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Quantify Aliasing a new approach to make resolution measurement more robust

Uwe Artmann, Image Engineering GmbH & Co KG; Kerpen, Germany

Abstract

Aliasing is a well-known effect in imaging which leads potentially to disturbing artefacts on structures. While the high pixel count of todays devices helps to reduce this effect, at the same time optical anti-aliasing filter are more often removed from sensor stacks to improve on system SFR and quantum efficiency. While the artefact is easy to see, an objective measurement and quantification of aliasing is not standardised or established. In this paper we show an extension to existing SFR measurement procedures described in ISO12233 which can measure and quantify the existence of aliasing in the imaging system. It utilises the harmonic Siemens star of the s-SFR method and can be included into existing systems, so does not require the capture of additional images.

Introduction

In dictionaries, aliasing is defined as *"the misidentification of a signal frequency, introducing distortion or error"*. Aliasing in images appears as an artefact where an object with fine repeated pattern is reproduced in broader pattern. This so called Moire effect is also shown in an example in figure 1. The high spatial frequencies generated by the chair are projected onto the image sensor and result in lower spatial frequencies. So the object has a certain spatial frequency f_o and the sensor can sample this with its own sampling frequency f_s. The Nyquist-Shannon sampling theorem states that the sampling frequency f_s needs to be at least double of the f_o to be sufficient. If the sampling frequency is not sufficient, higher spatial frequencies might lead to aliasing.

Figure 1. Detail of an image, showing the so called moire effect. The high spatial frequencies generated by the monochrome structures are reproduced as low frequency, coloured structures.

So avoid aliasing it needs to me made sure that the object frequencies are low pass filtered before the sampling to avoid too high frequencies. In signal processing this is performed by digital filter, in sensors this is done by optical low pass filter, also called anti-aliasing filter. To maximise the performance in terms of low light sensitivity and optical resolution, many sensors in todays cameras do not feature an anti aliasing filter, so aliasing might occur in the image.

While aliasing can be an annoyance for the user, it can also influence resolution measurement. Examples from a resolution measurement are shown in figure 2. The images show the center part of a sinusoidal Siemens star as used for the s-SFR measurement described in ISO12233[2]. In these images, three white circles are plotted. These represent 50%, 75% and 100% of the so called Nyquist frequency. This frequency is the theoretical highest spatial frequency that can rendered. Expressed in unit $^{cycles}/_{pixel}$ the Nyquist frequency is $f_{Nyquist} = 0.5$ as two pixel are required to represent one cycle, which means that a single pixel can represent a half cycle. With this assumption, we can calculate the Nyquist frequency for a given Siemens star as shown in Equation 1.

$$
\begin{aligned}
f &= \frac{n_{cycles}}{\text{circumference}} = \frac{n_{cycles}}{2\pi r} \\
f_{Nyquist} &= 0.5 \frac{cycles}{pixel} \\
r_{Nyquist} &= \frac{n_{cycles}}{\pi} \\
&with \\
r_{Nyquist} &= \text{radius of Nyquist frequency} \\
n_{cycles} &= \text{cycles per full circle (here: 144)}
\end{aligned}
\tag{1}
$$

It can be observed in these images, that aliasing can occur even for lower frequencies than the calculated $f_{Nyquist}$. The reason for this is, that the assumptions made here do not take into account that most sensors do not sample every color with every pixel, but use a color filter array and demosaicing, so the true sampling frequency is lower than the final pixel count in the image. A sensor that does not use a color filter array (like monochrome sensors) can show higher frequencies than assumed from equation 1 as we measure different orientations in a Siemens star. In diagonal orientation the sampling is different to the sampling in horizontal or vertical direction. As aliasing can lead to lower frequencies with high amplitude, this can interfere with the resolution measurement. To make the measurement more robust, we need a method to be able to tell if a measurement result is "trustworthy" or if it is influenced by aliasing.

Concept

The concept to measure aliasing is an extension to the s-SFR measurement procedure described in ISO12233. The used test pattern is shown in figures 2 and 3.

