https://doi.org/10.2352/ISSN.2470-1173.2018.13.IPAS-441

Enhancement of Underwater Color Images by Two-side 2-D Quaternion Discrete Fourier Transform

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Abstract

The research problem is to find an effective enhancement method for enhancing raw underwater color images. In underwater, as depth increases the high wavelength regions of the light spectrum are absorbed by the water and the light spectrum consists only of low wavelength regions such as green and blue and therefore, the image captured underwater looks green or greenish blue. This paper proposes an enhancement algorithm for improving the quality of raw underwater images by the method of alpha-rooting by two-side 2-D quaternion discrete Fourier transform (QDFT) with color correction done by multiscale retinex (MSR). The results of proposed enhancement are compared with the alpha-rooting method, by transforming color images to 2-D grayscale images. The enhancement are measured with reference to the metric color enhancement measure estimation.

Introduction

Underwater images are images captured by immersing cameras in the water while scuba-diving or swimming. In deep waters, most of the high wavelengths regions in the spectrum of light, such as red, yellow and orange, are absorbed by the water and so images captured in deep waters tend to appear more on the low wavelengths regions like green, greenish-blue and blue, depending on the depth from where the image is captured. Therefore, there is a need of enhancement algorithms which will improve the quality of underwater images. The proposed enhancement methods are based on quaternion approach of image processing, in which enhancement of images is done considering all channels simultaneously. Until recently, color images were enhanced by taking each channel separately [3]-[13], [25]-[29]. But, the color is the superposition of the color components and the enhancement of images could be done more effectively when considering all color components together. We have tested many algorithms which color-correct and enhance the image considering all channels simultaneously. This paper details an enhancement method of alpha-rooting by the two-side 2-D quaternion discrete Fourier transform and the results are compared with the alpha-rooting by the 2-D DFT on 2-D grayscale image of the color image transformed by the 2×2 transformation model[1]. The underwater images are pre-processed for color-correction on multiscale retinex. The enhancement is assessed with reference to the metric color enhancement measure estimation (CEME).

1. Alpha-rooting by Two-side 2-D Quaternion Discrete Fourier Transform (2-D QDFT)

1.1. Quaternion Numbers

Quaternion numbers are four dimensional hyper-complex numbers[3]-[17],[22]-[29]. The Cartesian form of quaternion number is represented as

$$\mathrm{q} = \mathrm{a} + \mathrm{ib} + \mathrm{jc} + \mathrm{kd}. \quad (1)$$

where the imaginary parts i, j, and k are related as

$$ij = -ji = k; jk = -kj = i; ki = -ik = j; \quad i^2 = j^2 = k^2 = -1. \quad (2)$$

1.2. Quaternion representation of Color image

The color image consists of three or four channels and can be represented by quaternion numbers. For example, a three channel color image of the RGB color model can be represented in quaternion arithmetic as

$$q = ir_{n,m} + jg_{n,m} + kb_{n,m}. \quad (3)$$

Here, $r_{n,m}, g_{n,m}, b_{n,m}$ are respectively the 2-D images in red, green, and blue channels. The quaternion representation of color image gives a superposition of all channels in the image.

1.3. 2-D Quaternion Discrete Fourier Transform (2-D QDFT)

The discrete Fourier transform of quaternion numbers is referred as quaternion discrete Fourier transform. The two-side 2-D quaternion discrete Fourier transform is defined as

$$F(p,s) = \sum_{n=0}^{N-1} W_j^{np} \left[\sum_{m=0}^{M-1} f_{n,m} W_k^{ms}\right], \quad (4)$$
$$p = 0,1,\dots,(N-1), \qquad s = 0,1,\dots,(M-1).$$

and it's inverse is defined as

$$f(n,m) = \frac{1}{NM} \sum_{p=0}^{N-1} W_j^{-np} \left[\sum_{s=0}^{M-1} F_{p,s} W_k^{-ms}\right], \quad (5)$$
$$n = 0,1,\dots,(N-1), \qquad m = 0,1,\dots,(M-1),$$

where the kernels W_j^t and W_k^t are respectively

$$W_j^t = \exp\left(-\frac{j2\pi t}{N}\right) = \cos\left(\frac{2\pi t}{N}\right) - \mathrm{j}\sin\left(\frac{2\pi t}{N}\right), \quad (6)$$
$$t = 0,1,\dots,(N-1),$$
$$W_k^t = \exp\left(-\frac{k2\pi t}{M}\right) = \cos\left(\frac{2\pi t}{M}\right) - k\sin\left(\frac{2\pi t}{M}\right),$$
$$t = 0,1,\dots,(M-1).$$

