Psychophysical Study of Color Verbalization Using Fuzzy Logic

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Abstract

We develop and implement a fuzzy logic algorithm able to determine which areas of the CIE-Lab color space correspond, with different weights, to which conventional color names (red, green, blue, yellow, magenta, purple, brown, etc.)Parameters of the underlying model are estimated using data derived from a psychophysical experiment. The resulting color naming application will benefit developers of computer vision and image classification applications, graphic designers, imaging hardware and software manufacturers, and other companies and individuals, including those involved in publishing, advertisement, and commercial photography.

Introduction

Knowledge in the area of color naming is valuable in the context of computer vision, image classification, and various other aspects of machine learning. Color naming is also important to graphic designers, imaging hardware and software manufacturers, image quality experts, color scientists, and other companies and individuals, including those involved in publishing, advertisement, and commercial photography.

Berlin and Kay [1] researched verbalization of color in numerous natural languages and postulated the existence of universal color categories. They described a set of 11 basic color categories found in most evolved languages. In English, these basic categories are named *white*, *gray*, *black*, *yellow*, *orange*, *red*, *pink*, *purple*, *blue*, *green*, and *brown*. We followed the fundamental approach of Berlin and Kay. However, with the needs of imaging scientists in mind, we have grouped *white*, *gray*, and *black* colors in one category named *neutral*, substituted *magenta* for *pink*, and expanded two of the original category names to *purple/violet* and *blue/cyan*.

Benavente et al. [2] used a nearly identical set of categories in their study that involved slicing the standard quasi-uniform CIE-Lab color space [3] at several constant values of L. (They kept *pink*. Our *neutral* category corresponds to their *achromatic* category.) Following the approach pioneered by Mojsilovic [4] and Seaborn et al. [5], they formulated the task of color categorization as a decision problem described within the framework of the fuzzy set theory proposed by Zadeh [6]. For the 2D L-slices, Benavente et al. built parameterized color membership functions $\mu(a,b)$. They pointed out that, ideally, color memberships "should be modeled by three-dimensional functions," i.e. functions from CIE-Lab to [0,1], and characterized finding such parametric functions as "a very complicated task."

Our paper aims to fill the gap using a novel technique that employs 3D *convex hull* [7], a well-known computational geometry algorithm utilized for the purpose of color gamut characterization by Guyler [8].

Benavente et al. based their learning data set consisting of 1617 samples on the classic Munsell Book of Colors [9] measured using the standard D65 illuminant. We chose a modern set of 1755 color solid chips coated by PANTONE [10] measured and viewed under the standard D50 (5000K) illuminant, which is closer to our regular white daylight most of the time, D65 being more consistent with the sunlight at noon. For details on fine distinctions between the D50 and D65 illuminants, along with the newer ID50 and ID65 illuminants, the reader is referred to the comprehensive monograph by Hunt [11].

Our paper is structured as follows. The next section characterizes our learning data set in more detail. The section after that expounds the fuzzy logic approach to color categorization. The psychophysical experiment used to generate the data for building the model is described next. We then explain our novel technique for generation of 3D color membership functions based on the 3D convex hull algorithm. Finally, the conclusions are reported and future work directions are discussed.

Learning Data Set

For our psychophysical experiment, we selected a 1755-color set of PANTONE solid chips coated. According to the manufacturer, the colors in the new edition are presented in a "chromatic arrangement." The modern PANTONE technology uses 18 ink bases to ensure very high reproducible gamut volume. A photograph of the book containing the chip set is shown in Fig. 1.

Figure 1. PANTONE solid chips coated

The chips were measured using a handheld GretagMacbeth Eye-One Pro spectrophotometer ran under the auspices of X-Rite ColorPort 1.5.4 software. The CIE 1931 Standard Colorimetric Observer (2-degree observer) measurement data was recorded for both D50 and D65 standard illuminants. The spectrophotometer was tested and calibrated prior to the beginning of measurements and re-calibrated after each sheet (approximately every 7 measurements). It is worth noting that the median ΔE_{76} color difference between the D50 and D65 data points for the set is 3.1, just above the approximate JND (just noticeable difference) of 2.3 [12], with the maximum at 11.6.

