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Compressed Sensing MRI using Curvelet Sparsity and Nonlocal Total Variation: CS-NLTV

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Abstract

Compressed sensing (CS) has been exploited for accelerating data acquisition in magnetic resonance imaging (MRI). MR images can be then reconstructed from significantly fewer measurements, i.e., drastically lower than that required by the Nyquist sampling criterion. However, the compressed sensing method usually produces images with artifacts, particularly at high reduction rates. In this paper, we propose a novel compressed sensing MRI method, called CS-NLTV that exploits curvelet sparsity (CS) and nonlocal total variation (NLTV) regularization. The curvelet transform is optimal sparsifying transform with the excellent directional sensitivity than that of wavelet transform. The NLTV, on the other hand extends the total variation regularizer to a nonlocal variant which can preserve both textures and structures and produce sharper images. We have explored a new approach of combining alternating direction method of multiplier (ADMM), adaptive weighting, and splitting variables technique to solve the formulated optimization problem. The proposed CS-NLTV method is evaluated experimentally and compared to the previously reported high performance methods. Results demonstrate a significant improvement of compressed MR image reconstruction on four medical MRI datasets.

Introduction

Compressed sensing [1, 2, 3] is another way of speeding the signal acquisition by providing means for smarter sampling. Since 2006, compressed sensing or compressive sampling has been receiving a considerable attention in theory and applications, and among them magnetic resonance imaging is one whose implied sparsity suggests the use of sparse sampling. Compared to the standard sampling theory, MRI images are speed-limited physically and constrained by the physiological nature [4], and thus they are featured by immense sparsity. Exploring this sparsity while preserving accurate reconstruction is critical for medical diagnosis, and thus it is a major goal and a challenge of the research in this area. The goal of the research is to speed up the scan time of magnetic resonance imaging and produce a high-quality imagery suitable for further accurate reading [5, 6].

Compressed sensing hypothesis [2] makes it possible to recover magnetic resonance images from vastly under-sampled k-space data without being constrained by Shannon/Nyquist requirements. The process includes encoding, sensing, and decoding processes. Most of the existing compressed MR imaging approaches are based on the linear model, i.e., $A\mathrm{x} = b, A = SF$, where S is a selection, or a sampling matrix; F is a 2D discrete Fourier matrix, and b is the observed k-space data which are significantly undersampled. Given the sparsity assumption about $\hat{\mathrm{x}}$ which is an estimate of x, one possible solution would be $\{\min_{\mathrm{x}} |\mathrm{x}|_0 : A\mathrm{x} = b\}$, but since ℓ_0-minimization problem is NP-hard [7], a reasonable alternative would be $\{\min_{\mathrm{x}} |\mathrm{x}|_1 : A\mathrm{x} = b\}$. Thus, the objective is to minimize absolute differences, i.e. the variation, and it is useful to penalize by finite differences, therefore in the general formulation the TV is used for checking the sparsity of the transform and the finite differences.

The review of the compressed sensing MRI methods shows how to solve the above problem with different regularization and penalty terms. Recent compressed sensing efforts in MR pursue a best combination of sparsifying transforms [8, 9, 10, 11, 12, 13] and a fast solution for obtaining a high-quality reconstruction. Various methods have been presented to reconstruct MR images from under-sampled data. Ma et al., introduced an operator-splitting algorithm, total variation (TV) compressed MR imaging (TVCMRI) [8]. By taking advantage of fast wavelet and Fourier transforms, TVCMRI can process MR data fast and accurately. Yang et al. [9] solved the same objective function presented in [8] by a variable splitting method (RecPF) which is TV-based $\ell_1 - \ell_2$ MR reconstruction. This method uses alternating direction method for recovering MRI images from incomplete Fourier measurements. A fast composite splitting algorithm (FCSA) [10] is proposed by Huang et al. FCSA is based on combination of variable and operator splitting. It splits the variable x, into two variables, and exploits the operator splitting method to minimize the regularization terms over the splitting variables. Nonlocal total variation for MR reconstruction (NTVMR) [11], and the framelet + nonlocal TV (FNTV) [12] methods have been proposed lately. It is analyzed in [13] that the use of the first order derivatives has two major shortcomings; it creates oil-painting artifacts and leads to the contrast loss. Out of those two methods which use nonlocal TV regularization, FNTV delivers a better quality. The FNTV is formulated to minimize the combination of nonlocal TV, framelet and the least square data tting terms. Recently, shearlet based methods are proposed generally for inverse problems and specifically for MR reconstruction [14, 15, 16]. In [16], a new framework, i.e., nonseparable shearlet transform iterative soft thresholding reconstruction algorithm (FNSISTRA), is presented by Pejoski et al. Along with the discrete nonseparable shearlet transform (DNST) [17] as a sparsifying transform, the authors used a fast iterative soft thresholding algorithm (FISTA) [18, 19]. The method has achieved a superior performance among all the above state-of-the-art methods.

