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Introduction to WLAN-Based Indoor Positioning of Mobile Devices

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Abstract

Due to limited availability of GPS-like signals indoors, and prevailing deployment of WLAN infrastructure in these environments, many proposed state-of-the-art indoor positioning techniques operate using a collection of WLAN signal measurements, called wireless fingerprints or just fingerprints that quite uniquely relate to user locations. As WLAN infrastructure was not historically designed for localization, the research community addressed several challenges to achieve robust operation of indoor positioning systems. While there are still other problems that hinder broad deployment of indoor navigators, an accumulated critical mass of scientific knowledge in this area is expected to drastically change indoor location-awareness, similar to the GPS revolution for outdoor navigation. This paper reviews main concepts of WLAN localization for a short introduction to this emerging transformative technology.

Introduction

The US Global Positioning System (GPS) transformed the human practice of navigation, by providing a free global service for outdoor location-awareness [1],[2]. Recently, other global navigation satellite systems (GNSS) emerged as well and provide similar services, such as GLONASS [3], Galileo [4] and Beidou [5]. The GNSS systems revolutionized outdoor navigation, and their receivers are broadly deployed in vehicle navigators and integrated into a variety of mobile and pervasive devices including smartphones and activity trackers. They are applied for defense applications and mandated for emergency service operations. GNSS systems eventually created a new global industry that provides various location-based services (LBS).

Existing GNSS signals cannot penetrate the majority of indoor areas, and an essential effort is applied by the research community to enable indoor location-awareness in anticipation of significant transformative impact similar to outdoor applications. An enhancement of GNSS, called assisted GNSS (aGNSS) expands the positioning service coverage to more indoor areas using a supplemental assistance information from wireless communication networks if GNSS receivers are integrated in devices supporting such connectivity. aGNSS is a standardized technology and is commonly integrated into smartphones [6]. But aGNSS is still not able to provide global indoor positioning coverage.

Alternative technologies are considered such as radio-frequency proximity sensors which are also called radio-frequency identification devices (RFID). They provide a beacon location reference when densely deployed [7],[8]. Other proposed signaling methods are Ultra Wideband (UWB) [9], Bluetooth [9], Zigbee [10] to name a few. Deployment of dedicated terrestrial signaling infrastructures for indoor positioning is very costly considering significantly higher accuracy requirements for potential indoor LBS and non-line-of-sight signal propagation in cluttered indoor environments. The new deployments will require substantial maintenance costs as well. For this reason, significant research effort has been concentrated on using already deployed cellular and Wireless Local Area Networks (WLAN) to avoid new deployments. The average positioning accuracy in cellular networks is very coarse, typically hundreds of meters. On the other hand, WLAN signals attracted considerable attention due to dense deployments, potential accuracy of one-two meters, or even more when using antenna arrays. There are many challenges in implementing accurate WLAN-based systems, and while many issues are already addressed, there are still open challenges preventing broad deployment of these methods. The paper will review these aspects in the following.

Conventional Localization

WLAN infrastructure is based on Access Point (AP) basestations that communicate with the users. These APs can serve as reference beacons for user localization applications. The user equipment can discriminate the reception of signals from individual APs as it acquires unique AP identifiers from the signals.

Conventional positioning algorithms can be generally categorized as the following methods: (a) proximity or nearest neighbor (NN); (b) direction-of-arrival (DOA) or angle-of-arrival (AOA); (c) time-of-arrival (TOA); (d) time-difference-of-arrival (TDOA); (e) Fingerprinting methods (FP).

If locations of beacons are known, the simplest localization technique is the proximity beacon, or nearest neighbor solution, i.e., the position of the user is assumed to be the position of the closest beacon as shown in Fig.1a, where $\mathbf{p}_i, \mathbf{p}_j$ denote location vectors of the beacons, and $\mathbf{p}$ is the location of the user. The closest beacon is either one with the strongest power of received signal similar to RFID techniques [7],[8], or the closeness is understood as user-beacon distances estimated from measured signal propagation times multiplied to the speed of light as described for TOA systems in the following. DOA/AOA positioning is based on determining the signal source direction using e.g., antenna array systems [12]. When the directions (angles) of arrivals are known, the position is obtained on the intersection of directional vectors as in Fig.1b.

TOA techniques use user-beacon distances (d_i) obtained from signal propagation times multiplied by the speed of light. Propagation times are half of round-trip-time measurements elapsed between a signal fragment communication and reception of the corresponding acknowledgement [13]. Fig. 1c illustrates the TOA technique, where the user-beacon distances and beacon locations are known, and trilateration system equations can be geometrically represented as finding the intersection of circles (in 2D) or spheres (in 3D) centered at the beacons. For L beacons the following set of equations should be solved for unknown $\mathbf{p}$:

$$d_i = \|\mathbf{p} - \mathbf{p}_i\|_2 , \ i = 1,..., L \tag{1}$$

The TDOA technique is similar to the TOA, except the trilateration equations are written for distance differences, i.e.

