

Non-blind Image Deconvolution using Sampling without Replacement

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Abstract

Image degradations can be modeled as a process of linear systems which are usually denoted by convolution. Deconvolution refers to a reverse operation of the linear system in which an original image is convolved with a blur kernel, which is also known as a point spread function (PSF) of the linear system. If the blur kernel, which can represent linear degradations such as an out-of-focus blur or motion blurs due to the shake of a camera, is known, we call this ill-posed problem the non-blind deconvolution problem. In this paper, we propose a non-blind deconvolution method using a convex optimization method in which a non-derivative approach is used to solve the ill-posed problem. The proposed method minimizes the objective function using the stochastic process in which the random variable selects the coordinate. The objective function is minimized along the selected coordinate direction at each iteration. If several coordinate directions are picked simultaneously, the cost can be decreased independently along each coordinate direction.

Introduction

Image deblurring refers to the process of estimating an intrinsic image from degraded observations using prior information which is associated with the degrading system or the original image. Image deconvolution plays a key role in various problems where the image degradation can be supposed to be a linear process. The PSF serves an amount of information about the degradation system. However, there still exist uncertainty with the PSF and we call this problem non-blind deconvolution problem. The non-blind deconvolution algorithm, which serves the solution to the non-blind deconvolution problem, is used for various image processing problems, especially in the blind deconvolution method in which the PSF is also unknown. Non-blind deconvolution methods can be extended to the kernel estimation which is a critical process in the blind deconvolution algorithms. Image deconvolution has an amount of applications in various areas such as image restoration, remote sensing, medical image processing, astronomical image processing, etc.

Many deconvolution algorithms assume that the degradation is a linear system. The image deconvolution problem to restore an intrinsic image from a single degraded image can be modeled using a linear convolution:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (1)$$

where $\mathbf{y}$ is a given observation which is usually a blurry image, $\mathbf{x}$ is the intrinsic image, $\mathbf{n}$ is an error term generally modeled as additive white Gaussian noise (AWGN), and $\mathbf{H}$ is a system matrix. The system matrix, usually having a form of a block circulant with

circulant block (BCCB) matrix, has an assumption that the degradation is a linear process based on the linear convolution. Even if the PSF of the system is known, there still exist uncertainty in this ill-posed problem. In the problem of non-blind deconvolution, noise which is usually modeled as AWGN and boundary problems are always taken into account to decrease the uncertainty. Additionally, there exist many non-linear factors in image processing modules such as dead pixels, hot pixels, non-linear in-camera processing, nonlinear camera response curve and the noise. Sometimes noise can be modeled as Poisson noise or film-grain noise which have different characteristics with AWGN [1]. Overcoming outliers mentioned so far is a difficult problem to be solved so various approaches have been tried for many years.

Various non-blind deconvolution methods have been proposed in last decades. Traditional image deconvolution methods include Wiener filter, constrained least square (CLS) filter, Richardson-Lucy (RL) deconvolution, etc. The Wiener filter, which is closely related to the mean square error between the estimated intrinsic image and desired image, is widely used to restore the noisy observation [4]. CLS filter uses the Laplacian operator as a prior knowledge with reducing the computational requirement by introducing the fast Fourier transform (FFT) [2]. RL deconvolution algorithm, which is also known as Lucy-Richardson deconvolution, uses Bayesian approach to estimate the intrinsic image [5, 6]. Nowadays, non-blind deconvolution algorithms try to estimate the intrinsic image by suggesting regularizations such as total variation. For example, stochastic process is used to estimate the intrinsic image and cost functions that contain the L1 norm is suggested [7].

In this paper, we propose the non-derivative image deconvolution algorithm in which a stochastic approach is used. First, in the section of related works, we categorize the image deconvolution algorithms into several groups and analyze gradient-based algorithms and non-derivative algorithms. In the section of the proposed method, a deconvolution algorithm, which does not use the gradient of the cost function, is introduced. In the section of experimental results, several results of simulations using test images are demonstrated and the values of peak signal-to-noise ratio (PSNR) are also shown to compare the performances of a conventional method and the proposed method. Lastly, in the section of conclusions, we conclude our paper and discuss the potential of our method.