In this paper, we propose a novel optimization scheme for MR image reconstruction. The method integrates the curvelet sparsity and the nonlocal total variation (NLTV); CS-NLTV. MR images exhibit a vast sparsity especially in the transform domain [20] such as for example, wavelets. Wavelet transform frequently cannot handle efficiently edges or curves or in general, singularities in higher dimensions. Therefore, we use the curvelet transform which provides sparsity, excellent localization properties and directional selectivity. On the other hand, the nonlocal total variation, which is the total variation extension to a nonlocal variant. The NLTV preserves fine structures, details and textures, prevents from oil-painting artifacts inherent to TV, and maintains the contrast. Such a framework is expected to yield a higher quality of MR images. The confirmation of this hypothesis is received by our experimental study wherein we compare TVCMRI [8], RecPF [9], FCSA [10], FNTV [12], and FNSISTRA [16] to the proposed method. The rest of the paper is organized as follows. The proposed CS-NLTV method is described in Section 2 and results and performance comparison are presented in Section 3; followed is Conclusion in Section 4.

CS-NLTV Method

Magnetic resonance imaging (MRI) model can be expressed as

$$A\mathrm{x} = b. \tag{1}$$

where $\mathrm{x} \in R^M$ is a MR image, $A \in R^{N\times M}$ is a measurement matrix with $N \ll M$, and $b \in R^N$ is the observed data. The MR data can be recovered by solving the following minimization problem

$$\underset{\mathrm{x}}{\text{minimize}}\ |\phi(\mathrm{x})|_1 \quad \text{subject to} \quad A\mathrm{x} = b \tag{2}$$

Here $|\phi(\mathrm{x})|_1$ is a regularizing functional and ϕ is a sparsifying transform. In the compressed sensing (CS) model of MRI, $A = SF$, where S is a selection or a sampling matrix, F is the 2D discrete Fourier matrix, and b is the observed k-space data. Assuming the sparsity of the model, the problem is ill-posed for minimizing the least-squares function. Therefore, the following cost function with a regularization term has been considered:

$$\min_{\mathrm{x}} |\phi(\mathrm{x})|_1 \quad \text{subject to} \quad \|A\mathrm{x}-b\|_2^2 \le \sigma \tag{3}$$

where σ is the variance of distortion in b. Here the ℓ_1-norm denoted by $|.|_1$ and the ℓ_2-norm by $\|.\|_2$. The constrained optimization in Eq. (3) is equivalent to the following unconstrained optimization problem as it is formulated in [21]:

$$\min_{\mathrm{x}} |\phi(\mathrm{x})|_1 + \frac{\lambda}{2}\|A\mathrm{x}-b\|_2^2 \tag{4}$$

where $\lambda > 0$ is a balancing constant which relies on the sparsity of the underlying MR image x under linear transformation. Considering the problem, we propose and formulate the optimization problem using a combination of both the nonlocal total variation and the curvelet as regularizers. The proposed optimization problem to obtain reconstruction $\hat{\mathrm{x}}$ as follows:

$$\hat{\mathrm{x}} = \underset{\mathrm{x}}{\mathrm{argmin}}\ \alpha|\nabla_w \mathrm{x}|_1 + \beta\sum_k |C_k(\mathrm{x})|_1 + \frac{\lambda}{2}\|A\mathrm{x}-b\|_2^2 \tag{5}$$

where $|\nabla_w \mathrm{x}|_1 = \sum_t |\nabla_w \mathrm{x}_t|_1$ is the nonlocal total variation norm and nonlocal weights w are computed from image estimate $\hat{\mathrm{x}}$. $C(\mathrm{x})$ is the combination of different subbands of curvelet transform. α and β are the weighting parameters stressing two regularization terms. The value of these two parameters in each loop, are adaptively derived based on the variance of noise present in reconstructed image from previous iteration. We stress more on the curvelet regulaizer term if the estimated variance in each curvelet subband is greater than a specified threshold. The variances of the signal in every curvelet subband are computed by exploiting the maximum likelihood estimator applied on the neighborhood (a square) areas of coefficients.