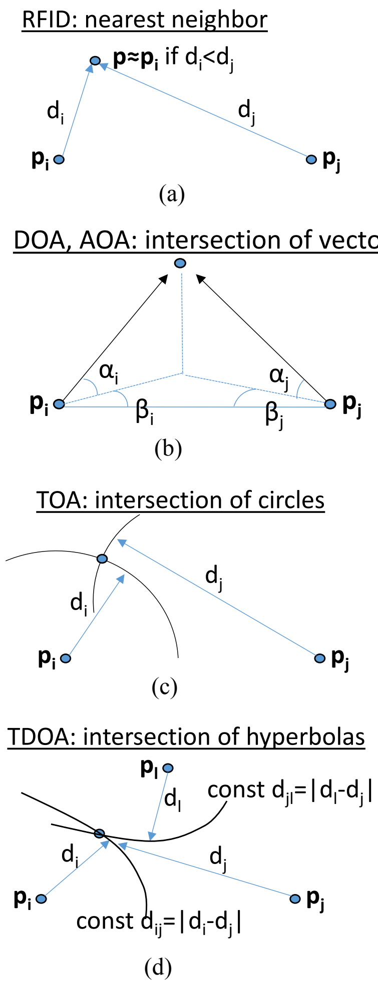

Figure 1. Conventional localization techniques: (a) Proximity AP or nearest neighbor; (b) DOA/AOA triangulation; (c) TOA-based trilateration; (d) TDOA-based trilateration

difference between the user distance from one AP and the user distance from another AP. In this case the system of trilateration equations can be geometrically represented as finding an intersection between hyperbolas (in 2D) or hyperboloids (in 3D) [14], see Fig.1d. The TDOA techniques are useful when the distance measurements have a common bias, e.g., sometimes resulting from a clock bias in the user equipment. Distance differences are free of such common bias.

In practice, extracting RTT measurements in WLAN systems is very challenging and requires consecutive connections of the user equipment with all APs [13]. Also indoor propagation environment is typically non line-of-sight, and the propagation time multiplied by speed of light does not accurately present user-beacon distance. Finally, alternative GPS-like TOA measurements in WLAN are not straightforward, as APs' clocks are also essentially biased.

Very often the distances are measured using signal propagation models, where they are estimated as a function APs' nominal transmission powers and user-received power measurements. Let d be the distance from the AP to the user, d_0 is a reference distance from AP with known nominal reception power P_0, and the received user signal is P. Then the following model can be used for linking all these variables [15]:

$$P = P_0 + 10\gamma \log_{10}\frac{d}{d_0}, \qquad (2)$$

where γ is the path loss coefficient provided by standard models or estimated using measurement calibration campaigns [16]. It represents the rate of fall of RSS in the vicinity of APs. Given measured P, known γ, P_0 and d_0, one can estimate d and use it in trilateration equation, if AP locations are known as well. Unfortunately these models are not very accurate and positioning accuracy is not up to expectations, and typically AP locations are not known as well.

To address the indicated problems a modification of this approach is proposed (EZ method) which assumes that path loss coefficients, transmission powers and APs' locations are not known [17]. A set of calibration equations is used to find APs' locations, path loss coefficients, and APs' transmission powers:

$$\begin{aligned} P_{ij} &= P_{0i} + 10\gamma_i \log_{10}\frac{d_{ij}}{d_0}, \\ d_{ij} &= \left\|\mathbf{p}_j - \mathbf{p}_{APi}\right\|_2 \end{aligned} \qquad (3)$$

where $\mathbf{p}_{APi}$ are unknown locations of APs, index $i = 1,...,I$ identifies APs, and the index $j = 1,...,J$ is used to denote a set of user locations $\mathbf{p}_j$. The quantity d_{ij} is the unknown distance between AP i and user location $\mathbf{p}_j$, while P_{ij} is the known received power measurement from AP i at user location $\mathbf{p}_j$.

Without loss of generality, one can assume that all APs are observable from each user location. The total number of equations is IJ. Considering the 2D localization problem ($2J$ unknowns), and that each AP has four unknowns ($P_{0i}, \gamma_i, \mathbf{p}_{APi}$), the total number of unknowns is $2J + 4I$. Given enough user locations $(IJ \geq 2J + 4I)$, the set of equations can be solved to find all unknowns. There exists ambiguity though that any solution is translation, rotation and reflection invariant. So four known user or AP locations can resolve ambiguities for this extra calibration. Once it's done, the AP-related parameters are estimated, and the system can use equations (3) with known AP parameters to solve the localization problem for the user positions. The EZ method improves the positioning accuracy with respect to the method (2), and it doesn't need parameters of APs. Still, its performance is not as good as it is reported for the fingerprinting methods which are described in the following.

Fingerprinting Localization

Most of the modern mobile devices are equipped with WLAN cards which provide Received Signal Strength (RSS) information that characterizes the power of received signals from observed APs. This information can be extracted using periodic OS-specific retrieval instructions, and there is no need to access the cards directly.

Fingerprinting methods exploit radio-maps of wireless channel specific measurements, i.e., databases of measurements where each entry is unique for each location. As WLAN APs are densely deployed indoors, and RSS measurements can be easily extracted, the sets of RSS readings from the observed APs at each location are commonly used as radio-map entries for WLAN fingerprinting localization. Radio-maps are designed during an offline phase by conducting RSS collection campaigns. Measurements are collected on a grid of locations, called Reference Points (RPs) or survey points, and saved. In the presence of measurement noise, it is preferable to collect multiple copies of fingerprints for application of advanced algorithms. During the online phase, the user equipment captures similar measurements at the user's location, and a positioning algorithm estimates the location by comparing the received data with the radio-map. The big advantage of the fingerprinting approach is that it is based on location-based evidence collection without a need to know APs locations or ensure line-of-sight signaling. The concept is illustrated in Fig. 2 [18].

