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Colour Visualisation of the Phase of Complex Signals

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Abstract

We propose a colour-based technique for the visualisation of the phase of complex signals, in particular of Fourier transforms and of analytic signals. Using the fact that both the hue and the angle are cyclic magnitudes, a one-to-one relationship can be established between angles and colour hues that respects the cyclic order: an order isomorphism. With the help of a Matlab GUI tool, called a chromatic profiler, *the correspondence is made so that the* cardinal points *[2] of the cyclic variables are preserved, and the rest of the points are mapped so that the result is best readable. What the technique may lack in accuracy is gained in readability.*

Introduction

For the natural domains of signals of time and space, it is rare to consider complex signals, nevertheless a main source of complex signals (e.g. functions of the time or frequency variables that take as values complex numbers) in engineering is the Fourier transform. Also, in going from a real signal (i.e. one that has null imaginary component) to is analytic signal (given by the sum of the original signal plus i times its Hilbert transform), you get a complex signal out of a real signal. The Fourier transform of a signal is a representation of the signal in the *Fourier frequency* (i.e. the frequency of a complex exponential) domain. The analytic signal allows for the computation of the instantaneous, Fourier frequency and phase.

Elaborating on the work presented in [2], using a *hue-angle correspondence* for the phase component[1] of a complex signal, we consider the colour visualisation of complex signals. The phase of the Fourier transform of a signal is closely related to its shape (in its original, e.g. time, domain) and is thus important when working with seismic signals, biomedical signals, images, etc.

In electrical engineering, there has been a traditional emphasis on the magnitude over the phase of the Fourier transform of signals, mainly because of its origins in telephony and the broadcasting of audio signals. Also, because the phase of a Fourier transform is a less intuitive component than the magnitude, and because the relation of the phase with the waveform or shape of the original signal (i.e. before being transformed) is indirect.

The hue-angle correspondence mentioned above can be a homeomorphism between circles since both magnitudes, hues and angles, live in circle spaces. In the following section, we review the definitions both of cyclic magnitudes and of circular signals, and the traditional ways of plotting their graphs, which present some difficulties. Afterwards, we present alternative techniques to visualise cyclic magnitudes such as the Fourier phase or the instantaneous phase using the hue of (chromatic) colour. In a later section, we consider the visualisation of the phase of the Fourier transform of images . The techniques we present allow for a ready visualisation of complex signals; the techniques are perhaps more qualitative than quantitative with respect to more traditional techniques. Whatever may be lost in numerical accuracy can be gained in intuition, which may prove useful for design purposes.

A word on nomenclature. Mathematically, a signal is a function $s : \mathbf{D} \to \mathbf{E}$, with a domain set $\mathbf{D}$ and a range set $\mathbf{E}$; it is a function that codes some information. Although most of the times the terms function and signal are interchangeable, some discrepancies in nomenclature exist, for example, a continuous signal (a function having as domain $\mathbf{D}$ a *contimuum*) may not be a continuous function (consider for example Heaviside's step function) and an *analytic signal* may not be an analytic function.

Cyclic Magnitudes and Circular Signals

As in [3], [4], [1], we define, somewhat redundantly from a logical point of view, a *cyclic ordering* for the elements of a set $\mathbf{A}$, as a ternary relation $R \subset \mathbf{A}^3$ (i.e. a subset of $\mathbf{A} \times \mathbf{A} \times \mathbf{A}$) that is

- *circular*:
  $(a,b,c) \in R \implies (b,c,a) \in R, \wedge, (c,a,b) \in R.$
- *antisymmetric*:
  $(a,b,c) \in R \implies (a,c,b) \notin R, \wedge, (b,a,c) \notin R, \wedge, (c,b,a) \notin R.$
- *transitive*
  $(a,b,c) \in R, \wedge, (b,d,c) \in R \implies$
  $(a,b,d) \in R, \wedge, (d,c,a) \in R.$
- *complete*
  $\forall a,b,c \in \mathbf{A}, (a,b,c) \in R, \vee, (a,c,b) \in R.$

Imagine the three elements a,b,c of a triple in the relation as 3 points on a circle and note that a cyclic ordering gives a direction to traveling the circle.