The quaternion multiplication is not commutative. Therefore, there are many different definitions of the quaternion discrete Fourier transform. We consider the two-side 2-D QDFT.

1.4. Alpha-rooting Method of Enhancement

The alpha-rooting method of enhancement modifies the magnitude of the transform with the power of alpha[14]-[17],[22]-[26]. The value of alpha varies from 0 to 1. The quaternion based alpha-rooting method is defined as

$$|F_{p,s}| \to |F_{p,s}|^{\alpha}, \quad (7)$$

where the magnitude of the transform is defined as

$$|F_{p,s}| = \sqrt{|F_{e_{p,s}}|^2 + |F_{i_{p,s}}|^2 + |F_{j_{p,s}}|^2 + |F_{k_{p,s}}|^2}$$

$F_{e_{p,s}}, F_{i_{p,s}}, F_{j_{p,s}}, F_{k_{p,s}}$ are the four components of the QDFT $F_{p,s}$.

2. Alpha-rooting by 2-D DFT on transformed 2-D grayscale image

2.1. Transformation Model

The color image can be transformed to a 2-D grayscale image and the image processing can be done on the transformed 2-D grayscale image. All known algorithms that work on the 2-D grayscale images can be used for processing the transformed image. There are many transformation models possible. One of the transformation models is the 2×2 transformation model. For the RGB color model, the 2×2 transformation model for the color image is given as

$I(n,m)$	$R(n,m)$	…
$G(n,m)$	$B(n,m)$	…
…	…	…

where $I(n,m)$ is the pixel-wise luminous of the image, which is calculated by $I = 0.3R + 0.59G + 0.11B$. For the color image of size $N \times M$, the transformed 2-D grayscale image by 2×2 transformation model is of size $2N \times 2M$.

2.2. Alpha-rooting Enhancement Method

The alpha-rooting method on transformed 2-D grayscale image is similar to the relation defined in Eq. (7) but, the magnitude of the transform used here is that of 2-D discrete Fourier transform. After performing the alpha-rooting method on the 2-D grayscale image, it is converted back to the color image.

3. Color Enhancement Measure Estimation (CEME)

The metric of color enhancement measure estimation (CEME) is the enhancement measure of color images and is based on contrast perception [31]. The metric CEME relates to the Weber's law that states that the perceived change in stimulus proportional to initial stimuli, and the Fechner's law, which states that the perception and stimulus are logarithmically related. That is, the visually perceived intensity value is proportional to the difference of the logarithm of the actual intensity. The CEME is calculated by dividing a discrete color image of the size $N \times M$ by k_1k_2 blocks of size $L_1 \times L_2$ each, where $k_n = \lfloor N_n/L_n \rfloor$, for n=1, 2. For the RGB color model, when the image is enhanced by the 2-D QDFT based enhancement algorithm,

$$f = (f_R, f_G, f_B) \to \hat{f} = (\hat{f}_e, \hat{f}_R, \hat{f}_G, \hat{f}_B), \quad (8)$$

where $\hat{f}_e$ is referring to the real component of the quaternion image, obtained after taking the transform. The CEME value is calculated by

$$CEME_{\alpha}(\hat{f}) = \frac{1}{k_1k_2}\sum_{k=1}^{k_1}\sum_{l=1}^{k_2} 20\log_{10}\left[\frac{\max_{k,l}(\hat{f}_e, \hat{f}_R, \hat{f}_G, \hat{f}_B)}{\min_{k,l}(\hat{f}_e, \hat{f}_R, \hat{f}_G, \hat{f}_B)}\right]. \quad (9)$$

