# Camera Resolution and Distortion: Advanced Edge Fitting 

Peter D. Burns; Burns Digital Imaging and Don Williams; Image Science Associates


#### Abstract

A frequently used method for camera imaging performance evaluation is that based on the ISO standard for resolution and spatial frequency responses (SFR). This standard, ISO 12233, defines a method based on a straight edge element in a test chart. While the method works as intended, results can be influenced by lens distortion due to curvature in the captured edge feature. We interpret this as the introduction of a bias (error) into the measurement, and describe a method to reduce or eliminate its effect. We use a polynomial edge-fitting method, currently being considered for a revised IS012233. Evaluation of image distortion is addressed in two more recent standards, ISO 17850 and 19084. Applying these methods along with the SFR analysis complements the SFR analysis discussed here.


## Introduction

Edge-gradient analysis is a well-established method for evaluating the capture of image detail by an imaging system. Originally developed for optical and photographic systems, it was adapted for the evaluation of digital cameras and scanners, when it was applied to slanted, or rotated, image features. The basic steps are shown in Fig. 1.


Figure1: Edge-gradient analysis steps

## Edge-SFR Measurement

For the ISO $12233^{1,2}$ method there are three basic operations: acquiring an edge profile from the (image) data; computing the derivative in the direction across the edge, and computing the discrete Fourier transform of this derivative array. If we interpret the slanted-edge Spatial Frequency Response (SFR) measurement as an estimation problem, several sources of error can be seen as introducing bias and/or variation into the estimated SFR.

For example, standard software programs do not require a precise alignment of the edge feature in the scene with image sampling array. The edge location is computed (estimated) from the data. An error introduced into the computed slope propagates as a bias error in the resulting SFR or MTF measurement. Most measurement error analysis focuses on variation. In this paper, we address a source of systematic error, and how to reduce it.

The estimation of the direction (slope) of the edge has a direct effect on the computed SFR. This has been modeled in much the same way as microdensitometer aperture misalignment. ${ }^{3}$ In the slanted-edge analysis, the processing of the image data by projection along the edge can be approximated by the synthesis of a slit of length $m$ pixels. The effective MTF due to the slope error is ${ }^{3}$

$$
\begin{equation*}
T(u)=\frac{\sin (\pi m s \Delta u)}{\pi m s \Delta u}, \tag{1}
\end{equation*}
$$

where $\Delta$ is the original data sampling interval, $s$ the slope misalignment error, and $u$ the spatial frequency.

Estimation of the edge slope can be influenced by image noise, which is an example of a source variation leading to a bias in the SFR measurement. The edge slope is computed from the set of line-by-line edge positions, which are computed from the firstderivative vectors of each image line in the region of interest (ROI).

## Image Distortion Interaction

With the adoption of several standard imaging performance measures, it is tempting to think of each as distinct. While many are aimed at measuring different imaging characteristics, however, it is instructive to see how one characteristic can influence measurement of a quite different attribute. This is the case with distortion and image resolution.

The edge-SFR method as widely practiced is based on a straight image feature. However, camera lens distortion will usually bend the edge so that it is curved when presented for analysis. When the normal SFR analysis is performed on such as edge, an 'error' introduced. The result is influenced by the curvature of the edge, because the computed edge response is no longer an image profile normal to the edge. So while the SFR measurement is evaluating the system output as presented, the measurement of one attribute is modifying the evaluation of another (SFR).

The influence of distorted edge features on SFR evaluation was discussed in Ref. 2. Analysis of residual edge-fitting errors was also suggested as part of automatic detection of the condition. Baer ${ }^{4}$ addressed the SFR evaluation of cameras by introducing circular test edges. For many cameras, a radial variation in image blur can be accommodated by evaluating a centered circular edge.

More recently, Cardei et al. ${ }^{5}$ also used circular edge features and polynomial fitting to sections (arcs) for their SFR analysis. They suggest an iterative method based on computing increasingorder models to the edge shape. The chosen SFR is based on the highest-order polynomial that is consistent with an estimate of over-fitting, based on the set of residual values.

In many cases the camera image processing path will include distortion correction of the image, and it is possible to perform the SFR analysis after this has been done. However, there can still be a residual curvature of the edge feature. In addition, the step of lens correction requires an interpolation and resampling of the image data. Since this is spatial processing, this will also influence the measured SFR.