Nonlocal Total Variation- NLTV

Nonlocal total variation (NLTV) in contrast to the TVs pixel-level correspondence establishes the patch-level correspondence [22]. For image x, the nonlocal weights can be formed for any two spatial nodes i and j as follows:

$$\varpi_{\mathrm{x}}(i,j) = e^{-\frac{\int_{R_1} G\sigma(t)(\mathrm{x}(j+t)-\mathrm{x}(i+t))^2 dt}{\sigma^2}} \tag{6}$$

where G is a Gaussian kernel with the variance σ^2; and R_1 is the spatial neighborhood of i and j for similarity consideration. The nonlocal gradient $\nabla_w \mathrm{x}(i,j)$ at i is described as a vector of all partial derivatives $\nabla_w \mathrm{x}(i,.)$ [23]:

$$\nabla_w \mathrm{x}(i,j) = (\mathrm{x}(j)-\mathrm{x}(i))\sqrt{\varpi_{\mathrm{x}}(i,j)},\ \forall j \in R_2 \tag{7}$$

where R_2 is the spatial neighborhood around i, whose nonlocal gradient $\nabla_w \mathrm{x}(i,j)$ is calculated. The adjoint of Eq. (7) is derived from the adjoint relationship with a nonlocal divergence operator div_w as:

$$\langle \nabla_w \mathrm{x}, v\rangle = \langle \mathrm{x}, div_w v\rangle \tag{8}$$

$$div_w v(i,j) = \int_{R_2} (v(i,j)-v(j,i))\sqrt{\varpi_{\mathrm{x}}(i,j)}\, dj \tag{9}$$

Given the image $\mathrm{x} \in R^M = R^{m\times n}$ with $R_1 = R^{(2a_1+1)(2b_1+1)}$, $R_2 = R^{(2a_2+1)(2b_2+1)}$, weights nonlocal total variation are defined as

$$\varpi_{\mathrm{x}}(k_1,l_1,k_2,l_2) = e^{-\frac{\sum_{z_1=0}^{2a_1}\sum_{z_2=0}^{2b_1} G\sigma(z_1,z_2)(\mathrm{x}(k_1-a_1+z_1,l_1-b_1+z_2)-\mathrm{x}(k_2-a_1+z_1,l_2-b_1+z_2))^2}{\sigma^2}}$$
$$k_1,k_2 = 1,...,m. \quad l_1,l_2 = 1,...,n. \tag{10}$$

Fig. 1 depicts the comparison of TV vs NLTV reconstruction from a noisy MRI image.

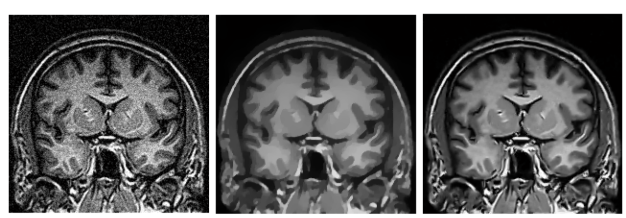

Figure 1: Left: Noisy MRI image, middle: recovered using TV, right: recovered using NLTV for respective image.