Related works

Many non-blind image deconvolution methods are based on least squares estimation which is a standard approach in regression analysis to the estimated solution. Least squares estimations can be categorized into two groups, i.e., linear least squares and

non-linear least squares estimations. In the case of linear least squares estimation, a unique closed-form solution is suggested and this solution can be regarded as unpleasant restored image in general. The singularity cannot be overcome by this approach. Especially in image processing, artifacts can be observed in the frequency region. In the case of the non-linear least squares estimation, however, the solution of the problem can be selected from the set of feasible solutions. Many iterative image deconvolution algorithms fall into the non-linear least squares estimation and select the solution by incorporating the prior knowledge as regularization. The most appropriate solution is selected from the set of feasible solution sets based on the suggested regularization.

The image deconvolution problem can be regarded as an ill-posed problem that contains uncertainties as mentioned before. The non-linear least squares estimation reduce the uncertainty by using the iterative approaches. Various iterative approaches can be constructed to solve the under-constrained problem incorporating information about the degradation system or the assumption of the intrinsic image.

Deconvolution algorithms often use the gradient of images because the gradient implies for the high frequency components in terms of Fourier analysis. An amount of image degradations such as defocusing problem or motion blur problem often degrades the regions of high frequencies more severely. Generally, the blur kernel works as a low-pass filter so the high frequency component is vulnerable to these degradations. Image deconvolution process requires inpainting, especially on edges, to restore the loss of high frequency components of degraded images. As in Fig. 1a, gradient-based methods look for the negative direction of the gradient at the current point. In this paper, the steepest gradient descent method was implemented to estimate the original image and was compared with the proposed algorithm that is a non-gradient-based method.

The cost function can decrease along the direction of coordinate as represented in Fig. 1b. The coordinate descent method solves the optimization problem by successively performing approximate minimization along coordinate directions. The minimization along the coordinate direction can be seen as a simplified version of the original ill-posed problem. By the simplification, we can update each pixel by considering the neighboring pixels. The optimization strategy based on the coordinate descent method is proposed. This method does not use the information of the gradient of the cost function and has similar complexity to the method with the steepest gradient method.

In this paper, we propose an iterative non-blind deconvolution method in which the stochastic process is used to estimate the original image. Our method would demonstrate the high resolution of intrinsic image with its own potential to decrease the processing time although its convergence property is similar to the steepest descent method. The proposed method also focuses on the recovery of high frequency components which have large loss during the degradation though the pixel-wise estimation.

Proposed method

A pixel has a higher correlation with its adjacent pixels rather than pixels apart from its coordinate in the image plane. The image degradation assuming a linear process considers that the degraded pixel is obtained as a weighted sum of adjacent pixels from the original images based on the estimated blur kernel. In

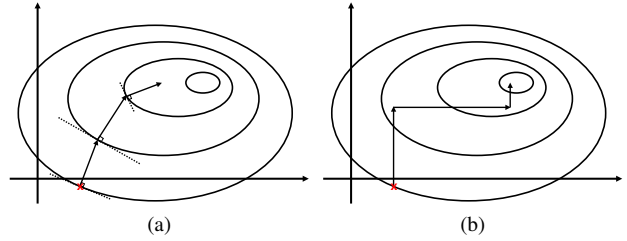


Figure 1: (a) Optimization method in which the gradient information is used. (b) Iterative algorithm which does not use the gradient information. The cost function decreases along the coordinates

this paper, we introduce an objective function which is differentiable. This differentiable cost function is minimized with local cost functions and the stochastic process. The objective function consists of local cost functions that are a set of adjacent pixels centered at every pixel in the image plane. The number of local cost functions is equal to the number of the image pixels. Once we select a coordinate (i.e., a pixel) to decided which local cost function to be decreased, the pixel will be updated via minimization of local cost functions. Stochastic strategies will be mentioned to decide which strategy can make the cost function much smaller.

Pixel-wise optimization

In this section, a convex optimization with a strictly convex cost function is derived by a non-derivative optimization method to compare with the gradient-based method. The cost function here in suggested is a data fidelity term with the quadratic regularization using the Laplacian[2]:

$$F(\mathbf{x}) = \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 + \lambda \|\mathbf{C}\mathbf{x}\|^2, \quad (2)$$

where $\|\cdot\|_2$ denotes the L2 norm, $\mathbf{C}$ denotes a Laplacian operator and λ is a regularization parameter. The suggested cost function is twice differentiable so second derivatives are available. This characteristic helps to compare the proposed method with the gradient based method mentioned in the previous section.