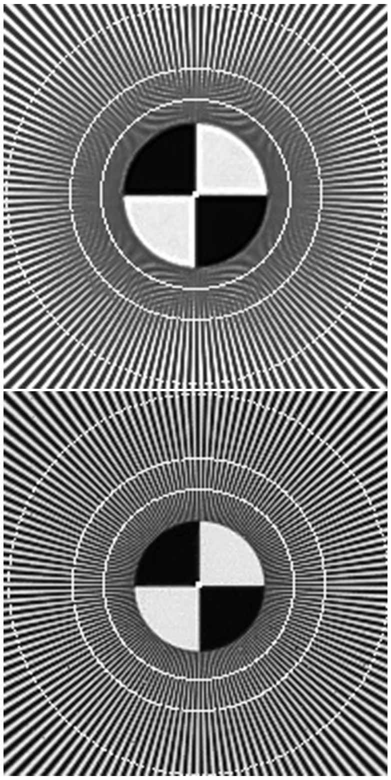

Figure 2. *Detail of center region of a Siemens star. White circles show 50%, 75% and 100% of Nyquist frequency.* ***Top:*** *A D-SLR camera with typical bayer pattern senso. One can observe aliasing artefacts well below $f_{Nyquist}$* ***Bottom:*** *A Foveon Sensor without bayer pattern. One can see resolution above $f_{Nyquist}$ in diagonal direction.*

To obtain the SFR from a Siemens star, the center of the star is located and digital values are read out for a single radius over angle ϕ. The function $I(\phi)$[1] describes the digital value I depending on the mean value a and the amplitude b which is scaled by the cosine of the frequency ($^{2\pi}/_{g}$) and the phase corrected angle $\phi - \phi_0$.

The standard document shows a fitting algorithm to obtain a and b, which is used to calculate the modulation. So the modulation is calculated for a given radius, the radius defines the spatial frequency. Calculating the modulation for all available radii will provide the s-SFR. The standard also states, that the phase is unknown and describes the method to obtain the phase by fitting a sinus and cosinus as in equation 3 and then calculate b as in equation 4.

$$I(\phi) = a + b \cdot cos\left(\frac{2\pi}{g}(\phi - \phi_0)\right) \quad (2)$$

$$I(\phi) = a + b_1 \cdot cos\left(\frac{2\pi}{g}\phi\right) + b_2 \cdot cos\left(\frac{2\pi}{g}\phi\right) \quad (3)$$

$$b = \sqrt{b_1^2 + b_2^2} \quad (4)$$

[1]Equations cited from standard document.

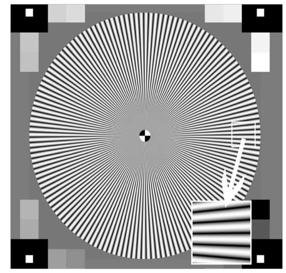

Figure 3. *The sinusoidal Siemens star for the S-SFR method described in ISO12233.*

So in the standard procedure the assumption is made that the frequency is known. In case of aliasing, this assumption is questionable. The core idea of this paper is to check if the assumption about the spatial frequency is correct or not. So if the real frequency differs significantly from the assumed frequency, we consider this as evidence for aliasing.

The first approach was to not only get the amplitude and phase from a fitting algorithm, but to also fit the frequency. We choose to implement the Gauss-Newton algorithm to minimise the residuals while estimating all relevant parameter of the equation 5.

$$I(\phi) = A_0 \cdot sin(B_0 \cdot \phi + C_0) + D_0 \quad (5)$$

Figure 4 visualises the iterative process of this algorithm. Starting from an initial estimation, the algorithm will minimise the residual error in every step, ideally approaching an ideal solution after a reasonable amount of iterations. While this worked well if the initial guess was good, we found many cases where this unconstrained method did not lead to a solution when the initial guess was not good. So after some improvement, we finally decided not to go forward in this direction.

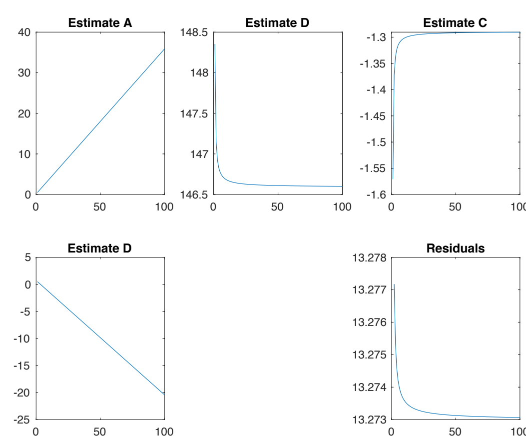

Figure 4. *Iterative estimation of the parameter in equation 4 The residuals shall minimise.*

The now used approach is to focus on the frequency only. The standard itself describes the method to obtain amplitude and phase. In the standards approach, the frequency is taken as given. For the aliasing analysis, the same fitting approach is applied for a sequence of frequencies with the assumed frequency as highest frequency.

