# Modeling and Estimation for Surface-Spectral Reflectance of Watercolor Paintings

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# Abstract

A method is proposed to estimate spectral reflectances of watercolor paint samples with different water ratios. We first suppose that watercolor painting is monochrome made by a single paint and water. An algorithm is developed based on the Kubelka-Munk approach. The optical constants of scattering and absorption are estimated from the measured reflectances of a standard sample painted with a known water ratio. A linear relationship was devised to predict the optical constants of watercolor paintings with different water ratios. The surfacespectral reflectances are then recovered with the predicted optical constants. Moreover, we extend the method to watercolor paintings with mixed color paints. The feasibility of the method is examined in experiments using a variety of watercolor paintings.

### Introduction

It is our essentially important issue that the valuable cultural heritage is preserved and taken over to posterity. It is no doubt that artworks such as art paintings will be lost or damaged in a long period of time. The purpose of digital archiving is to preserve, present, and hand down to future generations the irreplaceable art paintings in the form of digital images.

Surface-spectral reflectance is more important for digital archiving than color information. In fact, an RGB image creates some issues such as color gamut and light dependence. Although the use of spectra may increase the size of images, it allows us to perform precise color computations and obtain different impressions of a painting under different light sources.

Although there are different types of paintings including oil paint and watercolor paint, most target objects treated in digital archive of art paintings have been oil paintings. Tominaga et al. [1], [2] proposed a technique for viewpoint and illumination independent digital archiving of oil paintings. The surface-spectral reflectances of oil paintings were estimated using a multiband imaging systems. Sato et al. [3] estimated the surface shape and reflectance of oil paintings by using a light stripe range finder and a color CCD camera. The surface material of an oil painting consists of a thick oil layer, which is modeled to be inhomogeneously dielectric material with the dichromatic reflection property. Therefore, the surface includes gloss and specular reflection, depending on viewing and illumination angles.

On the other hand, watercolor paintings have essentially different characteristics from oil paintings. Watercolor paints easily soak into the paper. Little specular reflection is included. Rough surface of watercolor painting is not based on the paint itself but based on the drawing paper (backing paper). More important property of watercolor paintings is that the surfacespectral reflectance depends greatly on the drawing paper used and the ratio of paint to water. Figure 1 demonstrates images of watercolor paintings with different ratios of paint to water. The paint is called Viridian and the drawing paper is Canson MiTeintes. The surface appearance caused by surface-spectral reflectance is quite different with the water ratios.



Figure 1: Sample of watercolor paintings with different water ratios.

How can we estimate the spectral reflectance at different water ratios? In the field of computer graphics, a computer software was developed for digital water color painting (e.g., see [4]). However, measurement data from real paints were not used but computer simulation was performed to imitate various watercolor effects. The surface-spectral reflectance was not estimated but RGB color was predicted.

The present paper proposes a method to estimate spectral reflectances of watercolor paint samples with different water ratios. We first suppose that watercolor painting is monochrome which is made by a single paint and water. An algorithm is developed based on the Kubelka-Munk (KM) approach. The optical constants of scattering and absorption are estimated from the measured reflectances of a standard sample painted with a known water ratio. We devise a linear relationship to predict the optical constants of watercolor paintings with different water ratios. The surfacespectral reflectances are then recovered using the KM theory with the predicted optical constants. In experiments, the feasibility of the proposed method is confirmed by comparing the estimated reflectances with the measured ones for a variety of watercolor paintings. Part of the concept that we are discussing here was presented in a recent proceeding [5]. Moreover, we extend the method to reflectance estimation of watercolor paintings with mixed color paints. The surface layer of paintings is made of a mixture of multiple paints.

# **Modeling of Watercolor Painting**

Watercolor paint is composed of pigments, gum arabic as an adherence ingredient, and some auxiliary ingredient. Coloring principle of watercolor is shown in Figure 2 (see [6]). Pigments are placed on the paper, and the adherent component glues pigments to the paper. The appearance of translucency of watercolor is caused by simultaneously observing the color of paper and the color of pigments. Strictly speaking, watercolor is not a physically transparent material. The surface is not perfectly flat, but rough in microscopic point of view.