Cyclic magnitudes

A cyclic magnitude is a variable that takes values in a cyclically ordered set. An example of a cyclic magnitude is (planar) angle.Let $\mathbf{A} = [0, 2\pi) \subset \mathbf{R}$ (where $\mathbf{R}$ stands for the set of the real numbers, or real line); the cyclic ordering R being given as $(\theta, \phi, \psi) \in R \iff$

[1] Remember that a complex number has a rectangular-coordinate representation in terms of its real and imaginary parts and, in polar coordinates, a representation in terms of its magnitude and its phase, or angle.

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- $0 \leq \theta \leq \phi \leq \psi < 2\pi$, $\vee$,
- $0 \leq \psi \leq \theta \leq \phi < 2\pi$, $\vee$,
- $0 \leq \phi \leq \psi \leq \theta < 2\pi$.

In this canonic way, a cyclic ordering $(\mathbf{A}, R)$ is obtained from a linear ordering $(\mathbf{A}, <)$. If the linear ordering has a corresponding metric or measure (order $\rightarrow$ open intervals $\rightarrow$ length, topology), the set has a *total length*. In the case of the angle magnitude the total length is 2π. Likewise, cyclic magnitudes can be obtained out of an interval[2] $[x, y)$, its total length being $y - x$.

A metric for these cyclic orderings is obtained via a *tent function* $T : [x, y] \rightarrow [0, \frac{y-x}{2}]$. For example, for the angle space, the tent function is

$T : [0, 2\pi] \rightarrow [0, \pi]$, via
$T(\theta) = \theta$ if $0 \leq \theta \leq \pi$, and
$T(\theta) = 2\pi - \theta$ if $\pi \leq \theta \leq 2\pi$;
the metric being given by
$d(\theta_1, \theta_2) := T(|\theta_1 - \theta_2|)$,
see also [5].

Another case of a cyclic magnitude is that of the hue of the chromatic[3] colours and, by choosing a homeomorphism between the angle space and the hue space, a (hue) *colour code* for angles [2] results. This can be exploited to visualise plots of angle signals using colour, economising space and allowing for more readability.

Circular signals

A signal $x : \mathbf{B} \rightarrow \mathbf{A}$ is called *circular* if its range $\mathbf{A}$ is cyclically ordered. The *swing* of a circular signal is given by the total length of its range.

The graph of a signal $x : \mathbf{B} \rightarrow \mathbf{A}$ lives in $\mathbf{A} \times \mathbf{B}$. If both its domain $\mathbf{A}$ and its range $\mathbf{B}$ are linearly ordered sets, the graph naturally fits in the Cartesian plane. A circular signal $x : \mathbf{B} \rightarrow \mathbf{A}$, with a linearly ordered domain $\mathbf{B}$, has a graph that naturally embeds in a cylinder $\mathbf{B} \times \mathbf{S}^1$; call it a (circular) *cylindrical* signal; its graph in fact presents some difficulties, for it to be drawn on a (plane) page. On the other hand, a circular signal $x(t) : \mathbf{B} \rightarrow \mathbf{A}$, with a cyclically ordered domain $\mathbf{B}$, has a graph that naturally embeds in torus; call it a (circular) *toric* signal.

Several solutions have been adopted by the engineering community to circumvent the problem of the nonplanarity of the graphs of circular signals when plotting them. Consider first the plot of the phase component alone. One possibility is to continuously (with respect to the topology of the reals) *continue* the graph and to let the angle variable take any real number as value, considering two such numbers as *equivalent* whenever their difference is an integer multiple of 2π. This corresponds to a topological lift of the circle to the real line. Another solution is to slice open (and flat) the ambient cylinder of the graph, and to plot the phase function on such Cartesian plane, with a finite vertical axis from 0 to 2π (or from $-\pi$ to π), both heights corresponding to the position of the slice cut, and to implicitly assume that vertical jumps of magnitude 2π (are only artificial discontinuities and) correspond in fact to points of continuity.

[2] In similar ways, the intervals $(x, y]$, (x, y), $[0, \infty)$, $(0, \infty)$ and $(-\infty, \infty)$ can be given cyclic orderings.

[3] The achromatic colours being black, white and the greys.