For system analysis of a camera whose path includes this lens correction, SFR evaluation after this resampling is appropriate, since this is for the delivered image. For subsystem evaluation where the influence of distortion and, e.g., focus, motion blur need to be separated, this will not give the intended measure.

## Advanced Edge-fitting

For situations where there is residual curvature in the edge feature, or lens correction is not applied, we can modify the edgeestimation step of the SFR method. When the set of edge locations (computed from the line-derivative data) are used to find the edge, we can adopt a polynomial function, rather than the standard line (first-order). The second step is to use this fitted function when forming the super-sampled edge profile vector.

When investigating the effectiveness of this approach, it was useful to have a reference, noise-free image file with known edge profile. This was done, using the 'error function', or integrated Gaussian function

$$
\begin{equation*}
\operatorname{erf}(x)=\frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^{2}} d t \tag{2}
\end{equation*}
$$

An integrated Gaussian edge centered at $x=\mu$ can be written as,

$$
\begin{equation*}
e(x, \mu, \sigma)=\frac{1}{2}\left(1+\operatorname{erf}\left(\frac{x-\mu}{\sigma \sqrt{2}}\right)\right) \tag{3}
\end{equation*}
$$

The corresponding line-spread function and resulting SFR will have a Gaussian form.

Figure 2 shows the function plotted for $\mu=0, \sigma=1$ pixel. This represents the x -axis profile for a vertical edge feature. To generate a slanted-edge image array, each row in the array should offset, to achieve the desired edge-angle. The x-axis is plotted in units of pixels. In Eq. 3 the width parameter, $\sigma$, can be used to adjust the (spatial-frequency) bandwidth of SFR image array.


Figure 2: Computed edge function used in computed reference image.

Figure 3 shows a computed image ${ }^{*}$ with such an edge feature at a $5^{\circ}$ angle from vertical. Also shown superimposed is the fitted edge (line) and set of edge location data. The SFR computed from the image array of Fig. 3 is also shown, with the expected form.

Figure 4 shows a computed, distorted edge, and the result of a polynomial edge-finding method. The SFR computed from the uncorrected edge profile, i.e. standard method is shown in Fig. 5. The distorted edge has caused a widened edge- and line spread
function to be computed. The much lower SFR (Fig. 5) is the result. In other words, the distorted edge has introduced a negative bias into the SFR measurement.

However, when a polynomial edge-fitting step is employed, the SFR results show little if any influence of the distortion. Figure 6 shows the results from the corrected edge profile. In this noisefree case, the corrected results are almost identical to those for the undistorted image.


Figure 3: Computed Gaussian edge image with detected edge location, and resulting SFR.

## Residual Error Analysis

Up to now we have discussed the use of ideal, Gaussian edges. We now introduce a more realistic element into our SFR measurement - image noise. For our brief investigation here, we simply add a random noise array to the previously computed edge image arrays. The pixel-to-pixel variations are independent, and at a level consistent with well-exposed digital images. A Normal random variable, (standard deviation $=0.3$ for a [0-255] signal encoding) was added. Note that we expect that this will introduce a variation into the line-to-line edge-finding and the subsequent projection of the image data when forming the edge profile.

[^0] uncompressed TIFF files.


Figure 4: Computed Gaussian distorted edge image with detected edge location: linear (blue dash) and $3^{\text {rd }}$ order polynomial (red circles)


Figure 5: SFR based distorted edge of Fig. 4 computed without correction


Figure 6: Computed Gaussian distorted edge image with detected edge location and corrected SFR

Given this measurement variation, and the previously discussed bias due to image distortion leads us to a statistical approach. ${ }^{6}$ We consider the fitting of the edge location, whether to a first or higher-order function as an estimation problem. As for any statistical modelling effort, examining the remaining residual error is useful.

Figure 6 shows the results of the edge fitting step for the noisy, ideal edge in terms of the residual error. We compute the difference between each line-by-line edge location and the fitted edge (equation) in distance normal to the edge. We see a uniform apparently random error and symmetrical histogram consistent with a good edge model.



Figure 6: Edge location residual variation for ideal edge and linear fit, and corresponding histogram (lower).

For the distorted edge image array, we compute the SFR with an (incorrect) linear fit, and the third-order polynomial. Figure 7 shows both sets of residual error values. As expected, the residual for a linear fit to a distorted edge shows large, non-random variation. However, when the polynomial function is used, the results are similar to those of Fig. 6 for the straight edge feature. Figure 8 shows the probability histogram for these data.