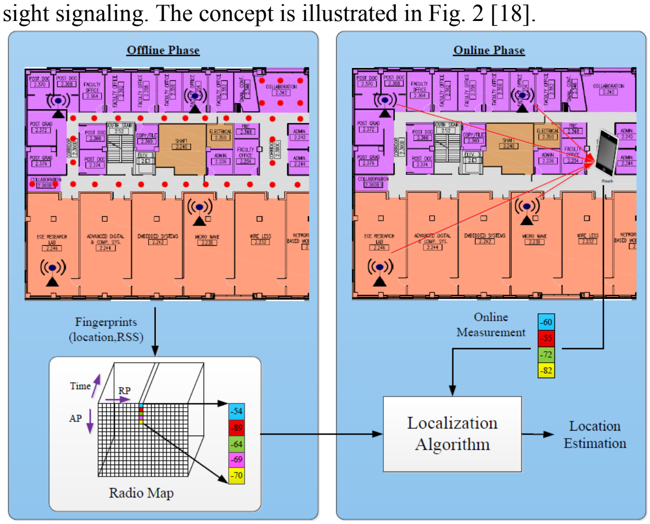

Figure 2. Schematic illustration of WLAN fingerprinting concept. Fingerprint vectors of length five are captured at each RP to design the radio-map, which is used then for localization [18].

The readers are referred to [18] for the more rigorous presentation of the subject, and the notation of [18] will be used in the following.

The grid of RPs is represented by their coordinates $\{\mathbf{p}_j = (x_j, y_j) \mid j = 1,\ldots,N\}$, and at each RP M fingerprint samples $r_j^i(t_m)$ are collected at time instants $t_m, m = 1,\ldots,M$ from access points $AP^i, i = 1,\ldots,L$. The radio-map components are designed as follows

$$\mathbf{R}(t_m) = (\mathbf{r}_1(t_m),\ldots,\mathbf{r}_N(t_m)) = \begin{bmatrix} r_1^1(t_m) & r_2^1(t_m) & \cdots & r_N^1(t_m) \\ r_1^2(t_m) & r_2^2(t_m) & \cdots & r_N^2(t_m) \\ \vdots & \vdots & \ddots & \vdots \\ r_1^L(t_m) & r_2^L(t_m) & \cdots & r_N^L(t_m) \end{bmatrix} \tag{4}$$

$$\mathbf{r}_j^i = \left[r_j^i(t_1),\ldots,r_j^i(t_M)\right]^T,$$
$$\mathbf{r}^i(t_m) = \left[r_1^i(t_m),\ldots,r_N^i(t_m)\right]^T$$
$$\mathbf{r}_j(t_m) = \left[r_j^1(t_m),\ldots,r_j^L(t_m)\right]^T$$
$$m = 1,\ldots,M$$

Also, the "averaged" radio-map is used:

$$\mathbf{\Psi} = (\boldsymbol{\psi}_1,\ldots,\boldsymbol{\psi}_N) = \begin{bmatrix} \psi_1^1 & \psi_2^1 & \cdots & \psi_N^1 \\ \psi_1^2 & \psi_2^2 & \cdots & \psi_N^2 \\ \vdots & \vdots & \ddots & \vdots \\ \psi_1^L & \psi_2^L & \cdots & \psi_2^L \end{bmatrix} \tag{5}$$

$$\boldsymbol{\psi}_j = \left[\psi_j^1,\ldots,\psi_j^L\right]^T,\ \boldsymbol{\psi}^i = \left[\psi_N^i,\ldots,\psi_N^i\right]^T$$
$$\text{where } \psi_j^i = \frac{1}{M}\sum_{m=1}^{M} r_j^i(t_m)$$

A median representative value of the fingerprints is used as well [19]

$$\psi_{med,j}^i = \underset{m\in\{1,\ldots,M\}}{\text{med}} r_j^i(t_m) \tag{6}$$

In online phase, after acquiring RSS measurement $\mathbf{y} = \left[y^1,\ldots,y^L\right]^T$, the positioning algorithm finds the user's location by comparing this measurement with the radio-map:

$$\hat{\mathbf{p}} = (\hat{x},\hat{y}) = g(\mathbf{R},\mathbf{y}) \tag{7}$$

Deterministic Fingerprinting Localization

Deterministic approaches use distance measures $d(*,\mathbf{y})$ to compare online measurements $\mathbf{y}$ with the entries of the radio-map [16]:

$$\hat{\mathbf{p}} = g(\mathbf{R},\mathbf{y}) = \underset{j=1,\ldots,N}{\arg\min}\, d(\breve{\mathbf{r}}_j,\mathbf{y}) \tag{8}$$

where $\breve{\mathbf{r}}_j$ is a representative fingerprint value at RP j such as $\boldsymbol{\psi}_j$ in (5). The Nearest Neighbor (NN) distance metric is a commonly used distance measure which exploits the Euclidean distance [16]:

$$d_{l2}(\breve{\mathbf{r}},\mathbf{y}) = \|\breve{\mathbf{r}} - \mathbf{y}\|_2 \tag{9}$$

As observability of APs varies at different locations, fingerprinting techniques typically use a low signal value (e.g. -90dBm) to substitute fingerprints of non-observable APs.

Distance modifications for deterministic approaches that are robust in the presence of outliers

Fingerprint measurement often includes outliers which degrade mean values of the radio-map $\boldsymbol{\psi}_j$. For example, due to transient effects in WLAN cards, some measurement components y^i might be missing from the online RSS reading $\mathbf{y}$. Such non-observable measurements appear as a common outlier type.