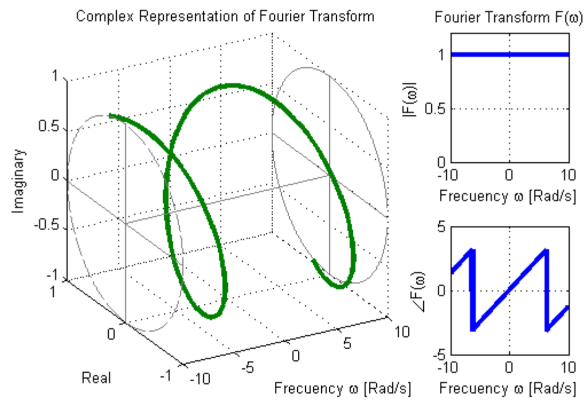

Figure 1. *The graphs of the signal $e^{j\omega} : \mathbf{R} \rightarrow \mathbf{C}$, of its magnitude and of its phase.*

It is important to stress the point that in such planar graphs of the phase component, a jump of value π is in fact a discontinuity, typically corresponding to a crossing of the origin of the complex plane of the graph of the function, while a "jump" of value 2π is no discontinuity at all; in this sense planar plots can be misleading; see Figure 1.

In this paper we present a visualisation methodology for the graph and the phase of complex signals that uses colour[4]. The plots are planar and concise and the problem of the artificial discontinuities is avoided.

Fourier Transforms

Even though, in engineering, it is usual to work with real signals, the Fourier transform of a (real or complex) signal typically has nonzero real and imaginary parts, that is, it is a complex signal. For illustration purposes, we consider the Fourier transform of continuous and discrete signals and, later on in Section , the Fourier transform of 2D discrete signals.

Consider the convolution system (a first-order low-pass filter) with characteristic function given by $h(t) = (1 - e^{-t\Omega_C})u(t)$, where $u(t)$ stands for Heaviside's step function [5]. The transfer function of the system is the Fourier transform of h, which is given by

$$H(\Omega) = F\{h\}(\Omega) = \int_{-\infty}^{\infty} h(t)e^{-j\Omega t}dt = \frac{1}{1 + j\Omega/\Omega_c}$$

which has as phase a cylindrical signal.

Likewise, consider a discrete convolution system (again a low-pass filter) with difference equation $y_n = 0.5(x_n + x_{n-1})$ with characteristic function $g_n = 0.5\delta_n + 0.5\delta_{n-1}$ (where δ is Kronecker's delta function[6]) and corresponding transfer function given by

$$G(\omega) = F\{g\}(\omega) = \sum_{n=-\infty}^{\infty} g_n e^{-j\omega n} = 1 + e^{j\omega},\ \omega \in [0, 2\pi)$$

whose phase $\angle G(\omega)$ is a toric signal.

[4] At this point, the technique is not suitable for colour blind people but it will be done so, if with limitations, in the future.

[5] $u(t) = 0$ if $t < 0$, and $u(t) = 1$ if $t \geq 0$, i.e. u is the indicator function of the interval $[0, \infty) \subset \mathbf{R}$.

[6] $\delta_n = 0$ in $n \neq 0$, and $\delta_0 = 1$.

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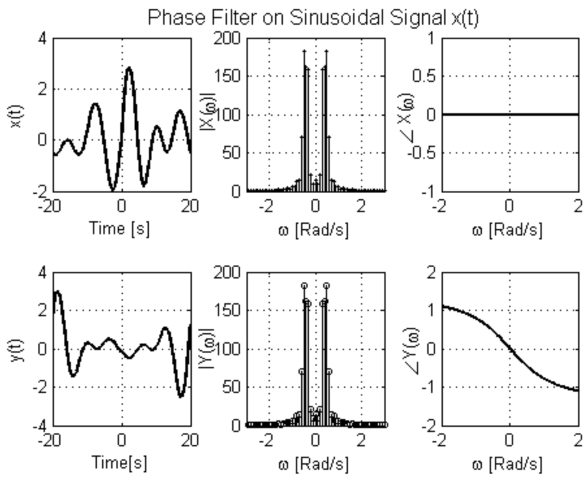

Figure 2. *Phase Filter All Pass Example on Sinusoidal Signals*

Likewise, for a discrete image, or a 2D discrete signal in general, $\{x_{n,m} : n,m \in \mathbf{Z}\}$, its Fourier transform is given by

$$X(\omega,\eta) = F\{x\}(\omega,\eta) = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} x_{n,m}e^{-j\omega n}e^{-j\eta m},$$
$$(\omega,\eta) \in (-\pi,\pi] \times (-\pi,\pi]$$

Fourier transforms are usually visualised with two graphics, one for the magnitude and the other one for the phase.