In our method, the suggested cost function decreases along the coordinate. Minimization of the cost function along a direction of the coordinate works by fixing all the components of the vector, $\mathbf{x}$, except for a pixel which corresponds to the selected coordinate as below

$$x_i^{k+1} = \arg \min_{\varepsilon} F(x_1^k, \dots, x_{i-1}^k, \varepsilon, x_{i+1}^k, \dots, x_N^k), \quad (3)$$

where k denotes the index of iterations, i denotes the index of components in the vector $\mathbf{x}^k$, x_i^k is the i^{th} component of vector $\mathbf{x}$ on k^{th} iteration, N is the dimension of the vector space or the size of images and F denotes the cost function suggested above. This minimization process of (3) can be regarded as an optimization for the single variable so the i^{th} component can be obtained by satisfying

$$\frac{\partial F(x_1^k, \dots, x_{i-1}^k, \varepsilon, x_{i+1}^k, \dots, x_N^k)}{\partial \varepsilon} = \frac{df_k(\varepsilon)}{d\varepsilon} = 0, \quad (4)$$

where f_k is the local cost function that measure the cost of the corresponding region that contains the selected pixel and its neighboring pixels. If the certain pixel can be regarded as a variable

to be calculated, other neighboring pixels can be considered as constants during the minimization of the corresponding local cost function.

We can estimate the local data fidelity and the local prior information using the local cost functions. Theoretically, if we minimize the cost function through optimizations of the local functions with the same number of access, convergence can be guaranteed[3]. Then the cost function decreases at every iteration as below

$$F(\mathbf{x}^0) > F(\mathbf{x}^1) > F(\mathbf{x}^2) > \dots, \quad (5)$$

where the $\mathbf{x}^0$ indicates the initial state which is the image that we obtained from the device, i.e., $\mathbf{y}$ in (1).

The pixel-wise optimization using the local cost functions requires the neighboring pixels of the certain pixel corresponding to the variable x_i^{k+1} for minimization. The neighboring pixels can be extracted by considering the PSF. When we extract neighboring pixels from the image plane, the pixels that have correlation with the selected pixel should be considered and the relationship between the original cost function F and the local cost function f_k is as below

$$F = \frac{1}{l \times l} \sum_{k=0}^N f_k(\epsilon), \quad (6)$$

where l is the size of the blur kernel N is size of the image.

The extracting operation can convert the optimization problem as the series of single variable optimizations by decomposing the multi-variable problem into many small problems. In this minimization mechanism of solving small problems consecutively, only one pixel is updated in one iteration. Thus far, as in (3), the ordering of the small problems is constructed by lexicographical ordering but the various orderings can be constructed. In the next subsection, the effect of constructing the sequence of the small problems is analyzed.

Sampling without replacement

As mentioned in the previous section, the ordering of solving small problems is a key factor to generate satisfying deconvolution results. In this subsection, an effective ordering of small problems is suggested. The easiest way of constructing the ordering is to solve the problems in lexicographical ordering (also known as cyclic ordering). In many applications the lexicographically constructed ordering is used for its simple structure which scans the image from top to bottom. Although lexicographically ordering is easy to be constructed, this ordering generate severe artifact as demonstrated in Fig. 3d. When the pixel-wise optimization works based on lexicographical ordering, the distribution of the updated pixels in the neighborhoods generates the artifact.

By using the stochastic process for constructing the ordering, our method generates more pleasant results than the result with lexicographical ordering. Fig. 3e demonstrates less artifacts than lexicographical ordering. The ordering can be constructed by sampling without replacement. If a pixel is updated once, the pixel has to be considered as constant until every pixel is updated, i.e., we deliberately have to avoid sampling any pixel of the image more than once. With this strategy, $N!$ ordering can be constructed. The satisfying restored images can be generated