Modifications of Euclidean distance are proposed to minimize outlier impacts. For example, in [19] RSS components y^i and $\breve{r}_j^i$ are compared, and if the online component y^i is not observed, i.e., the WLAN card is not able to detect AP i, but radio-map component $\breve{r}_j^i$ is available, then this component is excluded from the Euclidean distance computation (9).

Another modification of Euclidean distance is proposed in [20], which is using a switching between different distance measures as driven by an outlier detector.

Median-operator based distance measures are also used as more robust against outliers [21] such as

$$d_{med}\left(\breve{\mathbf{r}},\mathbf{y}\right)=\underset{i\in\{1,\ldots,L\}}{\operatorname{med}}\left\{\left(y^i-\breve{r}^i\right)^2\right\} \tag{10}$$

A hybrid approach [22] uses conventional NN approach (9) for position computation (9), and a combination of the median and min operators in a component-wise distance operator for measurement integrity checks in the presence of outliers:

$$d_{comp}\left(\breve{r}^i,y^i\right)=\min\left(|\,y^i-\psi^i\,|,|\,y^i-\psi^i_{med}\,|\right) \tag{11}$$

The Hampel filter [23] also exploits the median function for robustness. RP indices j are omitted in (9)-(11). Outlier-robust fingerprinting techniques exploiting sparse methods will be explained later in this paper.

Distance modification for heterogeneous devices for deterministic approach

One of the major obstacles for broader deployment of fingerprinting techniques relates to measurement differences on various wireless devices. There are many components, such as antennas and amplifiers, which will vary from device to device. It means that a radio-map collected using one device will not apply to another device without a preprocessing. An approach using rank-ordered fingerprints was proposed in [24]. In this approach, absolute values of fingerprints are not used and are replaced by their relative value order called rank order. It is assumed that while fingerprint readings may vary from device to device, and all fingerprints are scaled at the same time, relative strengths are invariant on various devices.

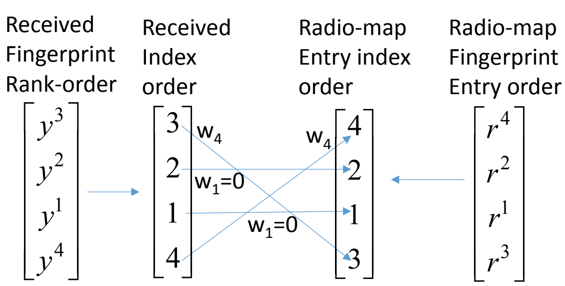

Figure 3. Illustration of distance computation for rank-ordered fingerprints.

In this approach, the uniqueness of fingerprints transforms to a unique ordering of entries. One can differently define distance metrics for rank-ordered RSS measurement sets. One approach is presented in [24]. A more generalized version can be defined as follows. Let us assume that the online reading $\mathbf{y}=\left(y^1,y^2,\ldots,y^L\right)^T$ is rank-ordered to $\left(y^{(1),i_{y,1}},y^{(2),i_{y,2}},\ldots,y^{(L),i_{y,L}}\right)^T$, where the first superscript shows the rank-ordered index, while the second one denotes the original index of that sample. The same rank-ordering will apply to a radio-map entry $\mathbf{r}=\left(r^1,r^2,\ldots,r^L\right)^T$ which converts to $\left(r^{(1),i_{r,1}},r^{(2),i_{r,2}},\ldots,r^{(L),i_{r,L}}\right)^T$, where only AP index is shown. Two sets of vectors will be compared now: $\left(i_{y,1},i_{y,2},\ldots i_{y,L}\right)^T$ and $\left(i_{r,1},i_{r,2},\ldots i_{r,L}\right)^T$. Component-wise distances are defined as follows. First, note that each i_{y,k_1} is equal to only to one i_{r,k_2} (Fig.3). A matching score w_{k_1} is defined as a function of $|\,k_1-k_2\,|$. It is equal to 0 if $|\,k_1-k_2\,|=0$, and it monotonically increases with $|\,k_1-k_2\,|$. The rank-ordered heterogeneous fingerprint distance can be computed as

$$d_{ro}\left(\mathbf{r},\mathbf{y}\right)=\sum\nolimits_{k_1=1}^{L}w_{k_1} \tag{12}$$

Probabilistic Fingerprinting Localization

RSS measurements are statistically random data that can be characterized by conditional probability $f\left(\mathbf{p}_j\,|\,\mathbf{y}\right)$, i.e., probability to observe a given received measurement $\mathbf{y}$ at a location $\mathbf{p}_j$, or $f\left(\mathbf{y}\,|\,\mathbf{p}_j\right)$ - the probability of receiving measurement $\mathbf{y}$ at a given location $\mathbf{p}_j$ [25],[26]. Maximum A Posteriori (MAP) estimation is formulated as follows

$$\hat{\mathbf{p}}=\underset{j=1,\ldots,N}{\arg\max}\,f\left(\mathbf{p}_j\,|\,\mathbf{y}\right) \tag{13}$$

As it is difficult to estimate probabilities $f\left(\mathbf{p}_j\,|\,\mathbf{y}\right)$, (13) transforms to Maximum Likelihood (ML) estimation by applying Bayes rule and assuming the uniform probability of a user location at all RPs:

$$\hat{\mathbf{p}}=\underset{j=1,\ldots,N}{\arg\max}\,f\left(\mathbf{y}\,|\,\mathbf{p}_j\right) \tag{14}$$

The probabilities $f\left(\mathbf{y}\,|\,\mathbf{p}_j\right)$ can be estimated from RSS data histograms at each location, or modeled using analytical functions such as Gaussian.