Visualisation of the Phase of Complex Signals Using Colour

A complex signal $s : \mathbf{A} \to \mathbf{C}$, where $\mathbf{C}$ stands for the set of the complex numbers, has a graph (i.e. the set of ordered pairs $(a, s(a))$) that lives (i.e. it is canonically embedded) in $\mathbf{A} \times \mathbf{C}$. If $\mathbf{A}$ is a linearly ordered set, such as the reals $\mathbf{R}$, topologically, the ambient space $\mathbf{R} \times \mathbf{C}$ for the graph of the signal is $\mathbf{R}^3$. Thus, for complex signals that are functions of a 1D linearly ordered domain, the graphs are naturally plotted in a 3D space, rather than on a plane. In particular, complex signals of constant magnitude are naturally plotted on a cylinder; see for example Figure 1, where the signal $F(\omega) = e^{j\omega}$, of magnitude $|F(\omega)| = 1$ and phase $\angle F(\omega) = \omega$, is plotted.

We present an alternative to the techniques of a plot in 3D space and to the more traditional practise of plotting separate, plane graphs of the magnitude $|s(a)|$ and phase $\angle s(a)$ components of a complex signal $s(a)$, $a \in \mathbf{A}$, consider the following. With the help of colour, a complex signal $s(a)$ can in fact be plotted on a flat screen or piece of paper. A simple coloured straight line (or band) can be used to indicate the phase [2] of the signal as a function of a by exploiting one of the already mentioned correspondences between hues and angles; moreover, by changing the luminance of the hue at different points on the line (or band), the coloured line (or band) can simultaneously indicate the phase and the magnitude of the signal.

It should be remembered that the complex number $0+0j$ has undefined phase so that, whenever the magnitude of a signal is null, the phase becomes undefined. The typical case of a discontinuity of the phase of a complex signal occurs when the graph of the signal continuously crosses the origin of the complex plane, the magnitude then becomes zero, while the phase abruptly changes from a given angle ϕ to the angle $\phi + \pi$; the magnitude remains continuous but (even if the function corresponding to the signal[7] is analytic) is not an analytic function anymore, as it is not differentiable anymore. Sometimes, since (for the case of planar plots) it is preferred, instead of plotting the magnitude and the phase of a complex signal, to plot its (real) *value function* (given by $\pm$ its magnitude) and its *angle function* (given by the phase +0 or $+\pi$) so that the value may be positive or negative so long as the angle is a continuous function, even when the signal crosses the origin.

Coding the Hue Circle in RGB Colour Space

In digital machines, colour is coded with the RGB, cubic, colour space. A hue circle can be derived in a straightforward fashion by considering the the *chromatic hexagon* [6] which is the (nonplanar) hexagon composed of the six edges of the RGB cube that do not contain [111] (white) nor [000] (black) as vertices. The points of the hexagon are at a fair distance from the black point, nevertheless their distance (i.e. the luminance) varies; likewise, the points of the hexagon are at a fair distance from the achromatic segment but the distance (i.e. the saturation) varies.

The chromatic hexagon can be parametrised with a number x in the interval [0, 6) as follows. Let $x = 0$ correspond to $[RGB] = [100]$, then, for $x \in [0,1)$, let the corresponding colour be $[1,x,0]$, for for $x \in [1,2)$, let the corresponding colour be $[1-(x-1),1,0]$, for $x \in [2,3)$, let the colour be $[0,1,x-2]$, for $x \in [3,4)$, let the colour be $[0,1-(x-3),1]$, for $x \in [4,5)$, let the colour be $[x-4,0,1]$ and, for $x \in [5,6)$, let the colour be $[1,0,1-(x-5))$. By assigning to each value of x the angle $\frac{2\pi}{6}x$, the hue circle of Figure 3 results. The variations in luminance and saturation are too big and the circle is not appropriate.