We have also measured an older set of 1123 PANTONE solid chips coated [13] with the same spectrophotometer. For the 1099 colors that the two sets have in common, the median ΔE_{76} color difference is 4.7, with the maximum value at 25.6 and other values above 20 found mostly in the yellow region, where the color differences were noticeable, but unlikely to impact color category attribution. The photographs reproduced in Fig. 2 illustrate this situation by showing two color strips detached from sheets of old chips and placed over the corresponding sheets of new chips.

Figure 2. Old PANTONE chips atop new ones

Different techniques for measurement of color gamut volume are known [14], but the absolute volume numbers are not as important for this study as sampling uniformity and complexity of the reproducible color gamut shape. Projection of PANTONE colors with 70≤L≤80 on the (a, b) plane is shown in Fig. 3.

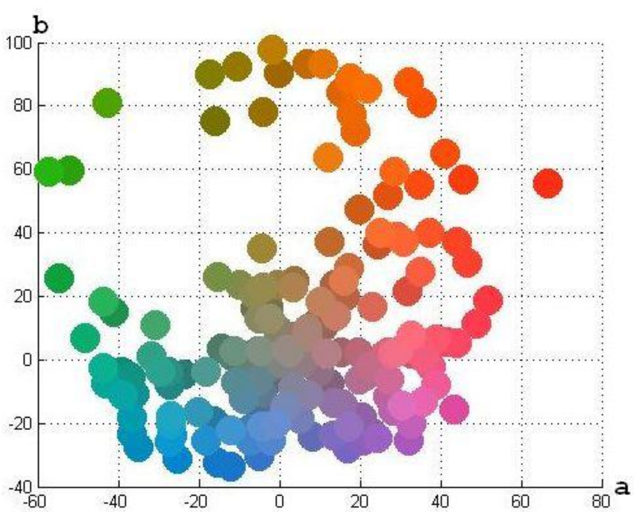

Figure 3. Projection of PANTONE colors with 70≤L≤80 on the (a, b) plane

Finally, we measured substrate white points for both chip sets in CIE-Lab coordinates. They proved similar: (95.95, 0.95, -5.5) for the new set and (94.7, -0.23, -0.3) for the old one. This measurement was performed to supplement the lack of *white* among PANTONE colors.

Fuzzy Logic Approach

The term *fuzzy logic* refers to the logic of fuzzy sets. The concept of a *fuzzy set* [6] is a natural generalization of the classical notion of a set. In fuzzy set theory, the classical sets such that each element either belongs or does not belong to a set are called *crisp sets*. A fuzzy set A consists of a crisp set X and a *membership function* μ_A: X→[0, 1], where 1 describes the situation when an element of X belongs to A, and 0 means the opposite. X is called the *universal set* of A. In our application, for example, fuzzy logic allows us to categorize a yellow-green color as 50% yellow and 50% green, or 60% yellow and 40% green, depending on its CIE-Lab color space coordinates. The more general restriction on the membership functions involved in formalization of the color naming task for n color categories C_k, k=1,…,n, can be expressed by the formula

$$\sum_{k=1}^{n} \mu_{C_k}(\boldsymbol{s}) = 1, \tag{1}$$

where a *sample* **s** corresponds to a point in the CIE-Lab color space. In our case, *n*=9 and

$$C_k \in \left\{ \begin{matrix} Neutral, Yellow, Orange, Red, Magenta, \\ PurpleViolet, BlueCyan, Green, Brown \end{matrix} \right\}. \tag{2}$$

Each observer in our psychophysical experiment was asked to assign exactly one color category to each PANTONE color. (More details about the psychophysical experiment will be provided in the next section.) The resulting histograms for the samples were then converted to estimates of the corresponding membership function values as follows, using the approach first proposed by Dubois and Prade [15].

If all observers agree that a sample ***s*** belongs to the category C_i, then we estimate $\mu_{C_i}(\boldsymbol{s}) = 1$, and the other membership function value estimates for the sample ***s*** are zeros. Otherwise, the percentages of responses placing ***s*** in color categories C_k, k=1,…,n, are normalized to [0, 1] to become member function value estimates meeting the criterion from Eq. (1).