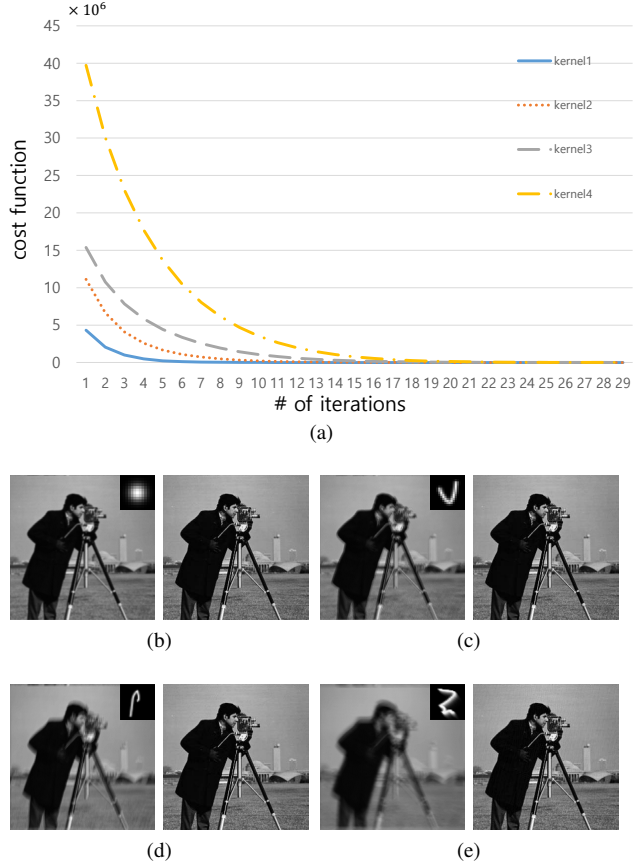


Figure 2: (a) Convergence of the proposed method with the image 'cameraman' and four blur kernels. The first blur kernel is of size 13 by 13. The second blur kernel is of size 17 by 17. The third blur kernel is of size 31 by 31. The forth blur kernel is of size 41 by 41. (b) With the first blur kernel. (c) With the second blur kernel. (d) With the third blur kernel. (e) With the fourth blur kernel.

by selecting ordering from the set of ordering based on sampling without replacement. (Note that lexicographical ordering is one in the set of orderings)

Experimental results

In the experimental results, the proposed method shows higher values of the measurement comparing with the other conventional methods that uses the gradient of the cost function [8]. The gradient-based method is implemented based on the steepest gradient descent method which has a similar computational complexity with our method. A same cost function is minimized by two different approaches, the gradient-based and the proposed algorithm. The first blur kernel in Fig. 4 is a Gaussian blur kernel whose standard deviation is two. The Gaussian blur kernel is used for modeling the degradation by errors in the focal distance of lens, i.e., problem of defocusing. The second blur kernel is modeled for the case of problems of motions during the time that shutter is open. The third and the forth kernels are constructed to generate complex deconvolution problems. The images convolved with these kernels also have regions with high frequency.

As shown in Fig. 5 which is a magnified version of the first

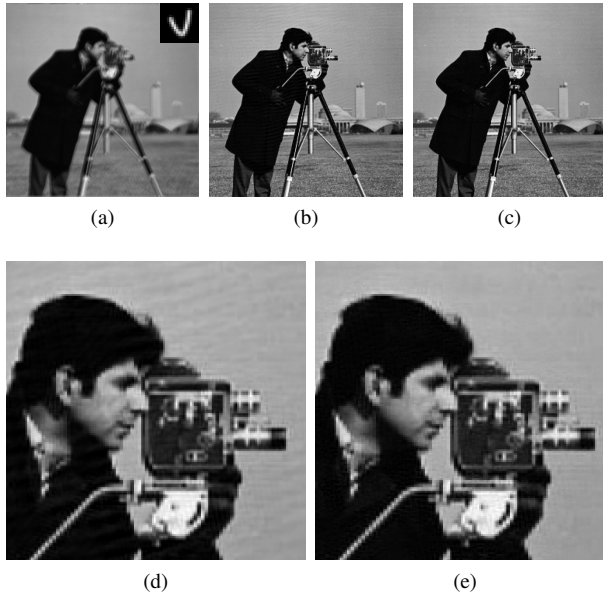


Figure 3: (a) Degraded image and the corresponding blur kernel. (b) Restored image using the sequence based on the lexicographical ordering. (c) Restored image using the sequence based on sampling without replacement. (d) Magnified image of (b). (e) Magnified image of (c).

experiment in Fig. 4, the proposed method demonstrated sharper edges than the conventional method. In the second experiment shown in Fig. 6, gradient-based method generated ringing artifacts near the edges but the proposed method showed less ringing artifacts. The images restored by the proposed method in Fig. 7 demonstrated much clearer flat regions near the edges than the conventional method. In the last experiment shown in Fig. 8, we could observe more distinguishable fingerprints.

The improved-signal-to-noise ratio (ISNR) is used to measure the quality of the deconvolved images. ISNR is closely related to the peak-signal-to-noise ratio (PSNR) as represented below

$$ISNR = 20 \cdot \log_{10} \left(\frac{\|\mathbf{x} - \mathbf{y}\|_2}{\|\mathbf{x} - \hat{\mathbf{x}}\|_2} \right), \quad (7)$$

where $\hat{\mathbf{x}}$ denotes the estimated image. Table 1 demonstrates values of two methods. Generally, as in the visual comparison before, the proposed method showed higher values than the conventional method.