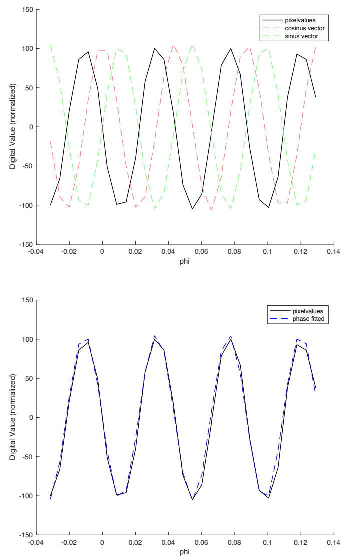

Figure 5. *As described in ISO12233, the amplitude and phase is fitted.* ***Top:*** *Fitting with sinus and cosines* ***Bottom:*** *The fitting with correct phase, obtained from sinus and cosines.*

For every frequency step (only assumed frequency and lower), the amplitude and phase is fitted and then the error between fitted function and the measured pixel values is calculated (see fig. 6). This algorithm, to check which frequency leads to the lowest error, is then applied for all radii of the Siemens star.

For every radius, the frequency with the lowest error is divided by the assumed frequency. So if the aliasing value equals one, the assumed frequency and the obtained frequency in the image are identical. The lower the value, the higher the differences.

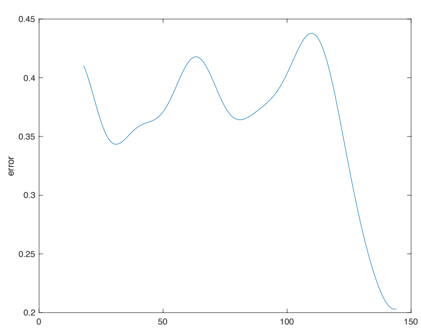

Figure 6. *Least square error between fitted function and pixel values for all spatial frequencies. In this example the measured and assumed minimum is identical.*

In figure 7 all three obtained functions from a range of radii are shown. The spatial frequency response is obtained according to standard procedure. The aliasing value is calculated as explained. The fitting error is shown in orange color, showing that even if the frequency changes, the resulting error is more or less stable. All values are calculated for a small segment of a Siemens star.

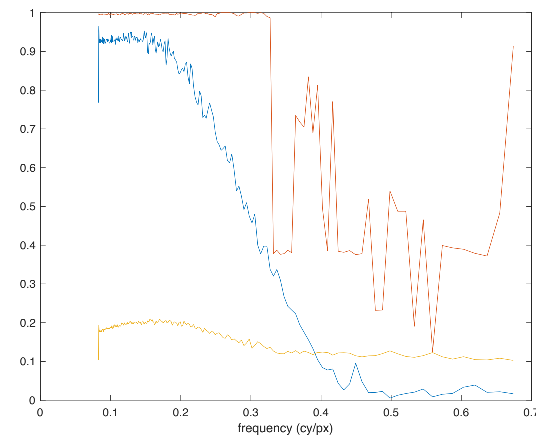

Figure 7. *Plot of all obtained functions vs. spatial frequencies. blue: spatial frequency response red: aliasing value yellow: fitting error*

Measurement

This calculation can now be applied to full Siemens stars. The star is divided into 24 segments, the calculation of SFR and aliasing value function is performed per segment. The naming convention of the 24 segments is shown in figure 8.

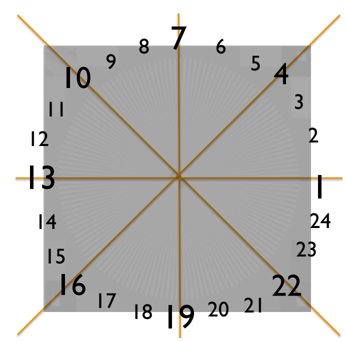

Figure 8. *All 24 segments of a Siemens star. This naming convention is also used for all other plots of this paper.*

For each segment the SFR and the aliasing value are obtained and visualised in figures 9, 10 and 12. The x-axes shows the segments as defined in figure 8. The SFR value and the aliasing value are color coded.

The corresponding measurement results to the images shown in the introduction are shown. The aliasing results reflect the visual impression. The image from the D-SLR shows significant aliasing well below $f_{Nyquist}$, so the bayer pattern sensor and the demosaicing process introduce some aliasing. The image taken with the Foveon sensor shows aliasing only for radii higher than $f_{Nyquist}$, this can also be shown in the aliasing results.

