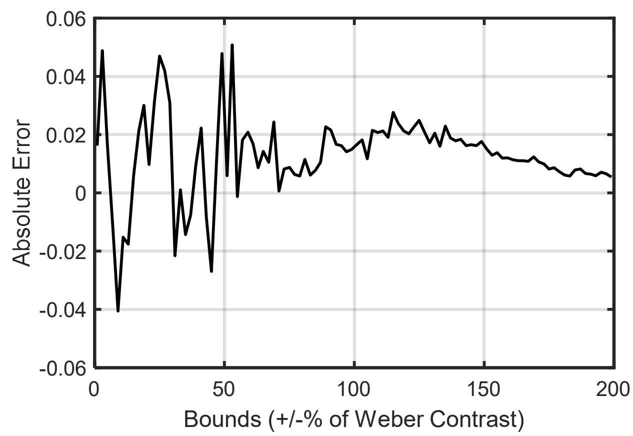

Figure 3(b) Absolute error of Weber CDP versus fast estimation of the same for patches of 50 versus 25 quanta with 1% to 200% nominal contrast bounds.

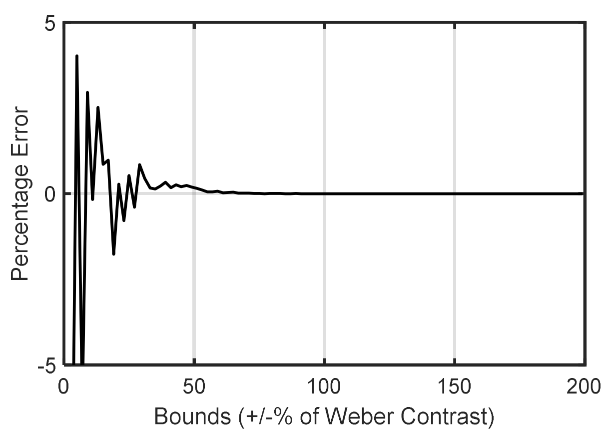

Figure 2© Percentage error of Weber CDP versus fast estimation of the same for patches of 500 versus 250 quanta with 1% to 200% nominal contrast bounds.

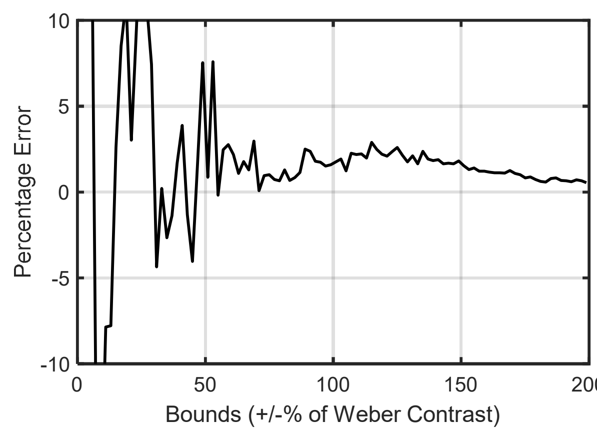

Figure 3(c) Percentage error of Weber CDP versus fast estimation of the same for patches of 50 versus 25 quanta with 1% to 200% nominal contrast bounds.

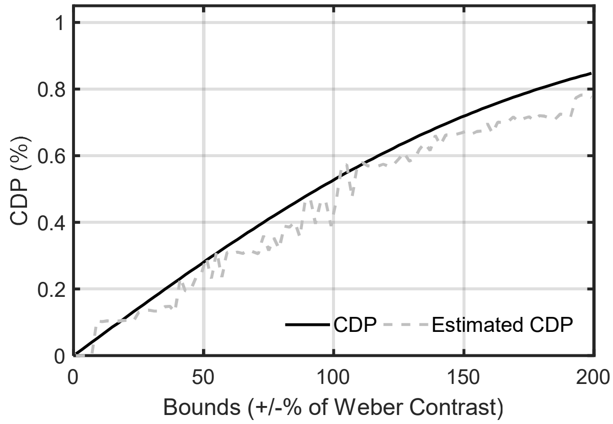

Figure 4(a) Weber CDP and fast estimation of the same for patches of 5 versus 2.5 quanta with 1% to 200% nominal contrast bounds.