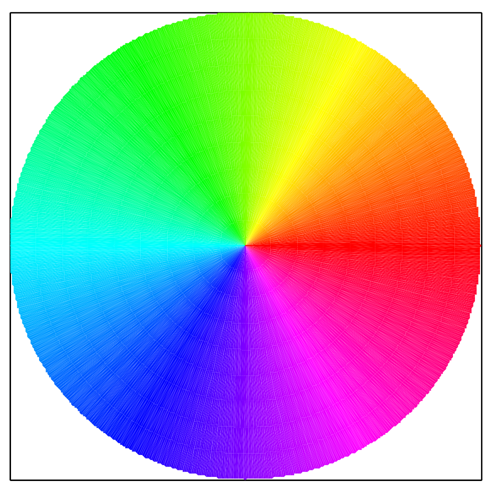

Figure 3. *Hue disk corresponding to the chromatic hexagon. Primaries R (having the hue of a spectral red than of unique red), G and B, being closer to the black point, have a smaller luminance than their pairwise combinations yellow (R+G), cyan (G+B) and violet (B+R).*

For the illustrations in the paper, rather than colouring a

[7] Strictly speaking, an *analytic signal* is a signal that results from the addition to a given real signal, of its Hilbert transform, times j $(=\sqrt{-1})$.

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(thin) circle, we color disks, with the understanding that the corresponding hue circle is at the boundary of the plotted disk.

Alternatively to a disk based on the chromatic hexagon, you can exploit a colour space of the type hue-saturation-luminance (such as Matlab's hsv colour space) to obtain a hue disk; by setting the luminance and saturation values as constant and letting the hue component sweep all possible values and then translating (as with Matlab's hsv2rgb) the hue-saturation-luminance values to corresponding RGB values, you also get a hue disk. See for example the one in Figure 4. In this case, to obtain a chromatic diagram, since Matlab's *hsv* colour space equally spaces in the circle the RGB primaries red green and blue, the function $h(\theta)$ rather that being linear, is preprocessed as in Figure 5 producing the circle in Figure 6.

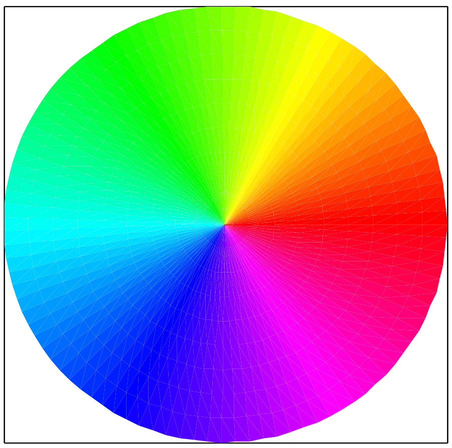

Figure 4. *Hue disk resulting from a linear variation of Matlab's hue h, followed by Matlab's hsv2rgb.*

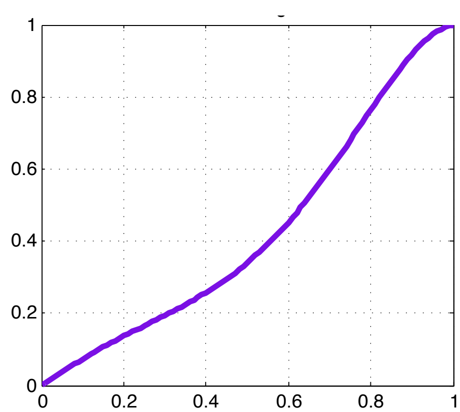

Figure 5. *An orientation-preserving homeomorphism of the circle to itself, that shifts the angles 0° (red), 60° (yellow), 120° (green) and 240° (blue) , to the angles 0°, 90°, 180° and 270°, correspondingly. The scale of the axes is shown normalized.*

Other hue circles result from appropriate functions $R(\theta)$, $G(\theta)$ and $B(\theta)$ of the angle variable $\theta \in [0, 2\pi)$; the aim is to get a circle where all hues appear continuously and the corresponding colours are of uniform saturation and luminance, Each such a set of functions is called an RGB *hue-angle code*, or correspondence and their graphs are called *profile curves* .

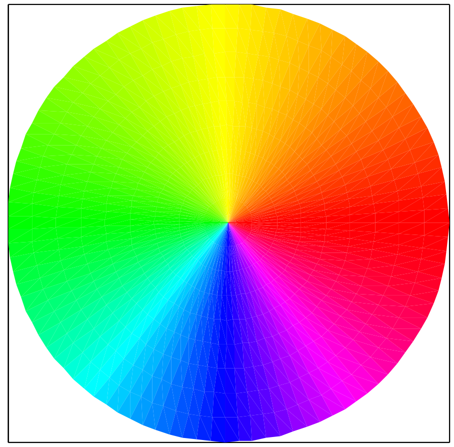

Figure 6. *The hue code in Figure 4 compösd with the homeomorphism graphed in Figure 5.*

In this vein, we have designed a computational tool which allows to manually design piecewise linear profile curves. This is achieved by placing a set of *hinges*, at a collection of θ positions, where the values of R, G and B are manually fixed; in between the hinges the variations are linear and, in a trial and error process where the corresponding hue disk is being displayed, a suitable hue disk is obtained.