4. Pre-processing for Color-correction by Multi-scale Retinex (MSR)

The retinex algorithm was proposed (Edward Land, 1986) as a model of human perception and color and it describes the difference between the digital images and images observed by human eyes[16],[30]. In retinex, it is assumed that the given image obtained from the camera, $f_{n,m}$ is in the multiplicative model.

$$f_{n,m} = l_{n,m}r_{n,m}, \quad n = 0,1,\dots,(N-1), m = 0,1,\dots(M-1), \quad (10)$$

where $l_{n,m}$ is the illumination component of the image at the pixel (n,m) and $r_{n,m}$ is the reflectance component of the image at the pixel (n,m). The multiplicative model in Eq. (10) becomes a summation model in the logarithm domain and the logarithm of the reflectance of the image can be obtained by $\ln(r_{n,m}) = \ln(f_{n,m}) - \ln(l_{n,m})$. In the retinex algorithm used in our proposed image enhancement, the illumination procedure is limited to Gaussian smoothing filters. The MSR algorithm, processing each channel of the image, can be expressed in a simple formula as

$$X_K(n,m) = \log(f_{K_{n,m}}) - \sum_{k=1}^{l} w_k \log\left(\left[(y_{\sigma_k} * f_K)_{n,m}\right]\right), \quad (11)$$

where
(a) K stands for color channels R, G, and B;
(b) $y_{\sigma_k}(n,m)$ is the Gaussian function

$$y_{\sigma_k}(n,m) = Ae^{-\left[\frac{(n-n_0)^2+(m-m_0)^2}{\sigma_k^2}\right]}, \quad A = \left[\sum_{n=0}^{N-1}\sum_{m=0}^{M-1} e^{-\left[\frac{(n-n_0)^2+(m-m_0)^2}{\sigma_k^2}\right]}\right]^{-1} \quad (12)$$

of different width (scales) defined by σ_k, where $k \in \{1,2,\dots,l\}$, $n_0 = \text{round}\left(\frac{N}{2}\right), m_0 = \text{round}\left(\frac{M}{2}\right)$. Here, $(y_{\sigma_k} * f_K)_{n,m}$ is the convolution of each channel with the Gaussian function.
(c) w_k are the weights of the k^{th} component of MSR output image with w_k satisfying the condition

$$w_1 + w_2 + \dots + w_k = 1. \quad (13)$$

The color-correction function is expressed as[32]

$$C_K(n,m) = \gamma + \beta\left[\log(f_K)_{n,m} - \log(f_{gray})_{n,m}\right], \quad \gamma = 125\log(125), \quad (14)$$

where $f_{gray} = (R + G + B)/3$.
The MSR with color correction is expressed as

$$\hat{X}_K(n,m) = C_K(n,m)\, X_K(n,m). \quad (15)$$

5. Experimental Results

Figure 1 shows the image results of the original image "fish.jpg" (Fig. 1(a)). The raw underwater image is pre-processed by retinex algorithm followed by color-correction method and the corresponding image result is given in Fig. 1(b). We see that the color in the image is corrected and the image is brightened.

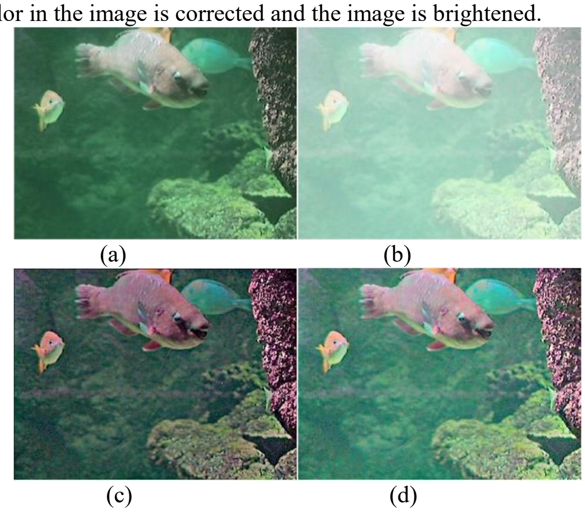


***Figure 1**: (a) Original Image "fish.jpg"; (b) color-correction of image in (a) by multiscale retinex; (c) Alpha-rooting by two-side 2-D QDFT with alpha = 0.73; (d) the alpha-rooting with alpha = 0.8 on transformed 2-D grayscale image and then converting back to color image. *Courtesy: Photo from [34].*