Given conditional probabilities $f\left(\mathbf{y}\,|\,\mathbf{p}_j\right)$, an averaged position estimation is also proposed [28]:

$$\hat{\mathbf{p}}=\sum_{j=1}^{N}w_j\mathbf{p}_j,\; w_j=\frac{f\left(\mathbf{y}\,|\,\mathbf{p}_j\right)}{\sum_{j=1}^{N}f\left(\mathbf{y}\,|\,\mathbf{p}_j\right)} \tag{15}$$

The probabilistic method is inherently able to isolate outliers if they are observable during the offline surveying phase, as then the system will expect the presence of outliers in the measurements. In practical applications, though, the surveyed data are collected during short periods of time, and may not represent outliers properly.

Probability density function estimation

Two main methods are applied for probability density function estimation: non-parametric and parametric. Typically, many samples of radio-map entries are collected at each location, which is reflected in (3) by indicating different time instants t_m in data collection as $\mathbf{r}_j^i=\left[r_j^i(t_1),\ldots,r_j^i(t_M)\right]^T$.

In non-parametric estimation, probability functions can be estimated using histograms of the RSS measurements at each location (Fig. 4). For practical applications, the histogram bin-granularity is controlled for implementation-viable solutions. Various aspects of such design are considered in e.g., [29] and [30]. This method depends on the number of samples used for histogram designs, and a large number of data might be needed for robust, practical applications.

Alternatively, in parametric methods, the data are represented through analytical distributions, such as Gaussian [28],[30] or more sophisticated models such as Kernel Density Estimation (KDE) [27] and mapping of RSS measurements to the domain of their principal components [31]. In particular, mixture Gaussian KDE design [27] assigns a probability function component, called a kernel, to each observation in the surveyed RSS data. For one observation component, y^i the distribution is written as:

$$f\left(y^i \mid \mathbf{p}_j\right) = \frac{1}{M}\sum_{m=1}^{M} K\left(y^i, r_j^i\left(t_m\right)\right) \tag{16}$$

where

$$K_{Gauss}\left(y^i, r_j^i\left(t_m\right)\right) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\left(y^i - r_j^i\left(t_m\right)\right)^2}{2\sigma^2}\right) \tag{17}$$

and assuming independence of measurements from all APs

$$f\left(\mathbf{y} \mid \mathbf{p}_j\right) = f\left(y^1 \mid \mathbf{p}_j\right) f\left(y^2 \mid \mathbf{p}_j\right) .. f\left(y^L \mid \mathbf{p}_j\right) \tag{18}$$

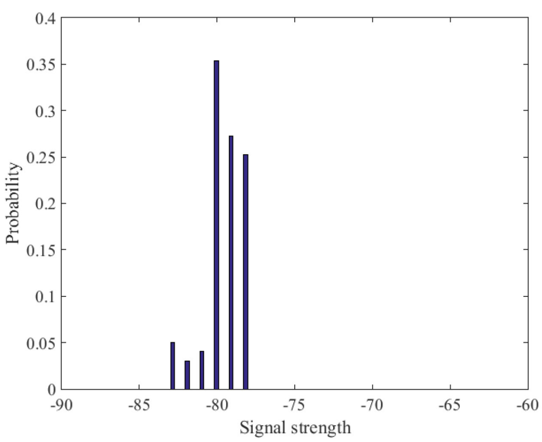

Figure 4. A typical RSS probability distribution obtained from a histogram of experimentally collected RSS measurements

Pattern Recognition Based Fingerprinting

The fingerprinting concept eventually addresses a pattern recognition problem, i.e., identifying radio-map entries which are the closest in some way to the online fingerprint observations, where the fingerprints serve as patterns. The radio-map will be used to train classifiers for proper recognition of fingerprints. One should note that pattern recognition algorithms typically estimate likelihoods of each possible outcome from possible options, which can be used to find an averaged location estimation similar to (14). Popular pattern recognition techniques exploit Support Vector Machines (SVM), Canonical Correlation Analysis (CCA), Neural Networks, and other methods [32], [33].

Sparsity-Based Fingerprinting

One of the newest trends in WLAN fingerprinting is the problem formulation through a sparse optimization problem. Define a location vector as

$$\boldsymbol{\theta} = [0,...,0,1,0,..,0]^T \tag{19}$$

where each element of $\boldsymbol{\theta}$ corresponds to an RP, and ideally only one nonzero element indicates the possible user location. The fingerprint measurement $\mathbf{y}$ is expected to be similar to one of the radio-map entries which is extracted by $\boldsymbol{\theta}$ from the radio-map $\boldsymbol{\Psi}$ as follows assuming a presence of errors ε:

$$\mathbf{y} = \boldsymbol{\Psi}\boldsymbol{\theta} + \varepsilon \tag{20}$$

The set of equations (20) is underdetermined, and possible solutions may not be as sparse as expected in (19). Additional constraints are imposed on $\boldsymbol{\theta}$ forcing it to be as sparse as possible. A Compressive Sensing solution [34],[35] selects the unique solution from (20) minimizing l_1-norm of $\boldsymbol{\theta}$:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \|\boldsymbol{\theta}\|_1 \text{ s.t. } \mathbf{y} = \boldsymbol{\Psi}\boldsymbol{\theta} \tag{21}$$