Curvelet Sparsity

Curvelet transform introduced by Candes [24, 25] is an efficient geometric multiscale sparsifying transform. Unlike wavelets, curvelets have directional sensitivity and anisotropy, optimal sparse representation, better ℓ_1-norm sparsity, and thus, they can efficiently characterize anisotropic features such as edges, arcs and curves. Curvelet transform has been used for image denoising, feature extraction and for solving inverse problems [26, 27, 28]. The curvelets at scale 2^{-j}, orientation θ_l, and position $k=(k_1,k_2)$ are defined as

$$\varphi_{j,l,k}(x)=\varphi_j(R_{\theta_J}(x-\rho_k^{(j,l)})) \tag{11}$$

where φ_j is a mother curvelet, $\theta_J=2\pi\cdot 2^{\lfloor -j/2\rfloor}\cdot l$, $J=(j,l)$ indicating the scale/angle, and

$$\rho_k^{(j,l)}=R_{\theta_J}^{-1}(k_1\cdot 2^{-j},k_2\cdot 2^{-j/2}) \tag{12}$$

$$R_\theta=\begin{pmatrix} cos(\theta) & sin(\theta) \\ -sin(\theta) & cos(\theta)\end{pmatrix}, R_\theta^{-1}=R_\theta^T=R_{-\theta} \tag{13}$$

The curvelet transform of function x is computed as

$$C_\varphi(x)(j,l,k)=\langle x,\varphi_{j,l,k}\rangle \tag{14}$$

The whole process can be performed in the frequency domain, by introducing 2D frequency window as

$$U_j(\omega)=2^{-3j/4}W(2^{-j}|\omega|)V(\frac{2^{\lfloor j/2\rfloor}\theta}{2\pi}) \tag{15}$$

where W is the Meyer wavelet window dividing the frequency domain into annuli $|x|\in[2^j,2^{j+1})$ and V is the angular window dividing the annuli into wedges θ_J. Then the curvelets can be defined in frequency domain as

$$\varphi_{j,l,k}(\omega)=U_J(R_{\theta_J}\omega)e^{-i\langle\rho_k^{(j,l)},\omega\rangle} \tag{16}$$

The discrete curvelet transform in the frequency plane has tiling in concentric squares and shears in Cartesian coronae. Due to the advantages of curvelets as sparsifiers with their detail representation properties, the transform is adopted for implementation in our method. We can implement $k-th$ subband of curvelet transform (C_k) as a mask in the frequency domain (Z_k) [29].

$$C_k(\mathrm{x})=F^*diag(vec(Z_k))F\mathrm{x}=P_k\ \mathrm{x} \tag{17}$$

where F is a vectorized form of the discrete Fourier transform operator. Thus, the curvelet regularization can be defined as ℓ_1 optimization term:

$$\min_{\mathrm{x}}|C(\mathrm{x})|_1=\min_{\mathrm{x}}\beta\sum_k|P_k\ \mathrm{x}|_1=\min_{\mathrm{x}}\beta\sum_k|C_k(\mathrm{x})|_1 \tag{18}$$

Solution

The proposed optimization problem is formulated as follows:

$$\underset{\mathrm{x}}{\operatorname{argmin}}\ \alpha|\nabla_w\mathrm{x}|_1+\beta\sum_k|C_k(\mathrm{x})|_1+\frac{\lambda}{2}\|A\mathrm{x}-b\|_2^2 \tag{19}$$

The proposed optimization problem has both ℓ_1 and ℓ_2-norm terms and thus is difficult to obtain the solution in a closed-form. The alternating direction method of multiplier (ADMM) [30] and splitting variables method are used to solve the formulated problem as follows:

$$\begin{gathered}\underset{\mathrm{x}}{\operatorname{argmin}}\ \alpha|y_1|_1+\beta\sum_k|y_2(k)|_1+\frac{\lambda}{2}\|A\mathrm{x}-b\|_2^2 \\ subject\ \ to\ \ \ y_1=\nabla_w\mathrm{x},\ y_2(k)=C_k(\mathrm{x})\end{gathered} \tag{20}$$

where $y_1\in R^M$, and $y_2(k)\in R^M$ are auxiliary variables. The Lagrangian function can be written as below:

$$\begin{aligned}\mathscr{L}(\mathrm{x},y_1,y_2,u_1,u_2)=\frac{\lambda}{2}\|A\mathrm{x}-b\|_2^2+\alpha|y_1|_1+\frac{\eta}{2}\|\nabla_w\mathrm{x}-y_1+u_1\|_2^2+ \\ \beta\sum_k|y_2(k)|_1+\frac{\Gamma}{2}\sum_k\|C_k(\mathrm{x})-y_2(k)+u_2(k)\|_2^2\end{aligned} \tag{21}$$

where $u_1\in R^M$ and $u_2(k)\in R^M$ are the newly defined scaled dual variables. The problem is solved by iterating over Eqs. (22-26) below:

$$\mathrm{x}^{(n+1)}:=\underset{\mathrm{x}}{\operatorname{argmin}}\ \mathscr{L}(\mathrm{x},y_1^{(n)},y_2^{(n)},u_1^{(n)},u_2^{(n)}) \tag{22}$$

$$y_1^{(n+1)}:=\underset{y_1}{\operatorname{argmin}}\ \mathscr{L}(\mathrm{x}^{(n+1)},y_1,u_1^{(n)}) \tag{23}$$

$$y_2^{(n+1)}:=\underset{y_2}{\operatorname{argmin}}\ \mathscr{L}(\mathrm{x}^{(n+1)},y_2,u_2^{(n)}) \tag{24}$$

$$u_1^{(n+1)}:=u_1^{(n)}+(\nabla_w\mathrm{x}^{(n+1)}-y_1^{(n+1)}) \tag{25}$$

$$u_2^{(n+1)}:=u_2^{(n)}+(C(\mathrm{x}^{(n+1)})-y_2^{(n+1)}) \tag{26}$$

The optimal solution for the sub-problem by Eq. (22) requires finding roots of its derivatives that leads to the following equations:

$$\begin{aligned}&\lambda A^TA\mathrm{x}-\lambda A^Tb+\eta\, div_w(\nabla_w\mathrm{x}-y_1+u_1)+ \\ &\quad\Gamma(\sum_k P_k^*P_k\mathrm{x}+\sum_k P_k^*(y_2(k)-u_2(k)))=0\end{aligned} \tag{27}$$

Minimization in Eqs. (23) and (24) can be attained by shrinkage operators such as:

$$y_1^{(n+1)}:=Shrink(\nabla_w\mathrm{x}^{(n+1)}+u_1^{(n)},\vartheta_1) \tag{28}$$

$$y_2^{(n+1)}:=Shrink(C(\mathrm{x}^{(n+1)})+u_2^{(n)},\vartheta_2) \tag{29}$$

where $\vartheta_1=\frac{\alpha}{\eta}$, $\vartheta_2=\frac{\beta}{\Gamma}$ and

$$Shrink(x,\xi)_n=sign(x_n)\ max\{|x_n|-\xi,0\} \tag{30}$$

Results

We test the proposed CS-NLTV method on 256×256 MRI images of brain, chest, artery, and the cardiac image presented in Fig. 2. Five high performance methods are chosen to be compared to the proposed method, i.e., TVCMRI [8], RecPF [9], FCSA [10], FNTV [12], and FNSISTRA [16]. The methods are studied with the random subsampling technique. We demonstrate results for the fixed number of iterations that is 50 iterations as the methods under comparison reported on their performance with this number as a stopping point and for four sampling ratios, i.e., 15, 20, 25 and 30%.

Fig. 3 and 4 show SNR plots, where SNR$=10\ log_{10}\frac{\|\mathrm{x}\|_2^2}{\|\mathrm{x}-\mathrm{x}^n\|_2^2}$, x is the original image and x^n represents the reconstructed image after n interations. SNR values are measured for the above quantities of subsampling ratios, with random subsampling. As it follows from the plots, the proposed CS-NLTV method has achieved a better performance compared to its counterparts and specifically, over a best among the reference methods that is by FNSISTRA. This high performance of CS-NLTV is derived from nonlocal total variation which locates sharper edges and suppresses artifacts; and it is due to the exceptional spatial localization and directional selectivity of the curvelet sparsity.

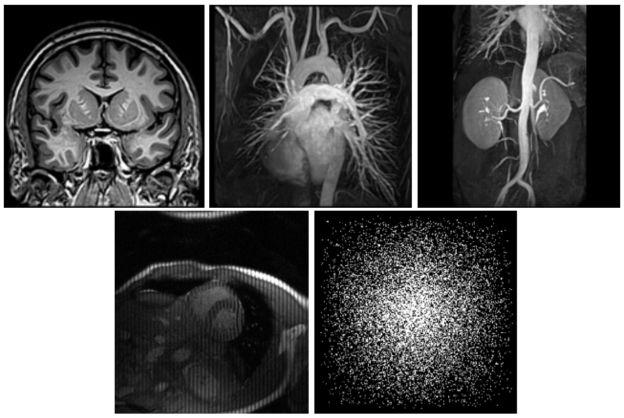

Figure 2: Top: left to right: brain, chest, artery images. Bottom: left to right: cardiac image and random variable subsampling.