Having looked at the details of the edge finding, we now compare the resulting SFR for both of these cases. Figure 9 shows
remarkably consistent results when polynomial edge fitting is used for the distorted data set. To aid the interpretation, every $5^{\text {th }}$ value for the distorted image has been plotted.


Figure 7: Edge residual for the distorted image of Fig. 4 after linear (left) and third-order (right) edge-fitting


Figure 8: Probability histogram of residual edge position error for distorted image after third-order edge-fitting

## Lens-distortion Correction

Digital processing is often used to correct image distortion introduced by lens aberration. This requires a resampling of the image array, based on known (or estimated) spatial characteristics of the image capture. Resampling involves interpolation of the sampled image, and can be expected to modify the effective SFR based on the output image. We can use the polynomial-based SFR analysis to quantify this effect in the following example.

We started with a computed straight edge image, as in Fig. 3. We then used Adobe Photoshop software to introduce geometrical distortion (modest barrel distortion). This saved image was taken as the distorted input. This was then 'corrected' by applying the inverse operation (pinhole) taken as the corrected image.


Figure 9: Results for the ideal and distorted images (with noise) after advanced edge-fitting SFR analysis.

Edge-SFR analysis was completed for the distorted image. When using the standard, the linear edge-fit result is shown in Fig. 10, labeled as 'Distort. first order.' As expected, the polynomial edge analysis shows higher SFR results, consistent with the previous results. We take this to be the desired SFR. The distortion-corrected image, with a now-straight edge, was then analyzed, and the results are also shown in Fig. 10.


Figure 10: SFR analysis with lens correction: distorted image and 1st-order analysis and polynomial analysis, and $1^{\text {stt}}$-order for corrected (resampled) image

Comparing the SFR results from this experiment allows us to compute the reduction in SFR due to the digital lens correction. This is the difference between the polynomial result for the distorted end, and the result for the corrected image. As we can see, for this example and software, the difference is relatively small (see arrows in the figure).

## ISO 17850 and 19084

While it is certainly possible to correct the above curved edge image feature to improve the SFR measurement, it is helpful to consider this in the context of other standard performance measures. ISO recently released the $17850^{7}$ standard for a
geometric distortion measurement, and ISO $19084^{8}$ for the wavelength (color) dependent nature of optical distortion. For camera system performance evaluation, these (macro) image measures will likely give context to their effects on our (micro) edge-based SFR results.

The methods used for the two distortion standards are based on a test chart with a regular array of dots. These are detected in the test image, and measures of dot-to-dot distance variation define the measure. Figure 11 shows the result of the evaluation in the Quiver and contour plots from the evaluation of a smartphone camera. The quiver plot is from Ref. 9. This (image) fielddistortion analysis gives us insight as to where, and to what extent, image distortion is most extreme. Note also that the left-to-right asymmetry of the apparent geometrical distortion can also indicate misalignment of the camera to the test chart and fixture (keystoning).


Figure 11: Quiver and contour plot representing measured geometric distortion. Each arrow (distance) length is drawn as 150\%. (Ref. 9)

## Conclusions

Thee method for ISO 12233 defines a method based on a straight edge element in a test chart. Image distortion, however, introduces a bias error into the result. We can both detect, and
correct the effect of, this image field distortion by generalizing the fit to the detected edge feature. Edge fitting can be based on polynomial model. Results indicate that the method can be effective, particularly when paired with analysis of the residual errors for the edge-location model.

In addition, this field-dependent distortion can be independently evaluated using two other ISO standard methods. For system testing, the results of such macro distortion can be used to identify image regions of serious distortion, likely to be concern. In addition testing fixture alignment can be evaluated and adjusted prior to full system testing.

## Acknowledgements

It is a pleasure to acknowledge several helpful discussions with members of ISO/TC42 standards teams, in particular Dietmar Wueller and Norman Koren.

## References

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## Author Biographies

Peter Burns (Burns Digital Imaging) is a consultant supporting digital imaging system and service development, and related intellectual property efforts. Previously he worked for Carestream Health, Eastman Kodak and Xerox Corp. He is a frequent conference speaker, and teaches courses on these subjects.

Don Williams is founder of Image Science Associates, a digital imaging consulting and software group. Their work focuses on quantitative performance metrics for digital capture imaging devices, and imaging fidelity issues for the cultural heritage community. He has taught short courses for many years, contributes to several imaging standards activities.


[^0]:    * All computed images were saved as monochrome, 8 -bit,