The sparsity of the solution to (21) depends on properties of the matrix $\boldsymbol{\Psi}$ and is sensitive to the noise component in (20). An alternative solution which overcomes these two issues is in relaxing the exact equation solution condition in (21) and including both components in the optimization. This method is called LASSO or l_1- penalized least squares [36]:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \left[\frac{1}{L}\|\mathbf{y} - \boldsymbol{\Psi}\boldsymbol{\theta}\|_2^2 + \lambda\|\boldsymbol{\theta}\|_1\right] \tag{22}$$

where $\lambda \geq 0$ is a tuning parameter. An extension of this approach considers the possibility of several nearby non-zero entries in (19), which may result from out of grid user locations, noisy measurements, and fingerprint correlations. It appears that the LASSO's extension, called GLMNET [37], improves the performance in that context:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \left[\frac{1}{L}\|\mathbf{y} - \boldsymbol{\Psi}\boldsymbol{\theta}\|_2^2 + \lambda\left((1-\alpha)\|\boldsymbol{\theta}\|_2^2 + \alpha\|\boldsymbol{\theta}\|_1\right)\right] \tag{23}$$

where $0 \leq \alpha \leq 1$ is an additional tuning parameter.

Fingerprinting techniques are often applied after a coarse localization stage to reduce computational burden. The localization area defined by the vector $\boldsymbol{\theta}$ is segmented to clusters first, and then a more accurate "fine" solution is searched within the clusters. Sparse techniques can be integrated with cluster prioritizing to avoid coarse localization errors. Different weights w_k can be assigned to each $k-$th cluster prioritizing the likelihood of user location occurrence in that cluster, and the sparse solution is formulated as:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \left[\frac{1}{L}\|\mathbf{y} - \boldsymbol{\Psi}\boldsymbol{\theta}\|_2^2 + \lambda_1\|\boldsymbol{\theta}\|_1 + \lambda_2\sum_{k=1}^{K} w_k\|\boldsymbol{\theta}_k\|_2\right] \tag{24}$$

where $\boldsymbol{\theta}_k$ is the $k-$ cluster of $\boldsymbol{\theta}$, and $\lambda_1, \lambda_2 \geq 0$ are tuning parameters.

Practical Considerations

Practical solutions take into account computational complexity and robust operation requirements for real-world measurements which often significantly deviate from analytical models.

Presence of outliers and varying reliability of APs

A major aspect of WLAN positioning is the presence of outliers which necessitates the development of corresponding robust techniques. WLAN systems are not designed for localization, and quality of RSS measurements is not guaranteed. There are two approaches addressing quality of measurements: (a) mitigating outliers' impact using more robust localization algorithms; (b) excluding measurements from unreliable APs.

For deterministic techniques, robust algorithmic extensions modify distance measure definitions for RSS measurements. Some of these robust metrics are listed in the deterministic fingerprinting section. Probabilistic techniques have a built-in immunity against outliers which are observable during the offline surveying stage. But they perform poorly in the presence of other online outliers. Pattern recognition methods do have some immunity in isolating outliers, but their performance varies. Recently the authors of this paper proposed new approaches on addressing outlier-immune fingerprinting in sparsity-based fingerprinting methods [36], [38].

Selection of reliable APs has also been addressed in the literature [39]. An intuitive selection of APs is to rely on strongest ones. But AP signal strengths vary from location to location, and it is preferable to consider signal stability as a more reliable criterion. A popular selection technique is based on Fisher Criterion which scores the APs based on associated measurement stability for an area of localization:

$$\zeta^i = \frac{\sum_{j=1}^{N}\left(\psi_j^i - \overline{\psi}^i\right)^2}{\frac{1}{M-1}\sum_{m=1}^{M}\sum_{j=1}^{N}\left(r_j^i(t_m) - \psi_j^i\right)^2}, \tag{25}$$

where $\overline{\psi}^i = \frac{1}{N}\sum_{j=1}^{N}\psi_j^i$ and $i = 1,\ldots,L$.

Computational complexity reduction of localization algorithms using coarse localization preprocessing

One of the most effective ways to reduce the computational complexity of fingerprinting techniques is to use a coarse localization preprocessing. The fingerprinting is essentially a comparison process of online measurements and radio-map entries as represented in (7). A two-tier processing which first identifies an approximate location area using computationally light algorithms followed by more sophisticated processing within the identified area saves significant computational resources. In other words, a clustering of the area of interest is first performed as a coarse preprocessing stage. The following describes representative coarse localization techniques.

For example, a simplified distance measure can be defined for computationally light processing, where APs' presence is used as the main indicator. An AP is considered "present" at an RP location if it is "visible" in terms of exceeding certain RSS threshold γ "most of the time" in the radiomap, e.g. commonly used 90% of time:

$$I_j^i = \begin{cases} 1 & \text{if } r_j^i(t_m) \geq \gamma \text{ for at least 90\% of } t_m\text{'s} \\ 0 & \text{otherwise} \end{cases}, \tag{25}$$

Similarly, one can define AP presence indicator for the online measurement:

$$I_{\mathbf{y}}^i = \begin{cases} 1 & \text{if } y^i \geq \gamma \\ 0 & \text{otherwise} \end{cases}, \tag{26}$$

And AP coverage vectors for radiomap and online measurements are binary and defined accordingly as

$$\mathbf{I}_j = \left[I_j^1,\ldots I_j^L\right] \text{ and } \mathbf{I}_{\mathbf{y}} = \left[I_y^1,\ldots I_y^L\right]. \tag{27}$$

The coarse distance measurement is taken as the Hamming distance between binary vectors:

$$d_H\left(\mathbf{I}_{\mathbf{y}}, \mathbf{I}_j\right) = \sum_{i=1}^{L}\left|\mathbf{I}_j - \mathbf{I}_{\mathbf{y}}\right|. \tag{28}$$

Given this computationally light distance, the clusters can be defined in different ways.

