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The pyramid of visibility

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Abstract

A fundamental limit to human vision is our ability to sense variations in light intensity over space and time. These limits were first described systematically a half century ago in a series of three seminal papers: de Lange [1], van Nes and Bouman [2], and Robson [3] measured the visibility of temporal, spatial, and joint spatio-temporal sinusoidal variations. Additionally, the first two papers described how sensitivity depended on the the light level from which the deviations occurred. Their results provided an enduring foundation for all subsequent studies of contrast sensitivity.

We have recently reanalyzed these reports and discovered a fundamental and remarkable simplification. In brief, we have found that for photopic retinal illuminances at moderate to high frequencies the log of human contrast sensitivity is a linear function of spatial frequency, temporal frequency, and the log of adapting retinal illuminance. As a surface in the space defined by spatial and temporal frequency, sensitivity thus forms a rectangular pyramid.

Elsewhere we have described the boundaries of this surface, where it intersects the plane defined by the maximum contrast limit of one, as the "window of visibility." [4, 5] The new linear formulation allows us to describe the complete surface as the "pyramid of visibility." The height of the pyramid rises linearly with the log of retinal illuminance. As a result, the window of visibility is always a diamond that grows and shrinks, linearly, with the log of retinal illuminance. Elsewhere we have shown that under typical conditions log retinal illuminance is a linear function of log luminance [6], in which case the pyramid model also applies for sensitivity as a function of luminance.

Almost 40 years ago, analyzing some of his own data, Kulikowski also noted the dependence of log contrast sensitivity on linear spatial and temporal frequency, and on the log of luminance [7]. His result appears not to have been widely understood, nor its practical significance appreciated.

This result has deep theoretical and practical significance. With respect to theory, the independence of spatial, temporal, and light level effects constrains models of processing mechanisms and strategies. The practical significance is that rendition of visual information for the human eye is ultimately governed by the pyramid of visibility. There is no need to render beyond these limits, and these limits determine the visibility of artifacts in rendered information [4]. Consequently this surface provides a critical guide to design of a wide variety of visual display technologies. In particular, these limits determine the ultimate number of pixels per degree and frames per second required in electronic displays of static or moving imagery.

Introduction

Contrast is the ratio of the luminance deviation to the luminance from which it deviates. Contrast threshold is the smallest contrast that can be detected reliably under given conditions. Contrast sensitivity is the inverse of the contrast threshold. A plot of contrast sensitivity as a function of frequency is called a contrast sensitivity function. In temporal frequency, this is the temporal contrast sensitivity function (TCSF). In spatial frequency, it is the spatial contrast sensitivity function (SCSF). When both spatial and temporal frequency are varied, it is the spatio-temporal contrast sensitivity function (STCSF). The contrast sensitivity function behaves very differently at low and high frequencies. At low frequencies, it may be flat or decline towards lower frequencies, and generally manifests the effects of light adaptation and gain control. At high frequencies, it falls steadily until reaching the upper limit of visible spatial or temporal frequency. In temporal frequency, this limit is called the Critical Fusion Frequency (CFF). In the remainder of this report we confine our attention to the high frequency portion of the contrast sensitivity function. We also confine our attention to photopic vision, and to retinal illuminances of 1000 Trolands and below. These limits still contain a large proportion of our daylight visual experiences, and especially our experiences conveyed by electronic displays.

Temporal contrast sensitivity

de Lange [1] measured the TCSF at a range of adapting retinal illuminances, for a 2 deg disk target on a 60 deg background for two observers. His data for observer V are shown in Figure 1, plotted against linear frequency. We have fit these data with the linear model

$$S_1(W,I) = c_0 + c_w W + c_I I \tag{1}$$

where S is log contrast sensitivity, W is temporal frequency in Hz, I is log retinal illuminance in log Trolands [8]. We have confined the fit to cases where $S < 1.5$. The parameter c_0 defines overall sensitivity in units of log contrast sensitivity; c_W describes the rate at which sensitivity changes with temporal frequency, and c_I describes the rate at which sensitivity changes with log retinal illuminance. The best fitting parameters are c_0 = 1.604, c_W = -0.0641, and c_I = 0.634. We will discuss these parameters below. The fit is reasonable, considering the range of frequency and