A *suitable hue disk* is one that, except at the center where a pinwheel singularity results, it continuously[8] displays all hues, with uniform luminance and saturation, that displays complementary hues as opposed and that displays the 4 unique hues red, yellow, green and blue at the four cardinal points.

Notice for example in Figure 7, that the use of sinusoidal RGB profile curves does not result in a suitable disk, mainly due to the virtual absence the unique yellow hue. Also, as mentioned, we want a hue circle that is a *chromatic diagram*, i.e. where opposing hues are complementary. [9] This provides more readability to the plots of angles based on the resulting hue code. Likewise, we want the 4 cardinal points of the circle (those at 0°, 90°, 180° and 270°) to correspond to the 4 unique hues red, yellow, green and blue. The aim of the design of the RGB profile curves is to achieve these goals.

In Figures 9 and 10, two RGB and their corresponding disks are shown; they were custom designed are presented with the colour profiler already mentioned, implemented as a Matlab GUI. The luminance is nearly constant, the uniques are placed at the cardinal points and opposing hues are nearly complementary.

In Figures 9 and 10 two custom made profiles are shown. They were designed with the help a GUI Matlab interface.

Phase and Magnitude Visualisation

Given a hue-angle correspondence, the phase of a complex signal (e.g. of the frequency variable) can be displayed as a coloured, horizontal ribbon, where the horizontal position measures frequency (or, more generally, the domain of the signal func-

[8] A spatial function of hue is continuous if the transitions are smooth, i.e. separated by just noticeable differences.

[9] Remember that two colours are said to be complementary if in an additive combination an achromatic colour results, e.g. blue and yellow.

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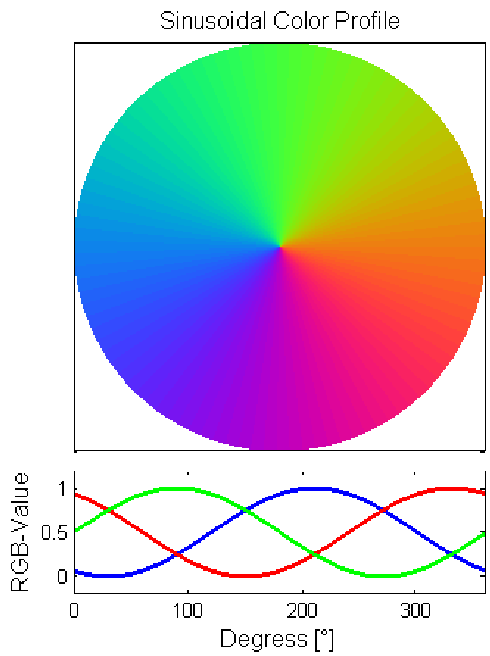

Figure 7. *Hue disk resulting from sinusoidal variations of the R , G and B components of the profile. The yellow hue is hardly visible. The corresponding RGB profile curves are shown below.*

tion), and the colour at each position represents the phase. If, in addition to the phase, the magnitude of the signal is to be plotted as well, it may be coded as changes of luminance of the coloured ribbon.

Visualisation of Fourier Transforms of Images

The phase of the Fourier transform of a 2D signal or image can be plot as a coloured rectangle, where each of its points has a colour that corresponds, via a chosen profile. As an example, a image compound by some black and white squares is showed in Figure 14.

As it was pointed several years ago by T.S. Huang, the phase of the Fourier transform of an image contains information about shapes and edges, while the magnitude contains information about texture. A colour phase plot is shown in Figure 15.

Instantaneous phase

The phase $\phi(t) = \angle(s(t))$ of the analytic signal $s_a(t) = s(t) + j\hat{s}(t)$ (where $\hat{s}$ stands for the Hilbert transform of s) of a signal $s(t)$ is called the *instantaneous phase* of the signal.