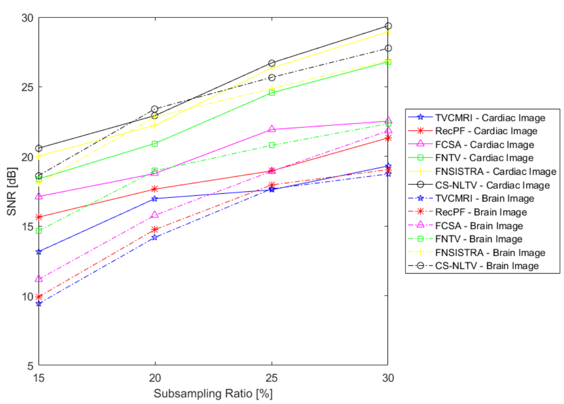

Figure 3: Performance of methods with random variable subsampling for cardiac and brain images.

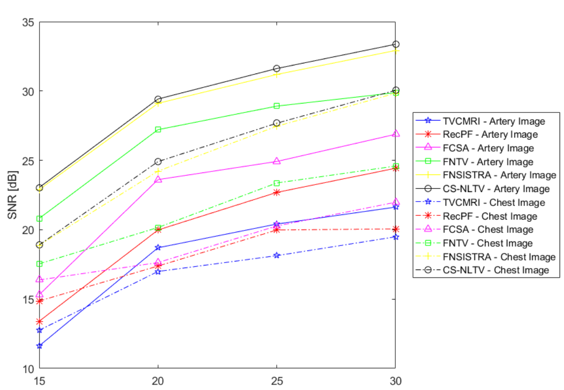

Figure 4: Performance of methods with random variable subsampling for artery and chest images.

Conclusion

In this paper, we have presented a new MRI compressed sensing method, the CS-NLTV. The method utilizes curvelet sparsity and the nonlocal total variation to gain on directional sensitivity and selective regularization at different levels. We have formulated the optimization problem for the reconstruction process and solved it originally, i.e., by combining alternating direction method of multiplier, adaptive weighting, and splitting variables technique. The method is able to reconstruct MR images with a high quality, which is assessed visually and using the objective quality metric. High SNRs yielded by the method quantify its high performance. The conducted experiments and the analysis of different reconstructed medical MRI datasets with a range of sampling ratios have demonstrated a superior quality of reconstruction by the proposed method in comparison to five high performance reference methods, including the state-of-the-art FNSISTRA method.

References

[1] E. J. Cands, J. Romberg, and T. Tao, Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information, IEEE Trans. Inf. Theory, vol. 52, no. 2, pp. 489509, 2006.

[2] D. L. Donoho, Compressed sensing, IEEE Trans. Inf. Theory, vol. 52, no. 4, pp. 12891306, 2006.

[3] E. J. Cands, J. K. Romberg, and T. Tao, Stable signal recovery from incomplete and inaccurate measurements, Comm. Pure Appl. Math., vol. 59, pp. 12071223, 2006.

[4] M. Lustig, D. Donoho, JM. Pauly, Sparse MRI: The application of compressed sensing for rapid MR imaging. Magn Reson Med, 58, No.6, 1182-1195. 2007

[5] M. Lustig, D. Donoho, J. M. Santos, and J. M. Pauly, A look at how CS can improve on current imaging techniques, IEEE Signal Process. Mag., vol. 25, no. 2, pp. 7282, Mar. 2008.

[6] E. Cands, J. Romberg, and T. Tao, Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information, IEEE Trans. Inf. Theory, vol. 52, no. 2, pp. 489509, Feb. 2006.

[7] B. K. Natarajan. Sparse approximate solutions to linear systems. SIAM Journal on Computing, 24:227234, 1995.

[8] S. Ma, W. Yin , Y. Zhang, A. Chakraborty, An efficient algorithm for compressed MR imaging using total variation and wavelets. In: Proceedings of CVPR. 2008.