In this paper, for simplicity, we suppose that watercolor paint in Figure 2 is a uniform turbid material, and fixed to the backing paper with a constant thickness. Then, Figure 3 shows an optics model of the KM theory to describe the watercolor painting. The symbol I is defined as the intensity of light traveling inside the layer towards the backing paper, and J is the intensity of light traveling in the reverse direction. The symbol X is the thickness. The upward light and downward light traveling through the thickness dx are described using optical constants of scattering and absorption as a system of two differential equations:

$$\frac{dI}{dx} = -(S+K)I + SJ$$
$$\frac{dJ}{dx} = -(S+K)J + SI$$
(1)

where *S* and *K* are, respectively, the coefficients of scattering and absorption in the paint. The surface spectral reflectance R of the painting is obtained as a standard KM equation by solving the above equations as follows [7]:

$$R = \frac{1 - R_W (a - b \coth(bSX))}{a + b \coth(bSX) - R_W},$$
(2)

where  $R_W$  is the reflectance of a white backing paper, and

$$a = \frac{SX + KX}{SX}, \quad b = \sqrt{a^2 - 1} \quad . \tag{3}$$



Figure 2: Coloring principle of watercolor.



Figure 3: Optics model for watercolor painting.

## **Estimation of Optical Constants**

#### Estimation based on reflectance measurement

We should note that it is difficult to measure the thickness X of watercolor painting from a real sample because the paints easily soak into the paper and the surface is not perfectly flat. Therefore, in this paper, we estimate SX and KX by combining the thickness and the optical constants instead of S and K in Eqs.(2) and (3).

Kubelka [6] proposed a basic formula to determine *S* and *K* from the measured reflectance under several conditions. He used

three types of reflectance of (1)  $R_B$  of a black backing paper, (2)  $R_{BC}$  of watercolor painting on the black backing paper, and (3)  $R_{\infty}$  of watercolor painting with unlimited thickness so that the substrate is completely hidden. This formula, however, is not available for watercolor painting. This is because measurement of the reflectance  $R_{\infty}$  at the unlimited condition is difficult for watercolor paintings with high water ratio. Also the direct measurement of thickness *X* is difficult.

Minato [8] proposed a formula to determine the scattering coefficient of color samples with the aid of two reflectance color samples. Two backings were used to create two reference color samples which are applied with the same paint. The color difference is then caused by the effect of two different backings. The scattering coefficient can be calculated from the color difference. This method can be called two-reference color method. The prior measurement information for  $R_{\infty}$  is not necessary in this method. We consider that this is useful for translucent material including watercolor paint.

The optical constants *S* and *K* are then given by the following equations:

$$A = R_{WC} \cdot R_B - R_{BC} \cdot R_W \tag{4}$$

$$B = (R_W - R_B)(1 + R_{WC} \cdot R_{BC}) - (R_{WC} - R_{BC})(1 + R_W \cdot R_B)$$
(5)

$$R_{\infty} = (-B + \sqrt{B^2 - 4A^2})/2A \tag{6}$$

$$(K/S) = (1-R)^2 / 2R_{\infty} \equiv (K/S)_{\infty}$$
<sup>(7)</sup>

$$S = \frac{\ln\left(\frac{(R_{\infty} - R_{W}) \cdot ((1/R_{\infty}) - R_{WC})}{(R_{\infty} - R_{WC}) \cdot ((1/R_{\infty}) - R_{W})}\right)}{X(1/R_{\infty} - R_{\infty})}$$
(8)

$$K = (K/S)_{\infty} \cdot S \tag{0}$$

where  $R_W$  is the reflectance of a white backing paper,  $R_B$ , is the reflectance of a black backing,  $R_{WC}$  is the reflectance of paint on the white backing,  $R_{BC}$  is the reflectance of paint on the black backing, and X is the thickness of the painting layer.

### Prediction of S and K at different water ratios

When we estimate the reflectance of a mixture of watercolor paints, we assume that the scattering coefficients and absorption coefficients for the individual paints are additive (see [9],[10]), that is, in proportional to their respective concentrations (water ratios) as

$$S_{M} = c_{1}S_{1} + c_{2}S_{2} + c_{3}S_{3} + \cdots$$
  

$$K_{M} = c_{1}K_{1} + c_{2}K_{2} + c_{3}K_{3} + \cdots$$
(10)

where  $S_M$  and  $K_M$  are the mixture's scattering and absorption coefficients;  $S_i$  and  $K_i$  are each paint's scattering and absorption; and  $c_i$  is the concentration (water ratio) of each paint. Thus, the scattering and absorption coefficients of a mixture paint are given in a linear combination of individual paints.