For example, consider the *rectangular pulse* signal

$$p(t) := u(t+1) - u(t-1) \tag{1}$$

where u stands for Heaviside's step function; its Hilbert transform $\hat{p}(t) = p(t) * \frac{1}{\pi t}$ is given by $\hat{p}(t) = \frac{1}{\pi} \ln |\frac{t+1}{t-1}|$. The instantaneous phase of $p(t)$, together with a colour-visualised version, is shown in Figure 16.

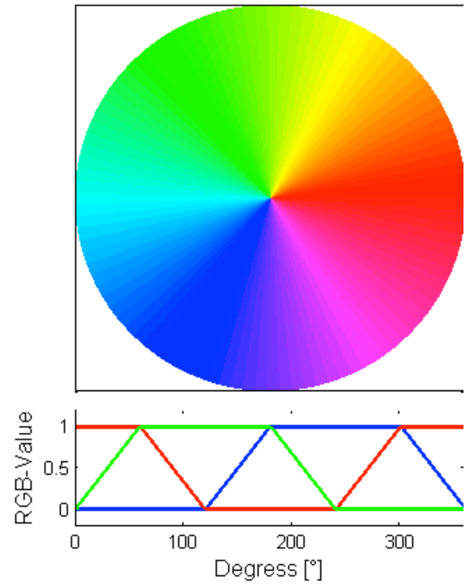

Figure 8. *Hue disk resulting from piecewise linear variations of the R, G and B components of the profile. Three, very apparent, strong variations of the luminance result. at yellow, violet and cyan hues.*

Conclusion and Further Work

The Fourier phase of a signal, as well as the phase of the analytic signal of a signal, are somehow less intuitive that the corresponding magnitudes; nevertheless, they are important aspects of the signal that can be exploited.

The circular aspect of the phase is usually ignored and the phase is assimilated to a linearly ordered magnitude. This also is an obstacle for the development of tools that process that important aspect of a signal.

A drawback of the proposed method, for visualisation using colour is that people with abnormal colour vision may not be able to read the plots. This is not a trivial issue as between about 0.5% and 10% of white people, more for the case of males than for females, perhaps less for nonwhite people, are either protanopes or deuteranopes. We plan to modify the technique so that it can be used for people with at least two types of cone. For this, in addition to designing appropriate profiles with the help of protanope and deuteranope people, we will place a reference frame near the corresponding plot.

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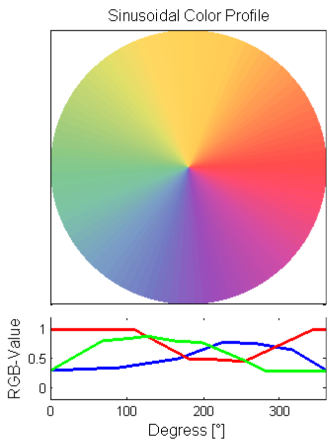

Figure 9. Hue disk resulting from the indicated variations of the R, G and B components of the profile.

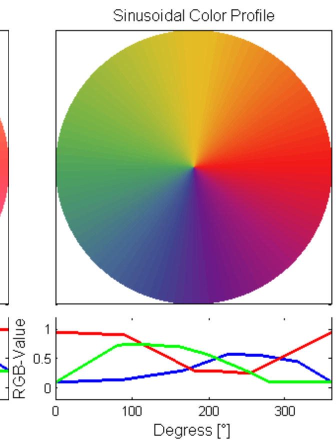

Figure 10. Hue disk resulting from the indicated variations of the R, G and B components of the profile.

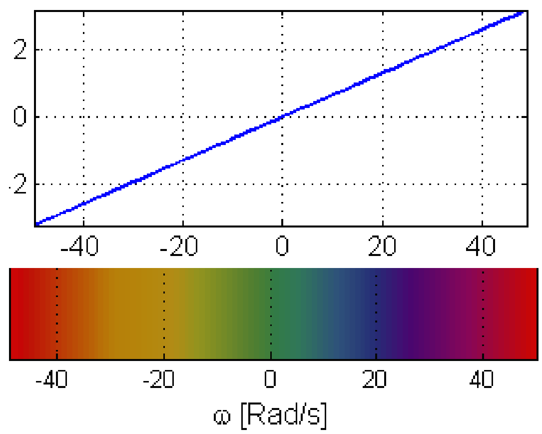

Figure 11. The lineal phase of the transfer function of the discrete filter is plotted above; below, plotted as a coloured ribbon. The phase varies from -2π to 2π (=-2π).