One approach is to set a distance threshold η which specifies the certain area in the vicinity of the online measurements, i.e., define the cluster using the online measurement as a centroid and condition $d_H\left(\mathbf{I}_{\mathbf{y}}, \mathbf{I}_j\right) \leq \eta$.

Another approach splits the location area into clusters offline, using radio-map data, and uses representative centroid measurements for each cluster as the entries of reduced radio-map for coarse positioning [39], [40]. K-means method provides such a technique. Denoting K clusters as $\mathbf{C} = \{C_1,\ldots,C_K\}$, the clustering is performed as the solution of an optimization problem:

$$\hat{\mathbf{C}} = \arg\min_{\mathbf{C}} \sum_{k=1}^{K}\sum_{\boldsymbol{\psi}_j \in C_k}\left\|\boldsymbol{\psi}_j - \boldsymbol{\mu}_k\right\|_d, \tag{29}$$

where $\|..\|_d$ is the used distance metric such as (28), and $\boldsymbol{\mu}_k$ is the cluster head, which can be one of the radio-map entries $\boldsymbol{\psi}_k$ of representative measurements (5), or an average (mean) of all $\boldsymbol{\psi}_j$ measurements within each cluster.

Typically, iterative solutions can be used to solve (29). For example, iterate sequentially the following steps until convergence:

(1) Select initial cluster-heads with corresponding RSS $\boldsymbol{\psi}_k$.

(2) Cluster all RPs by the nearest neighbor approach, associating them according to the closeness of their RSS $\boldsymbol{\psi}_j$ to a selected cluster-head $\boldsymbol{\psi}_k$.

(3) Adjust the cluster-head within each cluster by computing average distance between each cluster entry and other entries in the cluster. Select the entry with minimum average distance as the next cluster-head.

(4) For the new cluster-heads $\boldsymbol{\psi}_k$ repeat steps (2) and (3) until convergence.

The described algorithm extends to the means as well, by defining cluster-heads as averages over all cluster entries in Step 3. Other clustering techniques includ affinity propagation [41] and layered clustering [36], [38] among others.

Facilitating offline radio-map surveying

One of the major obstacles to the global deployment of WLAN fingerprinting techniques is the surveying process of laborious collection of radio-map data. It is both time-consuming and should take into account possible variations of the real radio-map in time.

A dynamic radio-map construction in [42] periodically acquires new measurements at several RPs for calibration and the previous radio-map to estimate the consecutive radio-map is upgrades.

Table I. Comparison of representative fingerprinting approaches

Methods	25% (ft)	50% (ft)	75% (ft)	100% (ft)	Comments	Time (ms)
WKNN [54]	6.69	12.86	27.34	190.62	K=10	0.138
KDE [55]	3.84	11.71	28.75	217.85	Gaussian Kernel is used	134
Contour-based [56]	56.84	125.31	187.25	261.5	Estimates path loss parameters	312
CS [35]	1.87	4.48	13.64	335.91	Solved via l1-magic [194]	2.21
GLMNET [36]	1.3	3.16	7.03	218	Developed approach solved via [195]	3.41
LASSO [36]	0.35	1.71	4.76	44.32	Developed approach solved via [195]	1.53
GS [38][53]	1.2	4.1	8.6	22	Developed approach solved via [196]	8.29

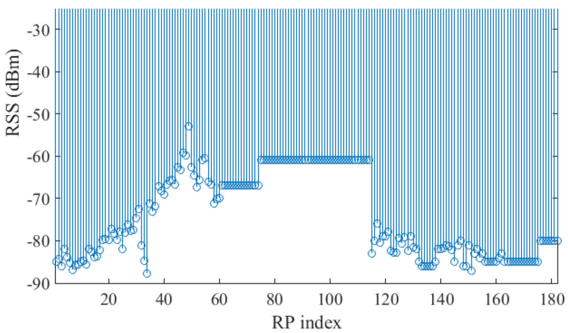

Figure 5. Relatively slow changes in RSS measurements on consecutive RPs justify the usage of interpolation techniques.

Crowdsourcing approaches involve volunteers for the collection of radio-map data [43], [44]. Some of them exploit known markers such as room identification, while others rely on previous radio-map data to extract new data and update the map. The major difference is in deviating from one-expert-surveyor-multiple-user model to multiple-volunteer-surveyor-multiple-user model. The data collected by volunteers are typically less reliable and collected during shorter periods of time compared with conventional fingerprint collection. There were attempts to identify more stable representative fingerprints from frequency of occurrence. For example, [43] ignores RSS records that are far away from the most-frequent RSS values and gives the maximum weight to the ones that have their peak value in an RSS histogram. Another challenge in data collection is that they are collected by various devices. Common approach is to use relative measurements which help to avoid heterogeneous source problem. In [43], the authors use the approach of [24] of rank-ordering RSS vectors and complement it with a set of weaker APs with a delta range of dBs.

Another approach to reducing the surveying effort is collecting measurements on coarser grids and interpolating measurements on the finer grid. One can accelerate the radio-map surveying several times. Early methods used radio-propagation path-loss models calibrated by the actual fingerprints measured at selected observation points.