An extreme example for aliasing was generated by downscaling the D-SLR image in vertical direction, then upscaling to original size. Both scaling steps with simple nearest neighbour algorithm, introducing strong aliasing. The results are shown in figure 11.

Figure 12 shows another example of image and the corresponding SFR and aliasing plot. The image was captured with a Canon Ixus 155 compact camera. The strong aliasing and especially the dependency on the orientation can be shown nicely.

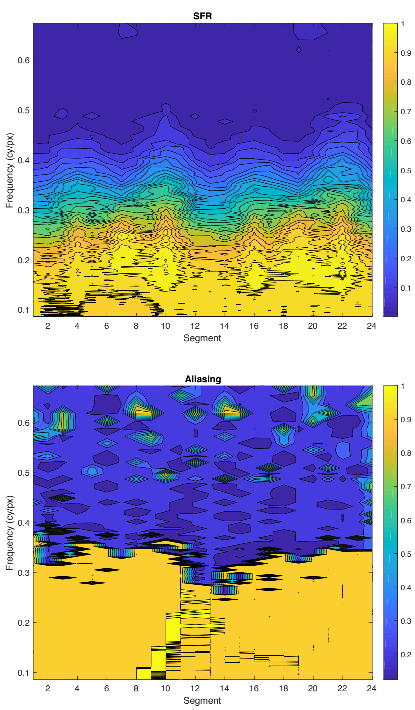

Figure 9. *SFR and aliasing per segment for D-SLR camera. Image shown in figure 2 (top)*

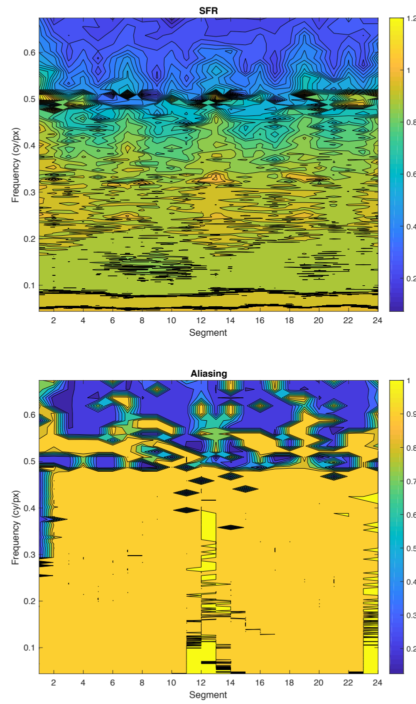

Figure 10. *SFR and aliasing per segment for Foveon sensor camera. Image shown in figure 2 (bottom)*

Conclusion and future work

We could show a new approach to extend the standard S-SFR method from ISO12233 for also measuring the aliasing.

The aliasing can be measured for different orientations, a benefit of the Siemens star in general.

More work is needed to visualise the results in a better way and to analyse more images to proof the method. The big benefit is, that existing images can be used as the method is fully compatible to existing test targets.

A psychophysical study to correlate the results with visual impression of naive observers is future work.

Acknowledgments

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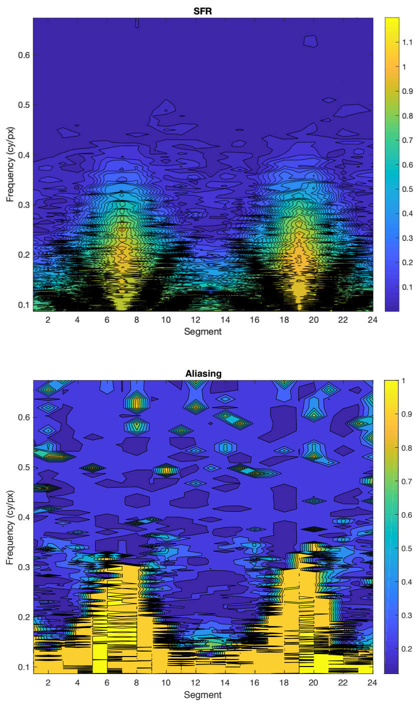

Figure 11. *SFR and aliasing per segment for D-SLR image with forced aliasing. Image was downscaled and upscaled in vertical direction with simple nearest neighbour algorithm.*