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Author Biography

Jorge Luis Mayorga was born in Bogotá, Colombia in 1993. He obtained his high school diploma from Gimnasio Vallegrande at Monteria, Cordoba in 2009; he then received his B.Sc. in electronics engineering from the Universidad de los Andes at Bogota, Colombia in 2015. His current research interests include signal processing, robotics, control systems and electronic embedded systems. Since 2014 is a chair member of the Industrial Applications Society IAS, Uniandes Student Branch. He has participated in the conferences Colombian Circuits and Systems Workshop (CWCAS), Simposio de Tratamiento de Señales, Imagenes y Vision Artificial (STSIVA 2015) and in IECON 2015 at Yokohama, Japan

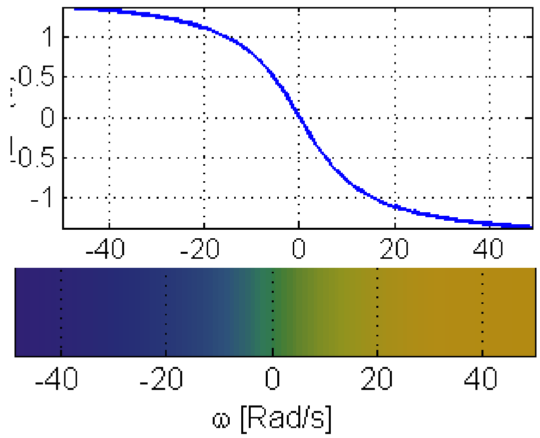

Figure 12. The sigmoidal phase of the transfer function of the continuous filter. Above, the graph of the function, below, plotted as a coloured ribbon. The phase goes from π to $-\pi$.

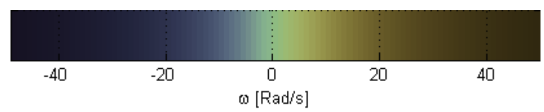

Figure 13. Magnitude (indicated with the luminance) and phase (indicated with hue) of the transfer function of the continuous low-pass Filter.

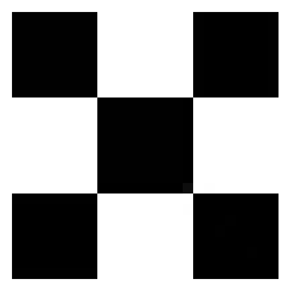

Figure 14. A simple monochromatic image.

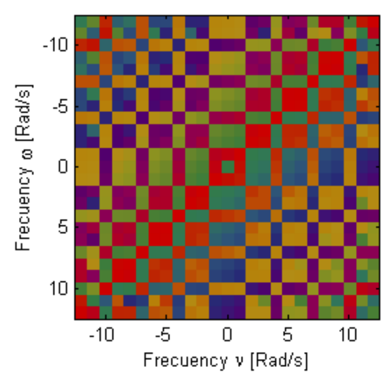

Figure 15. The phase of the Fourier transform of the image above.

Author Biography

Alfredo Restrepo received his B.Sc. in electronics engineering from the Pontificia Universidad Javeriana, at Bogotá, Colombia (1982) and his M.Sc. and Ph.D. in electrical engineering the University of Texas at Austin

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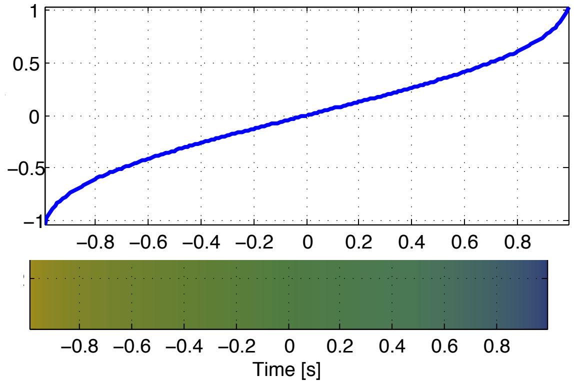

Figure 16. *The instantaneous phase of the pulse signal.*

(1986, 1990). Since then he has worked at the Universidad de los Andes, in Bogotá. He has been a researcher at the ENST, Paris, France and at the Universitá degli Studi di Trieste. His work has been on nonlinear signal processing, image processing and colour; also on geometric topology.