The pre-processed image is enhanced by the proposed enhancement algorithm, alpha-rooting by two-side 2-D QDFT and the image result is shown in Fig. 1(c). In Fig. 1(c) we see that the image objects are more defined than in Fig. 1(b). The pre-processed image in Fig. 1(b) is also enhanced by transforming the color image to the 2-D grayscale image with the 2×2 transformation model and afterward applying the alpha-rooting method by 2-D DFT and then converting back the enhanced grayscale image to the color image. The so enhanced image result is shown in Fig. 1(d). From Table 1 we see that CEME values of the enhanced image are far greater than the pre-processed image, and both the CEME of enhancement results show almost nearby values.

Figure 2 is another good example of raw underwater image that need enhancement. The color in the original image "sculpture.jpg" (Fig. 2(a)) is dominated by the green color and original color of the image objects are not clear in the captured image. After performing retinex algorithm and color-correction, the color of the sculpture and that of the goggles of the scuba-divers' are more close to the original color as shown in Fig. 2(b). The pre-processed image in Fig. 2(b) is further enhanced by the alpha-rooting method by the 2-D QDFT with alpha = 0.83. And we observe from the results in Fig. 2(c) that sharpness of the edges is more distinct after applying the enhancement method. Figure 2(d) shows the image of the alpha-rooting method by the 2-D DFT, after transforming the image to the grayscale image and then converting back to the color image. The alpha value in this latter approach is 0.85. The CEME value in Table 1 shows the visual perception metric for these different enhancement approaches. One can see that both CEME values of the enhanced image increase from the pre-processed image and, when comparing the CEME of both enhanced image, the values are almost close to each other.

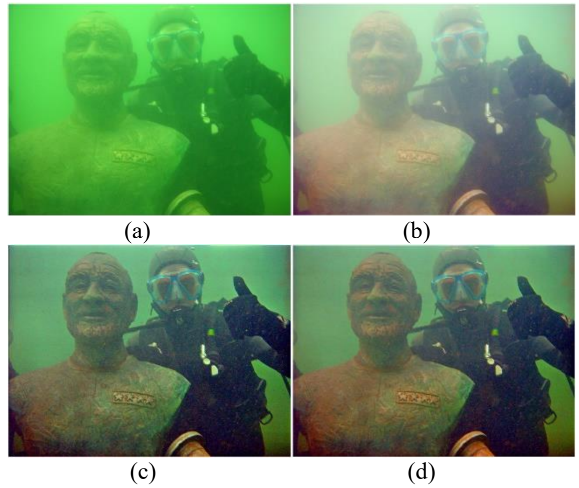


***Figure 2**: (a) Original Image "sculpture.jpg;" (b) color-correction of image in (a) by multiscale retinex; (c) the alpha-rooting by two-side 2-D QDFT with alpha = 0.83; (d) the alpha-rooting with alpha = 0.85 on transformed 2-D grayscale image and then converting back to color image. *Courtesy: Photo from [34].*

Figure 3 shows the raw underwater image and its enhanced images of the original image "planktons.jpg". The details of the original image (Fig. 3(a)) are obviously not clear. The retinex algorithm and color-correction is done for pre-processing the original image and the resulting image is given in Fig. 3(b). Figure 3(c) shows the image after enhancing the image by the alpha-rooting by the 2-D QDFT with $\alpha = 0.92$. Figure 3(d) shows the image result after enhancing the transformed grayscale image by the alpha-rooting method by the 2-D DFT with $\alpha = 0.93$, and then afterward converting back the enhanced image to the color image. The CEME values of the images are given in Table 1 and one can see that CEME of the enhanced imaged increases, as compared to the CEME of the image pre-processed by the retinex algorithm followed by color-correction method.