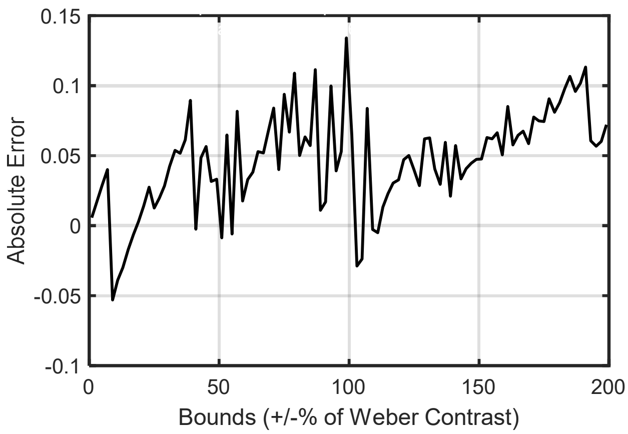

Figure 4(b) Absolute error of Weber CDP versus fast estimation of the same for patches of 5 versus 2.5 quanta with 1% to 200% nominal contrast bounds.

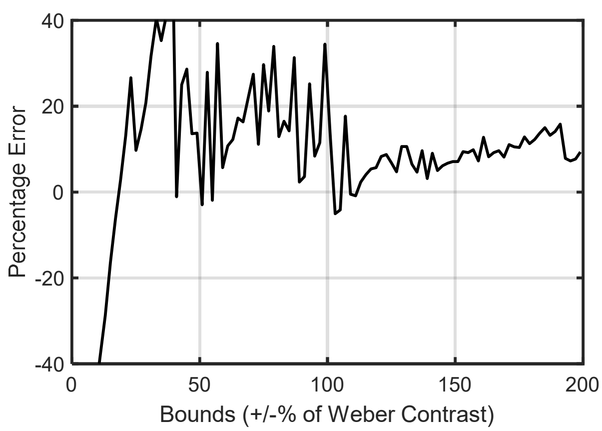

Figure 4(c) Percentage error of Weber CDP versus fast estimation of the same for patches of 5 versus 2.5 quanta with 1% to 200% nominal contrast bounds.

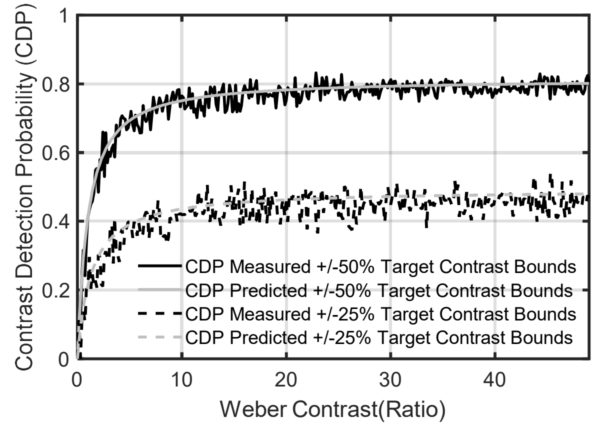

Figure 5, CDP calculated from a positive contrast sweep. The dark patch is held at 7 quanta while the bright patch is swept from 7 to 350.

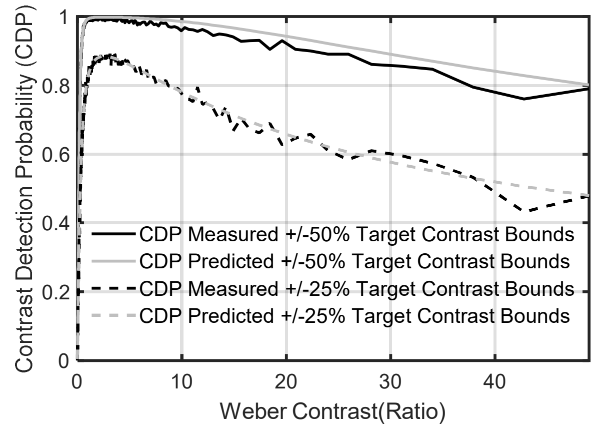

Figure 6, CDP calculated from a contrast sweep of the dark patch. The bright patch is held at 350 quanta while the dark patch is swept from 350 to 7.

Conclusions

A fast CDP estimation technique has been detailed which speeds calculation times by many 1000s for typical patch numbers and sizes. As a tangential benefit, because of the use of only the patch means and variance in the predictions, Weber contrast based CDP may be predicted mathematically rather than using full simulations. Increased errors are associated with calculations involving low quanta numbers or tight CDP bounds. It is recommended to keep quanta above 7 and bounds great than 10%. Above these levels errors remain well below an absolute value of 0.04 or 5%. The prediction results mimic behavior of CDP previously detailed by the author [4].

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