In order to derive continuous membership functions from the statistical estimates computed for our set of samples, we had to choose a curve to describe the transitions between color categories. The simplest approach would result in the membership functions plotted along the a axis looking like those shown in Fig. 4.

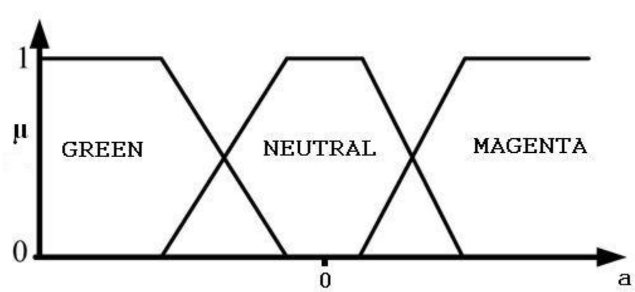

Figure 4. Simple one-dimensional membership functions

However, for the case of the border between two categories, we decided to adopt and adapt the approach of Benavente et al. that involves fitting the experimental data with a parameterized *sigmoid function*

$$S^1(x, \beta) = \frac{1}{1+e^{-\beta x}}, \quad (3)$$

where β is the parameter controlling the slope of the transition from 0 to 1 seen in Fig. 5, where dE denotes ΔE_{76} color difference measured along the x axis, which can be provisionally visualized as an imaginary "straight line" in the CIE-Lab color space.

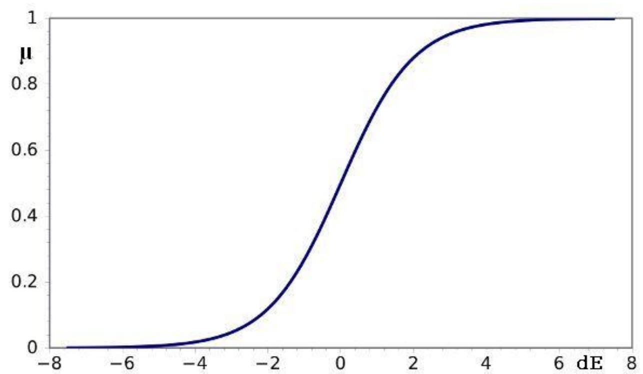

Figure 5. Sigmoid logistic function

This variety of sigmoid functions is also known as the *logistic function*. In our interpretation, the "line" actually consists of two straight segments, one of which connects the point of interest to the closest point that belongs to the closest color category, and the other one connects the former to the closest point that belongs to the second closest color category. More details on how the initially one-dimensional approach is extended to 3D will be provided later, in the section on 3D color membership function generation. Meanwhile, we'd like to point out that other S-shaped functions were available, including the *error function* erf(x) plotted in Fig. 6.

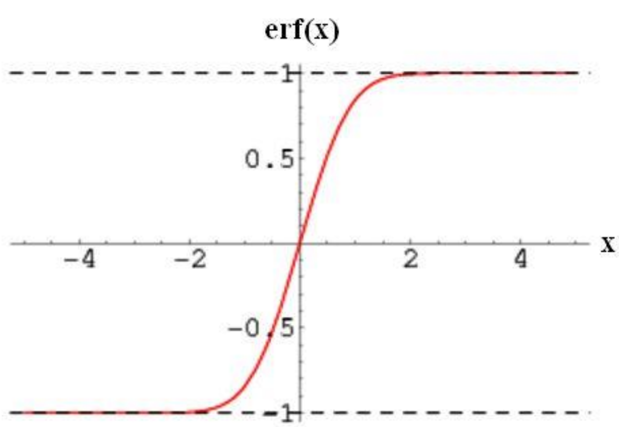

Figure 6. Error function

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt. \quad (4)$$

Now that the reader has the general idea of what we're going to do with the experimental data, let's describe our psychophysical experiment in more detail.

Psychophysical Experiment

In this experiment conducted in a controlled lab at Purdue's West Lafayette campus, the observers were instructed to examine a series of 254 PANTONE solid color chip sheets that were presented to them one-by-one in the order that the chip manufacturer characterized as "chromatic arrangement." Each chip sheet represented between 4 and 7 colors, displayed in the viewing booth under the standard D50 lighting conditions.