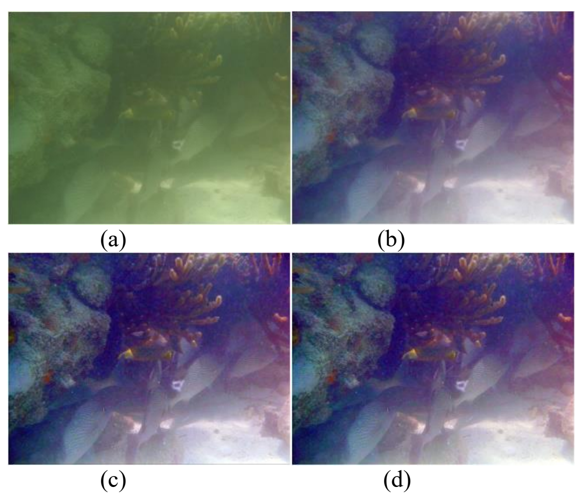


***Figure 3**: (a) Original Image "planktons.jpg"; (b) Color-correction of image in (a) by multiscale retinex; (c) Alpha-rooting by two-side 2-D QDFT with alpha = 0.92; (d) Alpha-rooting with alpha = 0.93 on transformed 2-D grayscale image and then converting back to color image. *Courtesy: Photo from [34].*

Figure 4 shows the details of the original image “tent.jpg” (Fig. 4(a)) and its enhanced images. The original image is taken underwater and in poor-lighting conditions. The Fig. 4(b) is the image result after performing retinex algorithm followed by color-correction method. The image in Fig. 4(b) gets brightened and color becomes closer to the original image-objects but it looks foggy in appearance. The image after enhancing by the proposed alpha-rooting method by QDFT with $\alpha = 0.86$ and by the alpha-rooting method by DFT with $\alpha = 0.87$ is shown respectively in parts (c) and (d) of Fig. 4. From Table 1 one can see that the CEME value of the image enhanced by the alpha-rooting method by the 2-D QDFT is higher than the CEME value of the image enhanced by the alpha-rooting method by 2-D DFT.

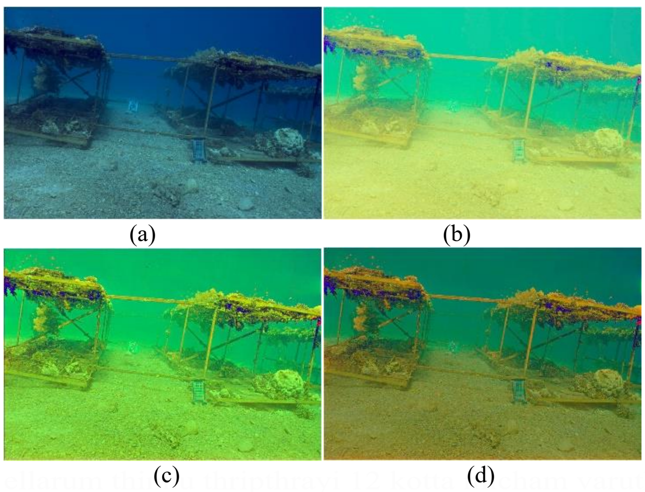


***Figure 4**: (a) Original Image “tent.jpg;” (b) Color-correction of image in (a) by multiscale retinex; (c) Alpha-rooting by two-side 2-D QDFT with alpha = 0.86; (d) Alpha-rooting with alpha = 0.87 on transformed 2-D grayscale image and then converting back to color image. *Courtesy: Photo from [34].*

Figure 5 shows the original image “ship_wreck.jpg” (Fig. 5(a)) and the results of enhancement. The original image has a prominent blue shade which implies that the captured image-object is deep down the surface of the water and all other color wavelengths of the light are absorbed by water in that depth. Figure 5(b) gives a good image results after pre-processing the original image by retinex algorithm followed by the color-correction method. The rust in the ship-wreck is obvious in Fig. 5(b) after pre-processing of the image is done. The pre-processed image when further enhanced by alpha-rooting method by QDFT with $\alpha = 0.93$ gives the image-objects a more distinct appearance [Fig. 5(c)]. As typical about the alpha-rooting method, both the smooth and sharp surfaces of the image are enhanced by the method. Figure 5(d) gives the image results when enhanced by the alpha-rooting method by DFT with alpha = 0.94 on the transformed grayscale image by the 2×2 transformation model and then converting back to the color image. CEME values of images enhanced by both enhancement methods are greater than the pre-processed image and the values are given in Table 1.