[9] J. Yang, Y. Zhang, and W. Yin, A fast alternating direction method for TVL1-L2 signal reconstruction from partial Fourier data, IEEE J. Select. Topics Signal Processsing, vol. 4, no. 2, pp. 288297, 2010.

[10] J. Huang, S. Zhang, and D. Metaxas, Efficient MR image reconstruction for compressed MR imaging, Med. Image Anal., vol. 15, no. 5, pp. 670679, 2011.

[11] Gopi V. P., Palanisamy P, Wahid KA, Babyn P, MR image reconstruction based on iterative split Bregman algorithm and non-local total variation, J Comput Math Method Med 2013:116, 2013.

[12] Gopi V. P., Palanisamy P, Wahid KA, Babyn P, MR image reconstruction based on framelets and nonlocal total variation using split Bregman method, Int Journal CARS, 9:459472, 2014.

[13] Osher S, Burger M, Goldfarb D, Xu J, Yin W, An iterative regularization method for total variation-based image restoration. Multiscale Model Simul 4:460489, 2005.

[14] W. Guo, J. Qin, and W. Yin, A new detail-preserving regularity scheme, Rice CAAM, Tech. Rep. 13-01, 2013.

[15] A. Pour Yazdanpanah, E. E. Regentova, G. Bebis. Algebraic itera-

tive reconstruction-reprojection (AIRR) method for high performance sparse-view CT reconstruction. Applied mathematics & information sciences. Vol. 10, No. 6, 2016.

[16] S, Pejoski, V, Kafedziski, and D Gleich, Compressed Sensing MRI Using Discrete Nonseparable Shearlet Transform and FISTA, IEEE Signal Processing Letters, vol. 22, mo. 10, 2015.

[17] W.-Q. Lim, Nonseparable shearlet transform, IEEE Trans. Image Process., vol. 22, no. 5, pp. 20562065, 2013.

[18] A. Beck and M. Teboulle, A fast iterative shrinkage-thresholding algorithm for linear inverse problems, SIAM J. Imag. Sci., vol. 2, no. 1, pp. 183202, 2009.

[19] W. Hao, J. Li, X. Qu, and Z. Dong, Fast iterative contourlet thresholding for compressed sensing MRI, Electron. Lett., vol. 49, no. 19, pp. 12061208, 2013.

[20] L. He, T.-C. Chang, S. Osher, T. Fang, and P. Speier. MR image reconstruction by using the iterative refinement method and nonlinear inverse scale space methods. UCLA CAM Report, pp. 06-35, 2006.

[21] E. Cand‘es, M. Wakin, and S. Boyd. Enhancing sparsity by reweighted L1 minimization. Preprint, 2007.

[22] A. Buades, B. Coll, J. M. Morel, A review of image denoising algorithms, with a new one, Multiscale modeling & simulation, 4, pp. 490-530, 2005.

[23] G. Gilboa, and S. Osher, Nonlocal operators with applications to image processing, Multiscale modeling and simulation, 7, pp. 1005-1028, 2008.

[24] Goldstein T., Osher S., The split Bregman method for L1 regularized problems. SIAM J. Imag. Sci. vol. 2, no. 2, pp. 323343, 2009.

[25] E. Cands, L. Demanet, D. Donoho, and L. Ying, Fast discrete curvelet transforms, Multiscale Model. Simul., vol. 5, no. 3, pp. 861899, 2006.

[26] E. Cands and D. Donoho, New tight frames of curvelets and optimal representations of objects with piecewise singularities, Commun. Pure Appl. Math., vol. 57, no. 2, pp. 219266, 2004.

[27] A. Pour Yazdanpanah, E. E. Regentova, Sparse-view CT reconstruction using curvelet and TV-based regularization. Image Analysis and Recognition, vol. 9730, pp. 672-677, 2016.

[28] Starck JL, Candes EJ, Donoho DL. The curvelet transform for image denoising. IEEE Trans Image Process. vol. 11, no. 6, pp. 67084, 2002.

[29] Wu H., Maier A., Hornegger J., Iterative CT Reconstruction Using Curvelet-Based Regularization. Bildverarbeitung fr die Medizin, 2013.

[30] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, Distributed optimization and statistical learning via the alternating direction method of multipliers, Foundations and Trends in Machine Learning, vol. 3, no. 1, pp. 1122, 2011.