An advertisement describing the goal and procedure of the experiment was posted on campus. It was also sent out to EISL members publically via email using the EISL mailing list.

Before the experiment, the observers were given an Ishihara colorblindness test. People who did not have normal trichromatic color vision or were not able to understand the instructions were excluded from subject pool. The number of subjects was capped at 30. The subjects signed the informed consent form. They were not paid for taking part in the experiment.

The viewers were asked to assign one and only one of several color categories to each PANTONE color examined. The assistant recorded their replies and presented the next sheet.

Each color chip sheet was shown once. Each session of the experiment lasted up to 30 minutes.

At the beginning of the session, each participant was assigned a unique "observer ID" that his/her answer set was associated with for the purpose of subsequent data processing and analysis.

The color categories were listed in the Charge to Observers document as follows.

a. NEUTRAL
b. YELLOW
c. ORANGE
d. RED
e. MAGENTA
f. PURPLE/VIOLET
g. BLUE/CYAN
h. GREEN
i. BROWN

The observers were informed that the NEUTRAL category grouped together WHITE, GRAY and BLACK colors.

The observers occasionally suggested other category names for future work, such as olive, beige, pink, terracotta, rosy, aquamarine, turquoise, teal, and navy blue.

3D Color Membership Function Generation

We began data processing by building nine 3D convex hulls of the CIE-Lab sample points corresponding to the PANTONE solid chips coated that all observers agreed to place in the same color category. The top-down view of the resulting convex hulls visualized using intuitive color coding to set face colors of the triangles that these surfaces consist of according to their respective color categories is shown in Fig. 7.

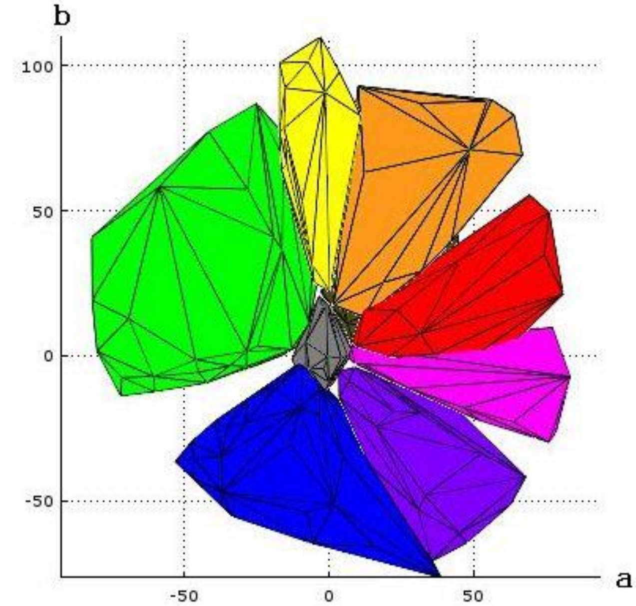

Figure 7. 3D convex hulls of color categories, top-down perspective

While the neutral colors predictably formed a spindle-like shape around the L axis, it's clear that some other divisions did not exactly follow the constant hue planes. It is especially difficult to see the convex hull for the brown color category under the orange and yellow ones in Fig. 7, so we provide the bottom-up view in Fig. 8 below to remedy that.