For the case of monochrome made of a single paint, the above equations are reduced into

$$S_{_{M}} = c_{_{1}}S_{_{1}}, \qquad K_{_{M}} = c_{_{1}}K_{_{1}}$$
 (11)

Let  $c_s$  be the water ratio for a standard reference sample, and  $(S_s, M_s)$  be the estimated coefficients from the reflectance measurements. Then, the coefficients at arbitrary water ratio c are predicted as

$$S = (c/c_S)S_s, \quad K = (c/c_S)K_s$$
(12)

### **Estimation Procedure**

Figure 4 shows the practical procedure of reflectance estimation for watercolor paintings. The first step is (1) reflectance measurement of white and black backing papers and (2) reflectance measurement of reference watercolor samples painted on the two papers.

However, correction of the measured spectral reflectances are often needed. This is because the effects of specular reflection and internal diffuse reflection caused between the paint layer and the air are neglected, and the ideal reflectances without such effects are discussed in the KM theory. A correction method by Saunderson [11] is used for correcting the measurements. Let R'and R be the measured reflectance and the ideal reflectance, respectively. Then we correct the R' and R to obtain an ideal reflectance from the measurement and inversely a realistic reflectance containing the interface effects from the ideal as follows:

$$R = \frac{R'}{(1-k_1)(1-k_2) + k_2 R'}$$
(13)

$$R' = (1 - k_1)(1 - k_2)\frac{R}{1 - k_2 R}$$
(14)

where  $k_1$  is the parameter of Fresnel reflection, and  $k_2$  is the parameter of internal reflection.

It is possible to improved estimation accuracy by choosing the parameters properly. In this paper,  $k_1$  and  $k_2$  are determined to minimize the average of root mean squared error (RMSE) in experiments using many watercolor painting samples of different watercolor paints and papers.

## **Experiments**

## **Reference Measurement**

We determined that the standard reference was the spectral reflectance of watercolor paint with the water ratio of 50%. The water ratio means the percentage of weight of paint in total. We deemed the weight of water mixed in paint at manufacturing as part of weight of paint. We used an applicator of 5 mil to paint on water color drawing papers. Four Holbein Artists' Watercolors paint sample were used in experiments as follows:



(1) Vermillion, (2) Permanent Yellow Light, (3) Viridian, (4) Cobalt Blue.

In addition, we used two drawing papers, Canson Mi-Tantes and Takeo Mermaid, to investigate the estimation accuracy due to the backing papers. The water ratio for reflectance estimation were 97.5%, 95%, 90%, 80%, 70%, 60%, 40%, and 30%.

### Sample Measurement

Figure 5 shows the measurement scene of watercolor samples. We prepared three white backing papers, and made three samples of the same watercolor paint on the white papers. Also we prepared a black backing paper, and made a sample of the same watercolor paint on the black paper. The spectral reflectances at



Figure 4: Practical procedure of reflectance estimation

several points on the painting sample surfaces were measured using Gretag Macbeth Spectrolino. The average reflectances were used for the analysis.



Figure 5: Measurement scene of watercolor samples.

## **Estimation Results**

Figure 6 shows the estimation results for the four color paints (Vermillion, Permanent Yellow Light, Viridian, and Cobalt Blue) drawn on the Takeo Mermaid paper. In the figures, the black bold curve and red bold curve represent the reflectance measurements of the white paper and the standard sample of 50%, respectively, and the symbols + and diamond represent, respectively, the measurements and the estimates at the respective water ratios. Figure 7 demonstrates the rendered images of the four color paints at different water ratios under Illuminant D65.

When comparing the rendered images between the estimated reflectances and the measurements in Figure 7, we find that the appearance of the reproduced watercolor paintings is very close to the original paintings. Table 1 shows the average RMSE and the average color difference for each color paint sample. The average values for the respective samples are about 0.01~0.06 and the average values of  $\Delta E_{ab}$  are about 2.5~5.5. Table 2 shows the best parameter values of  $k_1$  and  $k_2$  in the Saunderson's equations of Eq.(13) and Eq.(14).

#### **Extension to Mixed Watercolor Paints**

The method developed for watercolor paintings made by a single paint and water can be extended to the watercolor paintings by multiple paints and water. Let us consider two Watercolor paints 1 and 2 at different water ratios. Suppose that the optical constants S and K of Watercolor paints 1 and 2 are estimated at certain water ratios  $c_1$  and  $c_2$ . The S and K for the watercolor paint mixed by two paints with the water ratios  $c_1$  and  $c_2$  can be predicted based on Eq.(10).