Figure 6 shows another good example of underwater image. The details of the image-objects in the original image “corals.jpg” [Fig. 6(a)] are not very clear and needs enhancement. The raw image is first pre-processed by retinex algorithm and color-correction method and the image so obtained is shown in Fig. 6(b).

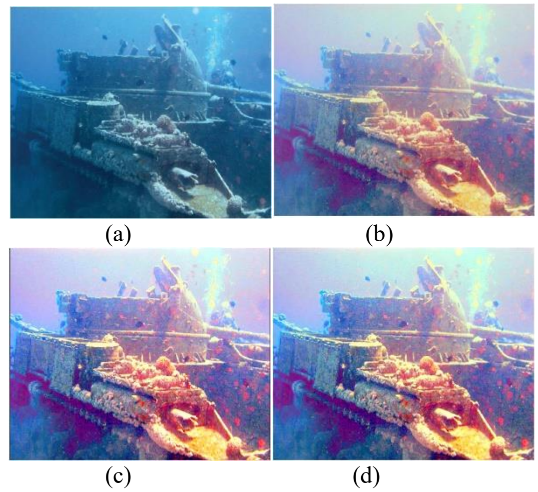


***Figure 5:** (a) Original Image “ship_wreck.jpg;” (b) Color-correction of image in (a) by multiscale retinex; (c) Alpha-rooting by two-side 2-D QDFT with alpha = 0.93; (d) Alpha-rooting with alpha = 0.94 on transformed 2-D grayscale image and then converting back to color image. *Courtesy: Photo from [34].*

In Fig. 6(b), it is interesting to note that the corals have the expected white color and fish are red after pre-processing. The pre-processed image is enhanced by both enhancement method and the results are given in parts (c) and (d) in Fig. 6. From Table 1, one can also see that the CEME value of the image enhanced by the proposed alpha-rooting method by the 2-D QDFT is higher than the CEME value of the image enhanced by the alpha-rooting method by 2-D DFT. The alpha chosen are respectively $\alpha = 0.87$ and $\alpha = 0.88$, which are the optimum choice of alpha values for the respective enhancement methods.

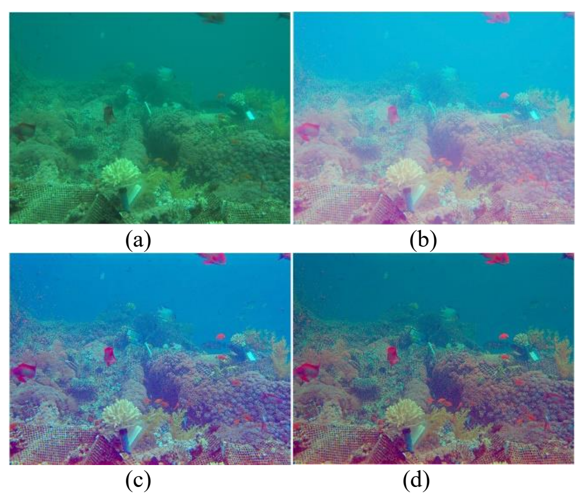


Figure 6: (a) Original Image “corals.jpg;” (b) Color-correction of image in (a) by multiscale retinex; (c) Alpha-rooting by two-side 2-D QDFT with alpha = 0.87; (d) Alpha-rooting with alpha = 0.88 on transformed 2-D grayscale image and then converting back to color image. *Courtesy: Photo from [34].*

Figure 7 illustrates another good example of underwater image enhancement results. The original image “rocks.jpg” (Fig. 7(a)) is first pre-processed by the retinex algorithm followed by color-correction; the resulting image is as shown in Fig. 7(b).