Table 1: Measurement using ISNR. CM denotes the conventional method and PM denotes the proposed method. experiment1 denotes the simulation with Lena; experiment2 denotes the simulation with Boat; experiment3 denotes the simulation with Shopping; experiment4 denotes the simulation with Fingerprints

	CM[8]	PM
experiment1	4.8317	5.5847
experiment2	10.4098	12.5403
experiment3	8.8867	11.5848
experiment4	12.8477	14.4292

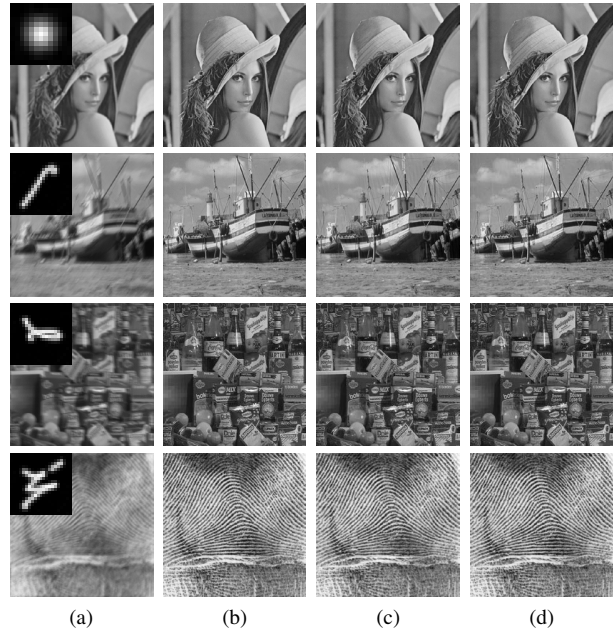


Figure 4: Final results with various images and blur kernels. (a) degraded images and the corresponding blur kernels. (b) original images. (c) demonstrates the restored images with conventional method. (d) demonstrates the restored images with the proposed method

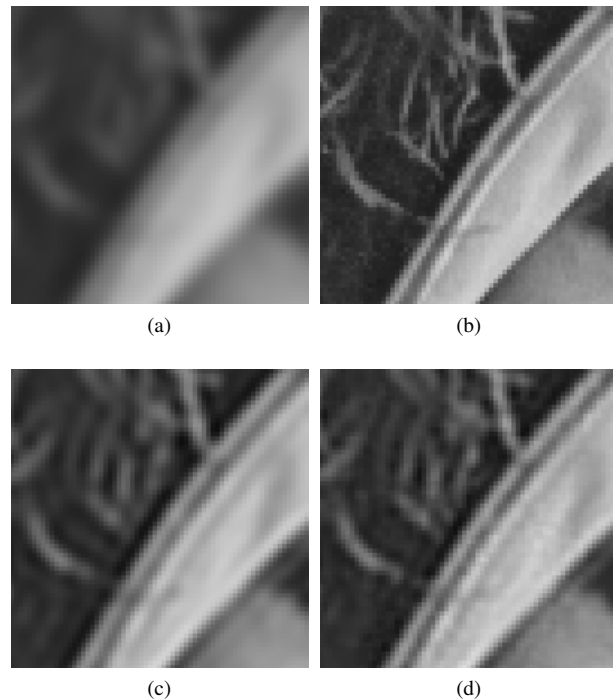


Figure 5: Cropped images from the results in the experiment with the image 'lena' in Fig. 4. (a) Degraded image. (b) Original image. (c) Conventional method[8]. (d) Proposed method.

Conclusions

The proposed method uses the approach of the pixel-wise optimization based on the sequences constructed by sampling with-