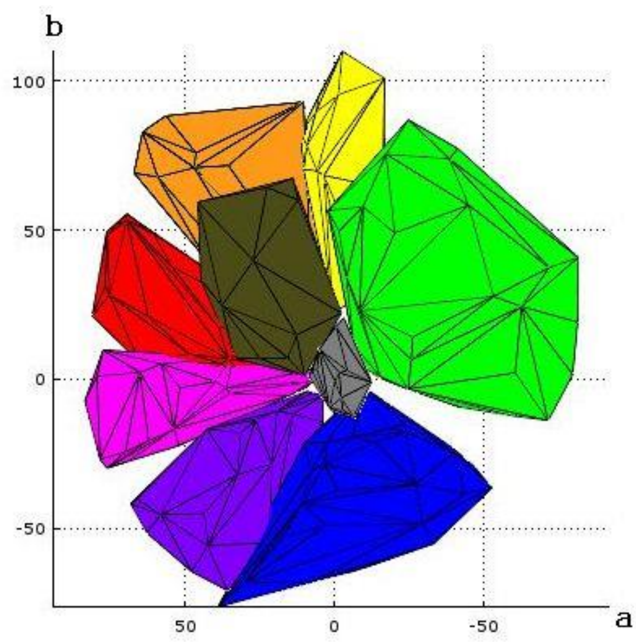

Figure 8. 3D convex hulls of color categories, bottom-up perspective

While the large gap between *green* and *blue/cyan* regions may indicate the need to introduce a separate *cyan*/*aquamarine*/*turquoise*/*teal* category of blue-green colors, some of the convex hulls shown in Figures 7 and 8 appear to intersect when, in fact, they don't. Fig. 9 shows the gaps that separate the convex hull for the *green* color category from those for *yellow*, *brown* and *neutral* categories. It also offers a better view of the border between *orange* and *brown*.

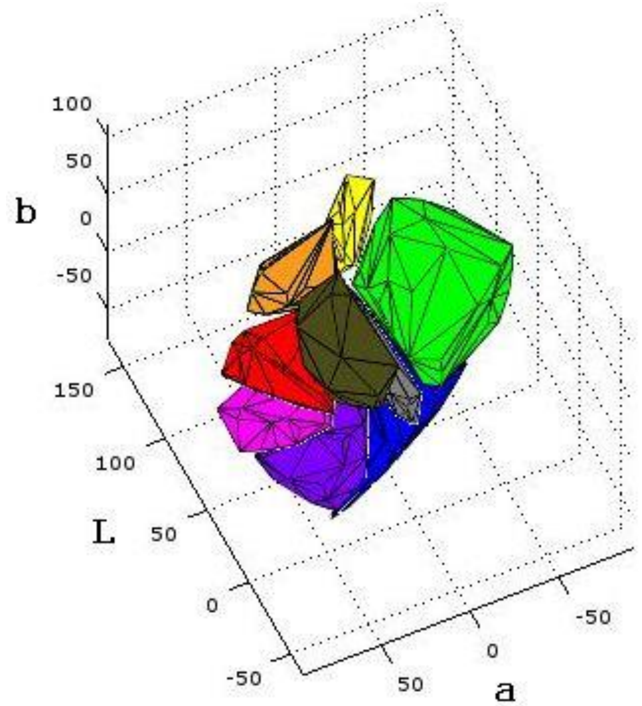

Figure 9. 3D convex hulls of color categories, an orthogonal projection

The illustrations above also tell us that most of the volume of the gaps can be "filled" by membership function transitions between two color categories, e.g., *yellow-green*, *orange-red*, *neutral-green*. At the same time, we see that, in some cases, up to four nonzero membership function values may be needed to characterize a color, say, *yellow-orange-brown-neutral*, *orange-red-brown-neutral*, or *red-magenta-brown-neutral*, the latter being something as mundane as black cherry.

Before we continue to handle the transitions, let's first observe that it is fairly straightforward to determine with good precision if a given point lies within a convex hull. Indeed, the center of mass of the vertices of the convex hull is guaranteed to lie inside it, by virtue of convexity. Furthermore, the sum of the volumes of all tetrahedra formed by the center of the mass and the triangular faces of the convex hull is the hull's volume. If a point is located inside the convex hull, then the sum of the volumes of all tetrahedra formed by the point and the faces of the convex hull is also equal to that volume. If the point is located outside, then the sum is larger than the hull's volume. To take computational errors into account, we test if the difference between the two aforementioned sums is below a very small finite threshold. If a point is inside a convex hull for the color category C_i, we assign $\mu_{C_i}(\boldsymbol{s}) = 1$. But what about the convex hull intersections?

Once the number of observers in our psychophysical experiment becomes sufficiently large, the convex hulls of the sample points such that $\mu_{C_i}(\boldsymbol{s}) = 1$, i=1,…,9, will no longer intersect. Nevertheless, we implemented a redundant check that would simply assign $\mu_{C_i}(\boldsymbol{s}) = 1/m$ to a point belonging to an intersection of m convex hulls, for the values of i corresponding to any of the hulls that formed the intersection. For all other values of i, we assigned $\mu_{C_i}(\boldsymbol{s}) = 0$ in order to meet the criterion from Eq. (1).