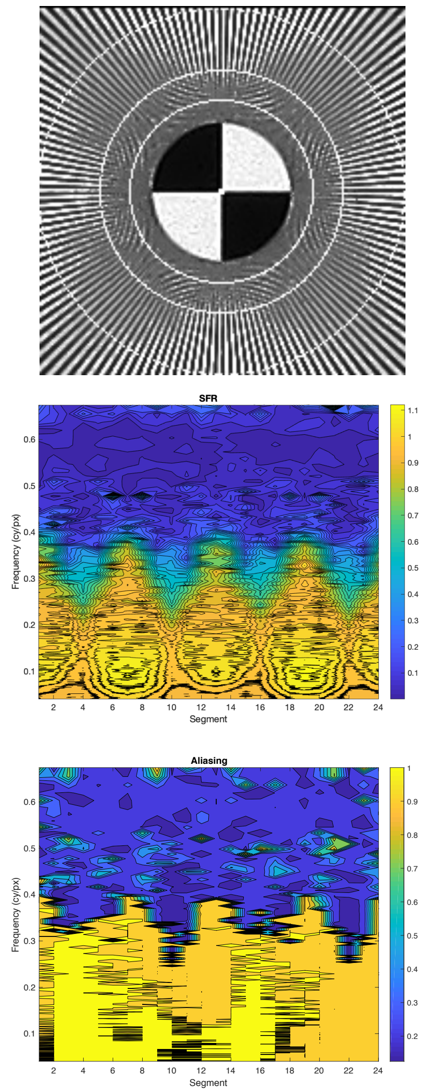

Figure 12. *SFR and aliasing per segment for Canon Ixus 155 compact camera. Image shown in top. Strong and orientation depending aliasing can be made visible.*

References

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[2] International Organization of Standardization, ”ISO12233 Photography - Electronic still picture imaging - Resolution measurements”, 2014

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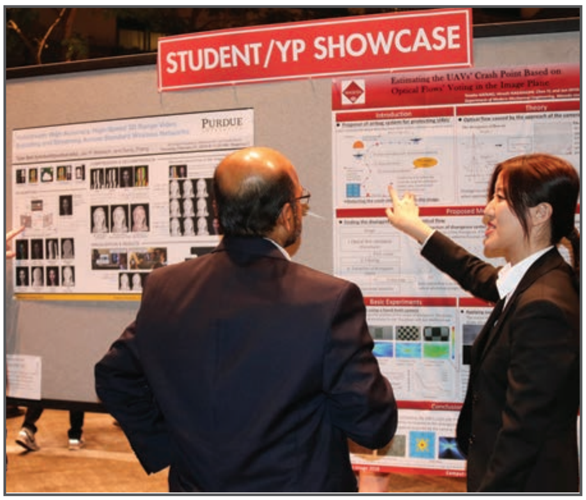
Author Biography

Uwe Artmann studied Photo Technology at the University of Applied Sciences in Cologne following an apprenticeship as a photographer, and finished with the German ’Diploma Engineer’. He is now CTO at Image Engineering, an independent test lab for imaging devices and manufacturer of all kinds of test equipment for these devices. His special interest is the influence of noise reduction on image quality and MTF measurement in general.



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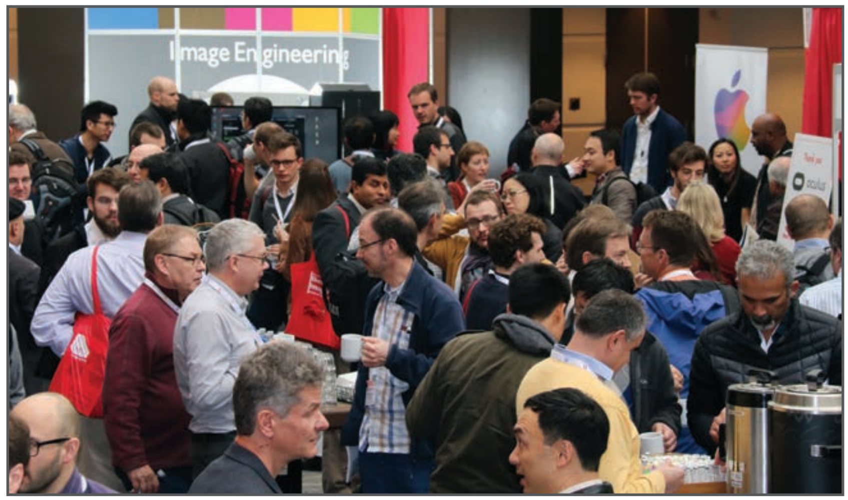

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