# Methods of describing a gamut boundary based on a face/vertex encoding 

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#### Abstract

Colour gamuts can be described as a list of vertices and a list of triangular faces connecting these vertices. This method of encoding a colour gamut is convenient for both gamut mapping and gamut volume calculation. Particularly where the vertices describe a surface that is non-convex, as in most print processes, it can be difficult to obtain a face list that produces a connected and nonoverlapping surface. Methods for obtaining a face list from characterization data were evaluated using data from a wide range of printing processes, and it was found that defining a mesh and corresponding triangulation in CMYK space gave consistent results across all the data sets.


## Introduction

In colour management it is increasingly common to represent a colour gamut as a set of vertices on the gamut surface in a Cartesian colour space, together with a set of indices into the list of vertices which define the faces of the surface. This method has been adopted for example in GamutID for RGB encodings [1] and in the iccMAX specification [2].

Once the face and vertex lists are defined, the gamut boundary can be communicated and used for a number of applications, such as gamut mapping, gamut volume calculation, and gamut comparison.

A number of procedures for defining gamut boundaries exist, notably the Segment Maxima Gamut Boundary Description [3, 9]. However, we believe there is a need for one or more simple and robust method that will permit a user to generate a face/vertex boundary description from either characterization data or an ICC profile, and which could be a candidate for standardization.

For displays, the additive mixing model and the threecomponent colour channels make it relatively straightforward to define a suitable face-vertex list [15]. However, for subtractive printing, with four or more channels, it is less straightforward to define a procedure which will work robustly in generating the description. For example, if a set of device values known to be on the colorant boundary is selected it may be non-trivial to define a unique triangulation of the set that leads to a smooth, connected and non-overlapping boundary description. In part this is caused by the existence of concave regions within the gamut.

## Applications

The goal of this work is to evaluate procedures for defining colour gamuts that support the following applications:

1. Gamut encoding using a face/vertex description
2. Gamut visualization
3. Gamut mapping
4. Gamut volume calculation
5. Comparison of the gamuts of different printers

The triangles encoded as the gamut boundary should be connected and non-overlapping. The procedure needs to be robust and accurate, while at the same time being possible for an end user to apply, ideally with no additional tools other than typical commercial software such as Adobe Photoshop.

Colour gamut vertices are commonly encoded as CIELAB coordinates, and the colour space of the data to be encoded as the colour gamut depends on the use case requirements. For surface colours it is usual to encode colorimetry relative to a perfect reflecting diffuser, but in the case of comparing surface colours with displays, where it is usual to map white point to white point, the data can be transformed to media-relative colorimetry. Comparing the gamuts of displays with different peak luminances cannot be done in the CIELAB space as there is no common reference white, and is not considered in this paper.

A further consideration is whether the gamut is encoded in terms of its characterization data, or in terms of a profile representing this data. A profile will have been built according to a set of separation rules including the total allowed coverage of the colorants (TAC) and the gray component replacement (GCR) strategy, which will constrain the available gamut especially in dark regions. Both measured and profiled gamuts are of interest, so need to be supported.

## Gamut encoding requirements

The main requirements of a gamut encoding procedure are that it should be:

Reproducible - while it is not essential that the same faces and vertices are always returned from a given data set, the faces and vertices should be functionally equivalent so that they give the same results in the applications listed above.

Connected - the faces must form a single continuous surface, with no overlaps or duplication.

Outward-facing - for each face, the vertices should be listed in clockwise order when viewed from outside the gamut

Accurate - all the vertices should lie on the gamut surface, within a small tolerance

Each face is encoded as a set of three indices into the vertex array which identify the vertices of the face in the three-dimensional space in which the vertices are encoded (most commonly CIELAB).

Although each face, and the set of faces which form the gamut, must conform to the requirements above, it is not essential for the faces or vertices to be arranged in any particular order.

In addition to the above requirements, the method should be straightforward to implement so that it can be used by any reader of the relevant standard without advanced specialist knowledge or tools.

## Gamut boundary encoding methods

Arbitrary vertices on a gamut surface cannot readily be triangulated to meet the criteria outlined above. As shown in Figure 1, even the well-defined gamut surface coordinates of the Perceptual Reference Medium Gamut [14] give rise to a poorly-connected set of faces when triangulated using an n-dimensional Delaunay algorithm.


Figure 1. PRMG gamut from Delaunay triangulation

If the surface vertices form a strictly-convex hull, they can be triangulated successfully using Delaunay triangulation. However, surface colour gamuts are rarely convex, and the convex hull typically has a volume $8-9 \%$ larger than that obtained from the actual surface. For some applications, such as a simple visualization, this may be sufficient but is not adequate for accurate volume calculation or gamut mapping.