In order to take care of the extrapolation to the outside area and the interpolations needed to fill the gaps between the convex hulls, we first observe that the computation of the minimum Euclidean distance from a point to a triangle in 3D is a well-known problem in computational geometry. Two practical solutions are given by Jones [16]. The surface of a convex hull consists of triangular faces, so we can now calculate the minimum distance from any point in CIE-Lab located outside of all convex hulls to each convex hull as the smallest of the minimum distances from the point to the convex hull's faces. By the definition of the ΔE_{76} color difference, the calculated distance is equal to the ΔE_{76} color difference between the point and the closest point that our model takes to belong to the color category corresponding to the convex hull.

To develop a complete 3D solution allowing up to four nonzero membership function values per point, we impose three ΔE_{76} thresholds, $T_1>T_2>T_3$.

If the ΔE_{76} color difference between the point and the second closest convex hull exceeds T_1, then our algorithm concludes that the second closest color category is too far away, so we assign $\mu_{C_i}(\boldsymbol{s}) = 1$ for the value of i corresponding to the closest color category and $\mu_{C_i}(\boldsymbol{s}) = 0$ for all other values of i. This provision solves the extrapolation part of the problem. We extrapolate by placing an outside point into the closest color category.

Otherwise, if the ΔE_{76} color difference between the point and the third closest convex hull exceeds T_2, then our algorithm concludes that the third closest color category is too far away, and we apply the parameterized fitting technique described earlier, in the section on the fuzzy logic approach. This case takes care of the gaps between two color categories. Nonzero values are assigned to the membership functions for the two closest color categories.

Otherwise, if the ΔE_{76} color difference between the point and the fourth closest convex hull exceeds T_3, then our algorithm concludes that the fourth closest color category is too far away. Let's denote the distances from the point to the three closest color categories $\Delta_1\leq\Delta_2\leq\Delta_3$. Clearly, $\Delta_1+\Delta_2\leq\Delta_1+\Delta_3\leq\Delta_2+\Delta_3$. We apply the technique for two categories to each of the three category pairs and then calculate weighted averages of the resulting membership function values so that the values computed for the two closest color categories taken separately are weighted by

$$w_{1,2} = \frac{\Delta_2+\Delta_3}{2(\Delta_1+\Delta_2+\Delta_3)}, \tag{5}$$

the values computed for the closest color category and the third closest color category taken separately are weighted by

$$w_{1,3} = \frac{\Delta_1+\Delta_3}{2(\Delta_1+\Delta_2+\Delta_3)}, \tag{6}$$

and the values computed for the second and third closest color categories are weighted by

$$w_{2,3} = \frac{\Delta_1+\Delta_2}{2(\Delta_1+\Delta_2+\Delta_3)}. \tag{7}$$

Finally, if the thresholding process indicates that we need to accommodate four closest color categories, then we can denote the corresponding distances $\Delta_1\leq\Delta_2\leq\Delta_3\leq\Delta_4$, apply the technique for three categories to each of the four possible category triplets, and then calculate weighted averages of the resulting membership function values so that the values computed for the three closest color categories taken separately are weighted by

$$w_{1,2,3} = \frac{\Delta_2+\Delta_3+\Delta_4}{3(\Delta_1+\Delta_2+\Delta_3+\Delta_4)}, \tag{5}$$

and so on. It is straightforward to extend our technique if there is a need to allow for five or more nonzero membership function values per point in the CIE-Lab space.

Conclusions and Future Work

We conducted a psychophysical study to determine how naïve observers verbalize colors (categorize color patches into several named color categories). The observers viewed a large subset of PANTONE color chips under standard D50 lighting conditions. The PANTONE color chips were measured using a spectrophotometer. A program was written to analyze the psychophysical and measurement data to characterize color space areas that correspond to color names. Another program was developed to interpolate/extrapolate based upon the previously obtained characterization in order to categorize CIE-Lab colors automatically, using fuzzy logic.