Recent approaches exploit conventional interpolation techniques for relatively smooth surfaces. For example, linear interpolations of RSS measurement are common [45]. Assume that there are three non-collinear vertices $\mathbf{p}_{j_1}, \mathbf{p}_{j_2}, \mathbf{p}_{j_3}$. Each point $\mathbf{p}_j$ located inside this triangle can be represented by so-called barycentric coordinates:

$$\mathbf{p}_j = \lambda_1 \mathbf{p}_{j_1} + \lambda_2 \mathbf{p}_{j_2} + \lambda_3 \mathbf{p}_{j_3}, \text{ where } \lambda_1 + \lambda_2 + \lambda_3 = 1, \quad (30)$$

The linear interpolation of fingerprints follows the same methodology using the barycentric coordinates of (30):

$$r_j^i = \lambda_1 r_{j_1}^i + \lambda_1 r_{j_2}^i + \lambda_1 r_{j_3}^i. \quad (31)$$

Fingerprint data at consecutive RPs are typically changing slowly (Fig.5) and can also be interpolated using other techniques as well including frequency domain methods [38]. In particular, the LASSO method applies. Let $\mathbf{b}^i$ be a set of fingerprints on RPs from AP i on a coarser grid, and the set of RSS on the finer grid is $\boldsymbol{\psi}^i$ from (5). Using a given subset selection matrix $\mathbf{A}$, one can have $\mathbf{b}^i = \mathbf{A}\boldsymbol{\psi}^i$. Interpolation algorithm uses a sparse representation of $\boldsymbol{\psi}^{f,i}$ in frequency domain, using Discrete Fourier matrix $\mathbf{F}$ as follows:

1. Represent: $\mathbf{b}^i = \mathbf{A}\mathbf{F}^{-1}\boldsymbol{\psi}^{f,i}$. (32)

2. Solve: $\hat{\boldsymbol{\psi}}^{f,i} = \arg\min_{\boldsymbol{\psi}^{f,i}} \left[\frac{1}{N^b} \left\| \mathbf{b}^i - \mathbf{A}\mathbf{F}^{-1}\boldsymbol{\psi}^{f,i} \right\|_2^2 + \lambda \left\| \boldsymbol{\psi}^{f,i} \right\|_1 \right]$

3. Interpolate: $\boldsymbol{\psi}^i = \mathbf{F}^{-1}\boldsymbol{\psi}^{f,i}$.

Heterogeneous devices

Diverse WLAN devices acquire different RSS measurements at the same location. It impacts performances of fingerprinting methods as offline survey data collected by one device will not match online measurements collected by another user-equipment [46]. To address this issue several directions are proposed in the literature: (a) use of mapping functions which align RSS measurements from different devices; (b) selection of device-independent features derived from conventional fingerprints; (c) normalization of RSS.

Mapping techniques typically assume radio-map measurements are captured by one device. Then a set of linear RSS conversion formulae is derived for other devices [47]. It was observed that the effects of hardware variation and some time-varying phenomena appear to be linear when RSS measurements are in dBs, and the calibration function can be of the following form for an RSS value r:

$$C(r) = C_1 \cdot r - C_2 \quad (33)$$

Thus, one should find parameters C_1, C_2 to transform RSS data to the desired format. E.g., one can collect some measurements at known locations and compute the least-squares fit between the observed values and the corresponding values from the radio-map. The issue is further progressed in [48] to automate mapping function design. The RSS readings from the uncalibrated device are first labeled with a coarse location exploiting correlation ratio computed from the Pearson product-moment correlation coefficient, which is computed from a radio-map entry and online measurements. Then learning algorithms are applied to train the mapping function.

Selection of device-independent features can include relative information which is not dependent on the devices. In particular, rank-order based distance computation in [24] and in (12), are device independent. In [43] rank-ordered RSS are complemented with a set of weaker APs within a given delta range in dBs.

As RSS measurements are collected in dB scale, any hardware variations in terms of multiplying to various gains appear as varying additive components. Signal Strength Difference (SSD) between RSS measurements from various APs eliminates this additive gain component and is invariant to heterogeneous platforms [46]. Considering L RSS readings from all APs, there are only $L-1$ independent SSDs. A similar technique exploiting RSS difference, called DIFF, was proposed in [50]. A method called hyperbolic location fingerprinting (HLF) [51] utilized signal strength ratios as fingerprints. A variation of SSD is proposed on using both SSD and RSS values in the Delta-Fused Principal Strength (DFPS) approach [49]. DFPS multiplies a combined vector of SSD and RSS to a linear projection matrix to extract L features. The matrix is obtained through principal component analysis (PCA).

So-called standardized fingerprint modification was introduced in [52], which operates on $\boldsymbol{\psi}_j = \left[\psi_j^1,...,\psi_j^L\right]^T$. For simplicity, we omit the RP index j and use $\boldsymbol{\psi} = \left[\psi^1,...,\psi^L\right]^T$ instead. Then the standardized fingerprint is defined as:

$$\begin{aligned} \hat{\boldsymbol{\psi}} &= \frac{1}{\hat{\sigma}}\left[\psi^1 - \overline{\psi},...,\psi^L - \overline{\psi}\right]^T \\ \overline{\psi} &= \frac{1}{L}\sum_{i=1}^{L}\psi^i;\quad \hat{\sigma} = \sqrt{\frac{1}{L}\sum_{i=1}^{L}\left(\psi^i - \overline{\psi}\right)^2} \end{aligned} \tag{34}$$

Conclusion

This paper introduced major aspects of WLAN fingerprinting localization. This includes main approaches in localization algorithms along with several aspects of practical design such as two-tier coarse-fine localization, reduced complexity surveying approaches, and using heterogeneous devices for data collection and user-localization. Table I compares several popular fingerprinting-based methods including three last entries proposed by authors which outperform conventional methods. More detail description is provided in [18].

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