Figure 8 shows the estimation results for the mixture of Viridian of water ratio 50% and Cobalt Blue of water ratio 50% painted on the Mermaid paper. In this figure, the red and blue bold curves represent the estimated reflectance and the measured one, respectively, and the two broken cuvees represent the estimated reflectance curves of the single watercolor paintings. The rendered color images under D65 are shown in the lower part in Figure 8. The mixture watercolor paintings are well recovered.



0 └─ 400 450 500 550 600 650 700 Wavelength (d) Cobalt Blue

44

R\_WC(50%

0.1

Figure 6: Estimation results for four paints on Takeo Mermaid paper.

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| Estimated reflectances.<br>(a) Images for Vermilion   |           |        |  |  |  |  |  |  |
|   |           |        |  |  |  |  |  |  |
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| Direct measurements.                                  |           |        |  |  |  |  |  |  |
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| Estimate  | ed reflec | tances |  |  |  |  |  |  |
| (b) Images for Permanent Yellow Light                 |           |        |  |  |  |  |  |  |
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| Estimated reflectances.<br>(c) Images for Viridian    |           |        |  |  |  |  |  |  |
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| Direct measurements                                   |           |        |  |  |  |  |  |  |
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#### Table 2: Best parameters in Saunderson's equations.

|              | Parameters       |                |         |
|--------------|------------------|----------------|---------|
| Samples      |                  | k <sub>1</sub> | $k_{2}$ |
| Vormillion   | Canson Mi-Tantes | 0              | 0       |
| verminon     | Takeo Mermaid    | 0              | 0       |
| Permanent    | Canson Mi-Tantes | 0              | 0.04    |
| Yellow Light | Takeo Mermaid    | 0              | 0       |
| Viridion     | Canson Mi-Tantes | 0              | 0       |
| Vindian      | Takeo Mermaid    | 0              | 0       |
|              | Canson Mi-Tantes | 0              | 0.02    |
| Copait Blue  | Takeo Mermaid    | 0              | 0       |



Reflectance estimation results



Figure 8: Estimation results for mixture of Viridian of water ratio 50% and Cobalt Blue of water ratio 50% painted on Mermaid paper.

| Samples      | RMSE             | ∆Eab  |      |
|--------------|------------------|-------|------|
| Vormillion   | Canson Mi-Tantes | 0.017 | 2.49 |
| verminon     | Takeo Mermaid    | 0.030 | 4.93 |
| Permanent    | Canson Mi-Tantes | 0.016 | 3.57 |
| Yellow Light | Takeo Mermaid    | 0.021 | 4.86 |
| Viridion     | Canson Mi-Tantes | 0.040 | 3.20 |
| vindian      | Takeo Mermaid    | 0.060 | 4.63 |
| Cobalt Blue  | Canson Mi-Tantes | 0.020 | 2.64 |
|              | Takeo Mermaid    | 0.035 | 4.01 |

#### Table 1: Average values of RMSE and color difference $\Delta E_{ab}$

Illuminant D65

Figure 7: Rendered images of four color paints at different water ratios under

# Conclusions

This paper has proposed a method to estimate spectral reflectances of watercolor paint samples with different water ratios. We first supposed that watercolor painting was monochrome made by a single paint and water. An algorithm was developed based on the KM approach. The optical constants of S and K were estimated from the measured reflectances of a standard sample painted with a known water ratio. A linear relationship was devised to predict the optical constants of watercolor paintings with different water ratios. The surface-spectral reflectances were then recovered using the KM theory with the predicted optical constants. In experiments, the feasibility of the proposed method was confirmed in experiments using a variety of watercolor paintings with mixed color paints.

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# **Author Biography**

Shoji Tominaga received the B.E., M.S., and Ph.D. degrees in electrical engineering from Osaka University, Osaka, Japan, in 1970, 1972, and 1975, respectively. In 2006, he joined Chiba University, Japan, where he was a Professor (2006-2013) and Dean (2011-2013) at Graduate School of Advanced Integration Science. He is now a Specially Appointed Researcher, Chiba University. His research interests include digital color imaging, multispectral image analysis, and material appearance modeling. He is a Fellow of IEEE, IS&T, and SPIE.