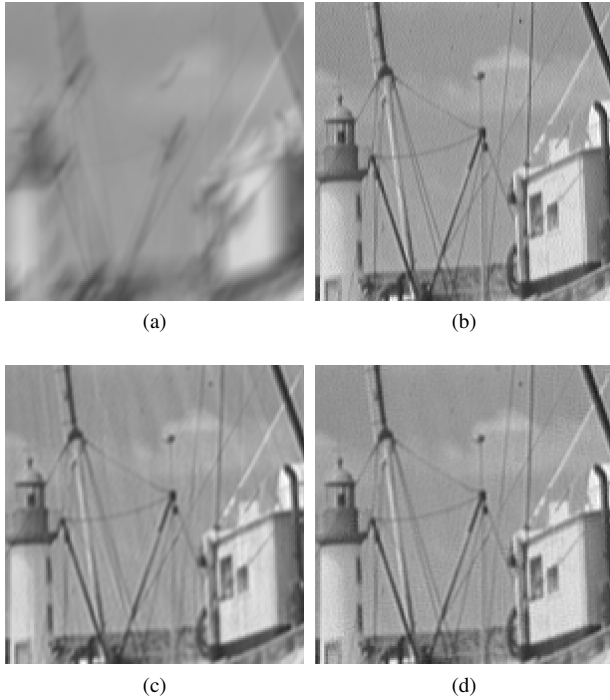


Figure 6: Cropped images from the results in the experiment with the image 'boat' in Fig. 4. (a) Degraded image. (b) Original image. (c) Conventional method[8]. (d) Proposed method.

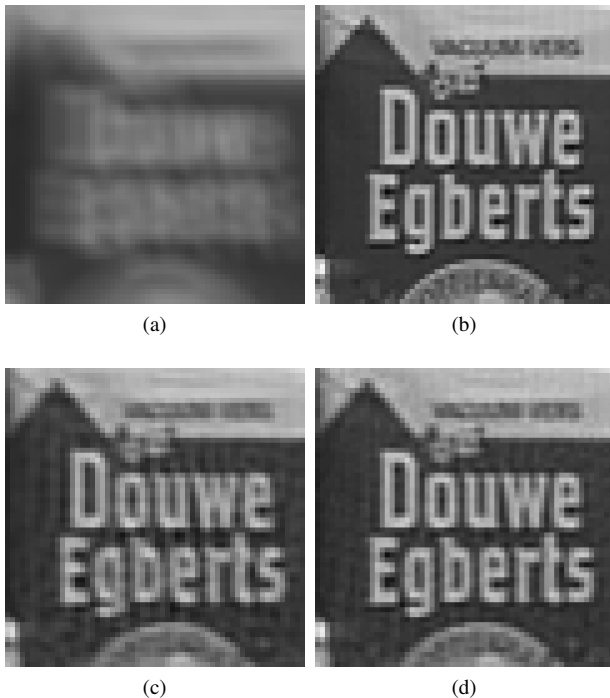


Figure 7: Cropped images from the results in the experiment with the image 'shopping' in Fig. 4. (a) Degraded image. (b) Original image. (c) Conventional method[8]. (d) Proposed method.

out replacement for image deconvolution. The pixel-wise optimization in which the deconvolution processes can operate inde-

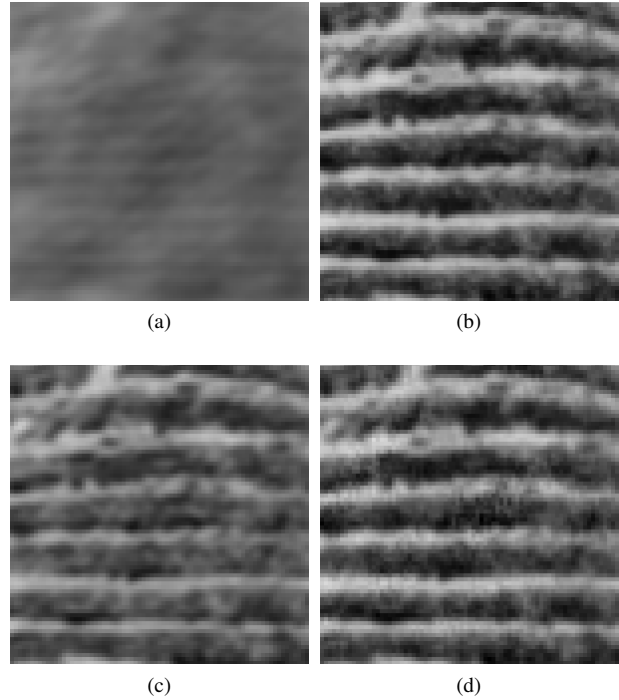


Figure 8: Cropped images from the results in the experiment with the image 'fingerprints' in Fig. 4. (a) Degraded image. (b) Original image. (c) Conventional method[8]. (d) Proposed method.

pendently according to each coordinate requires the simple structure to be implemented. This operation does not require the information that can be obtained by scanning the whole images such as gradient information. The independent deconvolution processes based on the randomly selected coordinates are easy to be applied a parallel computing to our method. The proposed method which is combined with the parallel computing can show the fast convergence rate according to the number of CPUs in the multi-core processor. The required computation can be divided into the number of CPUs because iteration processes based on the selected pixels is independent each other.

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