To meet the criteria above it is necessary to generate a regular mesh where the triangulation is defined by the mesh structure. Two basic approaches are considered here:

1. Defining the mesh in device coordinates and using the corresponding CIELAB coordinates to define the vertex array and
2. Defining a mesh in CIELAB coordinates and clipping this to the device gamut.

## Device coordinate mesh

The device coordinates that lie on the gamut surface for a CMYK process will be those where one or two of the CMY primaries is zero and black is zero (for vertices at and above the cusp lightness) and those where one or two of the CMY primaries is $100 \%$ and black is non-zero (for vertices below the cusp). This principle is followed in the CMYK Gamut Boundary Target [6], which is arranged in dimensions of varying lightness and primary colorant combinations, forming a mesh in colorant space. The target can be printed and measured to obtain the gamut vertices. It is also possible to assign an ICC profile for a printing process and convert the target image to CIELAB colorimetry.

In many situations characterization data exists for the process whose gamut is required. The CMYK values in the Gamut Boundary Target do not match those in standard characterization data sets [13]. Using the gamut surface coordinates in ISO 12642 (as defined in ISO 12647-7), it is possible to extract an array of gamut vertices following the principle described above for the Gamut Boundary Target.

## CIELAB mesh

A regular mesh can be defined in CIELAB space that can be clipped to the gamut surface in question. The regular geometries spheroid and cuboid could be considered for this initial mesh, but have disadvantages which arise from the difference in their geometry relative to a typical print gamut: a spheroid is unlikely to enclose all colours in a surface colour gamut, particularly in the yellow region; and a cuboid risks projecting nodes from the corners in an irregular fashion to the gamut surface, leading to faces being rotated or incorrectly located. Another approach is to use an initial gamut which is a superset of print gamuts, which will undergo less distortion when clipped to the gamut surface. The Perceptual Reference Medium Gamut, defined in ISO 12640-3, is a candidate for such a superset gamut. In high-chroma printing processes it is possible that some colorants may lie just outside the PRMG boundary, but taking the media-relative CIELAB coordinates as defined in ISO 12640-3 and clipping them to the gamut surface of the target gamut defined in CIELAB coordinates relative to the perfect reflecting diffuser ensures that this problem does not occur.

CIELAB spherical coordinates are also used in the Segment Maxima Gamut Boundary Descriptor [9], which encodes the surface intersection along rays originating from a centre point within the gamut solid.

## Applications

## Gamut encoding using a face/vertex description

A gamutBoundaryDescriptionType is defined in iccMAX. The gamut is encoded as a $v \times 3$ array of vertices in PCS coordinates, an $f \times 3$ array of indices into the vertex array which identify the faces, and an optional $v x c$ array of device coordinates, where $v$ is the number of vertices, $f$ is the number of faces and $c$ is the number of device channels. The iccMAX Reference Implementation [12] includes the IccXml tool to convert between binary and humanreadable XML representations of a profile, and an example XML encoding of a gamutBoundaryDescriptionType is provided at http://www.color.org/iccmax/gamut.

## Gamut visualization

Graphics libraries such as OpenGL [10] support the rendering of patch objects by means of arrays of faces and vertices.

## Gamut mapping

Gamut mapping can be performed with a face/vertex gamut boundary description, using the intersection of the mapping vector with the plane of the gamut face. A detailed description was given in Green [7].

## Gamut volume calculation

Given a face/vertex gamut boundary description, the volume of the gamut can be computed by adding a point at the gamut centroid, and computing the volumes of the resulting tetrahedra, as described in Deshpande [4].

## Gamut comparison

A method for comparing two gamuts in terms of their volume and intersection was described in Deshpande [4].

## Verifying connectedness

Euler's formula is used to verify that the faces connect to form a closed polyhedron. This formula states:

$$
\begin{equation*}
E=v-e+f \tag{1}
\end{equation*}
$$

where $v, e$ and $f$ are respectively the number of vertices, edges and faces. $E$ is the Euler number and is always 2 for a simplicial polyhedron.

For a given face array $T$ and vertex array $V, e$ can be determined as follows:

1. Verify that all the faces are unique,
2. For the three columns of $T, T_{1}, T_{2}$ and $T_{3}$, form the three $t \times 2$ arrays $\left[T_{1} T_{2}\right],\left[T_{2} T_{3}\right]$ and $\left[T_{1} T_{3}\right]$, representing the set of edges,
3. Sort these arrays so that each row is in ascending (or descending) order
4. Count the number of unique rows.

If the faces are not unique, the Euler number may not be 2 and while the faces may appear connected when visualized, duplication will lead to errors in a volume calculation.