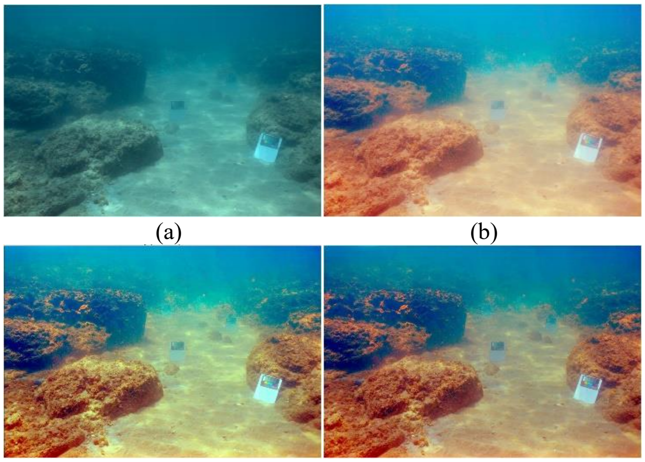
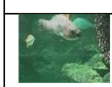
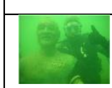
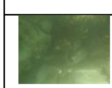
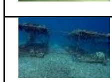
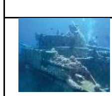
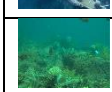
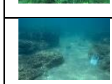


Figure 7: *(a) Original Image "rocks.jpg"*; (b) Color-correction of image in (a) by multiscale retinex; (c) Alpha-rooting by two-side 2-D QDFT with alpha = 0.9; (d) Alpha-rooting with alpha = 0.9 on transformed 2-D grayscale image and then converting back to color image. *Courtesy: Photo from [33].*

Table 1: CEME values of original & enhanced underwater images

Original Images	Original Image	Color-correction by Multiscale Retinex	Alpha-rooting by 2-D QDFT	Alpha-rooting method on transform-ed 2-D grayscale image model
	CEME			
	18.0237	6.1704	20.3362 (alpha = 0.73)	20.9267 (alpha = 0.8)
	27.9550	9.4850	17.5209 (alpha = 0.83)	17.4997 (alpha = 0.85)
	11.6103	15.8916	23.3601 (alpha = 0.92)	23.1925 (alpha = 0.93)
	21.9634	12.9805	19.3646 (alpha = 0.86)	16.7009 (alpha = 0.87)
	37.0770	13.4596	17.7915 (alpha = 0.93)	17.7398 (alpha = 0.94)
	22.4217	7.9381	11.7513 (alpha = 0.87)	10.9437 (alpha = 0.88)
	17.7172	10.6056	12.1283 (alpha = 0.9)	12.3562 (alpha = 0.9)

It is obvious to notice in Fig. 7(b) that the colors in the color-chart are more close to the original colors, after performing pre-processing methods. Particularly the red color, which is a high-wavelength color of visible light, is seen as black in the original raw underwater image and after performing the pre-processing methods the red color in the chart looks exactly red color. The alpha-rooting method of enhancement is applied on the pre-processed image. The image in Fig. 7(c) is the result obtained by using the quaternion approach, and part (d) in Fig. 7 is the image after enhancing by the transformation-model-method approach. The enhanced images in part (c) and (d) are more distinct than the pre-processed image, which has a foggy appearance. Table 1 gives the CEME values of the enhanced images for these processing methods. The CEME values of the enhanced image are seen greater than the CEME value of the pre-processed image. The CEME values of both enhancement methods have values close to each other. Also, CEME values of both enhancement methods are higher than the CEME value of the image pre-processed by the retinex algorithm, which is done to take care of the poor-lighting conditions when image was captured, followed by color-correction method, which is done to take care of unavailability of the higher wavelengths regions of the light while capturing the image.

Conclusions

The application of the enhancement method of alpha-rooting method by the two-side 2-D quaternion discrete Fourier transform on underwater images is novel. Also, the pre-processing for color-correction by multiscale retinex gives an additional benefit in enhancing the quality of images. The alpha-rooting on transformed 2-D grayscale image is also found to be an effective method for image enhancement. The current comparative study of enhanced images shows that the proposed quaternion approach of enhancement has higher CEME values than for the image enhanced by transformation-model approach.

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