Our results to date confirm that substantial portion (> 90%) of the reproducible CIE-Lab volume can be covered by about a dozen common color category names specified. One of the possible directions for future work involves adding more color categories to the model.

More work is also needed to complete comprehensive validation of the model. By that, we intend to confirm that computational geometry can be used as a basis to fit a model from which color classification can be conducted with an improved efficiency/accuracy tradeoff than other methods available, for both the training and prediction tasks.

We developed a novel technique for definition of 3D membership functions to model color categories in CIE-Lab. To the best of our knowledge, we're also the first to investigate the use of the 3D convex hull in the CIE-Lab space for the color naming task. Given that the actual reproducible color gamuts are well-known to have concavities [14], another direction for future work would be toward making an attempt to relax the convexity requirement.

Our methodology for psychophysical analysis over a large reproducible color gamut appears to be unique, along with our selection of PANTONE chips for the training set. Thus we believe our work will also produce a new dataset for use in future research.

So far, we have utilized the older ΔE_{76} color difference metric, which was easy to incorporate into our model thanks to its geometric simplicity (Euclidean distance). Yet another direction for future work would be to investigate if the newer ΔE_{00} color difference metric can be used to improve the performance of our fuzzy logic algorithm.

Finally, part of our work involved analyzing the PANTONE colors under the D50 illuminant. It may be useful on its own and was previously unavailable.

References

[1] B. Berlin and P. Kay, Basic Color Terms: Their Universality and Evolution, Berkeley CA: University of California Press, 1969.

[2] R. Benavente, M. Vanrell, and R. Baldrich, "Parametric Fuzzy Sets for Automatic Color Naming," J. Opt. Soc. Am. A, vol. 25, no. 10, pp. 2582-2593, 2008.

[3] CIE Colorimetry - Part 4: 1976 L*a*b* Colour Space, Joint ISO/CIE Standard, ISO 11664-4:2008(E)/CIE S 014-4/E:2007.

[4] A. Mojsilovic, "A Computational Model for Color Naming and Describing Color Composition of Images," IEEE Trans. Image Processing, vol. 14, no. 5, pp. 690-699, 2005.

[5] M. Seaborn, L. Hepplewhite, and J. Stonham, Fuzzy Colour Category Map for the Measurement of Colour Similarity and Dissimilarity, Pattern Recognition, vol. 38, no. 2, pp. 165-177, 2005.

[6] L. Zadeh, "Fuzzy Sets," Information and Control, vol. 8, no. 3, pp. 338-353, 1965.

[7] C. Barber, D. Dobkin, and H. Huhdanpaa, "The Quickhull Algorithm for Convex Hulls," ACM Trans. on Mathematical Software, vol. 22, no. 4, pp. 469-483, 1996.

[8] K. Guyler, "Visualization of Expanded Printing Gamuts Using 3-Dimensional Convex Hulls," American Ink Maker, vol. 79, no. 9, pp. 36-56, 2001.

[9] Munsell Color Company, Munsell Book of Color — Matte Finish Collection, Grand Rapids MI: Munsell Color Company, 1976.

[10] Pantone, Inc., PANTONE GP1606 Plus Series Solid Chips Coated and Uncoated, 2 Book Set, Carlstadt, NJ: Pantone Inc., 2015.

[11] R. W. G. Hunt, Measuring Color, 4th Edition, Chichester, West Sussex UK: John Wiley & Sons Ltd, 2011.

[12] G. Sharma, Digital Color Imaging Handbook, Boca Raton, FL: CRC Press, 2003.

[13] Pantone, Inc., PANTONE GP1203 Solid Chips Coated and Uncoated, 2 Book Set, Carlstadt, NJ: Pantone Inc., 2005.

[14] D. Gusev, "Color Gamut Characterization Via Ray Tracing for Device Profile Generation," in PICS 2003: Image Processing, Image Quality, Image Capture Systems, Rochester, New York, 2003.

[15] D. Dubois and H. Prade, "Fuzzy Cardinality and the Modeling of Imprecise Quantification," Fuzzy Sets and Systems, vol. 16, pp. 199-230, 1985.

[16] M. Jones, "3D distance from a point to a triangle," Technical Report CSR-5-95, Department of Computer Science, University of Wales, Swansea, February 1995.

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