## Implementation

The following methods were tested on a range of different print characterization data sets and profiles:

- Arrangement of a data set as a spheroid
- Arrangement of a data set as a cuboid
- Compression from PRMG coordinates

For each case a triangulation was performed, and vertices obtained. The resulting face/vertex arrays were used to visualize the gamuts and to compute the volumes. The visualizations were tested to identify any incorrectly-connected faces, and the gamut volumes were calculated.

## Profiles

Profiles were generated from 20 different standard characterization data sets provided by CGATS, FOGRA and JapanColor, representing a large range of gamut sizes. Three largegamut ink-jet characterization data sets were also used. The profiles were made using ProfileMaker 5, using the same separation settings throughout with medium GCR and a TAC of 320 .

## Device coordinate mesh - spheroid arrangement

The CMYK Gamut Boundary Target [6] is arranged so that each column has a constant CMY colorant ratio, varying in 24 steps from a light tint to the cusp where one or more colorants is at $100 \%$, and below the cusp where black is progressively added until it reaches $100 \%$. There are 18 columns with different CMY colorant ratios. The layout thus approximates spherical coordinates of lightness and hue.

The triangulation of the target was described in Green [7]; the faces step through the rows of the target until the faces of all rows are defined. To close the spheroid, upper triangles in the first row and bottom triangles in the last row are not included.

Figure 2. CMYK Gamut Boundary Target

The CMYK Boundary Target image was assigned to different CMYK profiles, and subsequently transformed to CIELAB using the ICC-Absolute Colorimetric rendering intent.

An example of this triangulation applied to the FOGRA51 data set is shown in Figure 3.


Figure 3. FOGRA51 data set triangulated using the spheroid CMYK mesh

## Device coordinate mesh - cuboid arrangement

In the cuboid arrangement described by Mahy [11], the gamut surface is segmented into different $2-\mathrm{ink}$ and 3 -ink faces, covering the different permutations of $\mathrm{C}, \mathrm{M}, \mathrm{Y}$ and K on the surface, as shown in Figure 4. Each face comprises steps from zero to $100 \%$ colorant.


Figure 4. Faces of a CMYK cube sampling the outer gamut

A gamut boundary image and a corresponding triangulation were generated according to this description. The image was assigned to different CMYK profiles and transformed to CIELAB as above.

An example of the CMYK cuboid triangulation applied to a data set is shown in Figure 5.


Figure 5. FOGRA51 data set triangulated using the CMYK cuboid mesh

## CIELAB mesh

Gamut boundary data for the Perceptual Reference Medium Gamut is given in ISO 12640-3 [14] and in the ICC specification [8]. The PRMG coordinates form a regular mesh in CIELAB space, and a triangulation for this mesh was derived.


Figure 6. Triangulated Perceptual Reference Medium Gamut

A 16-bit CIELAB image representing the PRMG coordinates were converted to the colour encoding of each of the test profiles, and then converted back to CIELAB so that the PRMG coordinates are clipped to the CMYK gamut. As in the cognate triangle method [7], the triangulation defined for the PRMG also applies to the CIELAB values clipped to the CMYK gamut.

An example of this triangulation applied to the FOGRA51 data set is shown in Figure 7.


Figure 7. FOGRA51 data set triangulated using the PRMG CIELAB mesh

## Results

All of the triangulation methods described above have face, edge and vertex numbers which give an Euler characteristic number of 2, indicating that the tessellation represents a closed volume in 3D-space, and there were no visible incorrectly-connected faces.

The volumes calculated using the sphere and cuboid CMYK triangulation methods agreed closely, varying by an average of $0.3 \%$. The convex hulls of the two triangulations differed by an average of only $0.1 \%$.

The PRMG-based method predicted consistently smaller gamut volumes, on average $90 \%$ of the other methods. This may possibly be due to the extra conversion from CIELAB to device coordinates. The gamut vertices of all the data sets were inside the PRMG, including the ink jet printers and clipping was avoided.

Applying the triangulations to the CIELAB image data and computing the volumes for the 20 data sets took approximately 3 seconds on a Windows 7 machine.

## Conclusions

Three methods have been evaluated for describing a gamut boundary using a face/vertex encoding. The three methods are all very simple and fast to implement using the given gamut images and triangulations. The two CMYK-based methods are in close agreement, while the PRMG-based method, which has an additional conversion step, predicts smaller gamut volumes. The PRMG-based method also suffers from clipping of high-chroma ink-jet printer colours. Both CMYK-based procedures can be recommended for determining and encoding the gamut of a printer using a face/vertex description.

The methods described apply only to the gamut that is obtained from a workflow where the available colours are restricted by the profile used. The CMYK cuboid method could be adapted to use a mesh based on colours found in characterization data sets, so that the printer gamut can be evaluated directly from data without the effect of the profile.

It is also noted that CIELAB is not an ideal space for analyzing gamut volumes, and a more perceptually uniform space would give a more realistic prediction of apparent gamut volume.

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