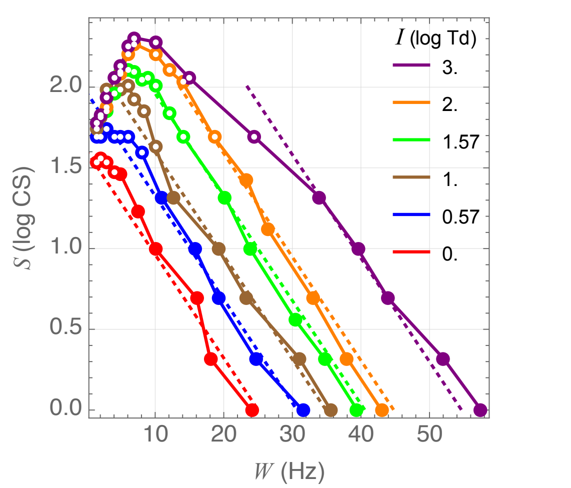

Figure 1. Linear model fit to TCSF. Points are contrast sensitivities of observer V measured by de Lange [1]. The dashed lines are fits of the linear model to the filled points.

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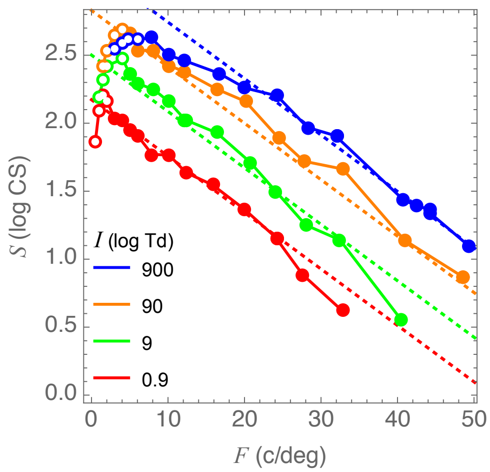

Figure 2. Linear fit to SCSF. Points are contrast sensitivities measured by van Nes and Bouman [2]. The dashed lines are fits of the linear model to the filled points.

retinal illuminance. The RMS error is 0.086 (1.7 dB). We note the lack of an interaction between F and I. Elsewhere we explored the value of an interaction term FI and found it improved the fit very little [8].

Spatial contrast sensitivity

van Nes and Bouman [2] measured contrast thresholds for stationary sinusoidal gratings 4.5 degree wide by 8.25 degree tall, with a dark surround. They used a 2 mm artificial pupil and retinal illuminances of between 0.0009 and 900 Trolands in steps of a factor of 10. Here we consider only the photopic values of 0.9, 9, 90, and 900 Td. The data are plotted in Figure 2 as log contrast sensitivity versus linear frequency.

We fit these data with the linear model

$$S_2(F,I) = c_0 + c_F F + c_I I \quad (2)$$

where F is spatial frequency in cycles/degree, and c_F is the rate of change in log sensitivity with spatial frequency. We used only data for which $F > 4$. The best fitting parameters are c_0 = 2.19, c_F = -0.0415, and c_I = 0.329. Again the fit is reasonable given the range of frequency and retinal illuminance included. The RMS error is 0.091 (1.8 dB).

Spatio-temporal contrast sensitivity

Robson [3] measured contrast sensitivities for targets that were sinusoidal in both space and time, with various spatial and temporal frequencies. The target was 2.5 x 2.5 degrees on a 10 degree square uniform background of 20 cd m^{-2}. In one case, temporal frequency was fixed at one of several values, and spatial frequency was varied. In the second case, spatial frequency was fixed at one of several values, and temporal frequency was varied. In Figure 3 we plot the data against a linear frequency.

We fit these data with the linear model

$$S_3(W,F) = c_0 + c_w W + c_f F \quad (3)$$

Because we are only interested in the high-frequency portion of the CSF, we confined the fit to points for which |{W,F}| <= 6. The resulting fits are shown in Figure 3 along with the data. The best fitting parameters are c_0 = 2.71, c_W = -0.0603, c_F = -0.0647. The

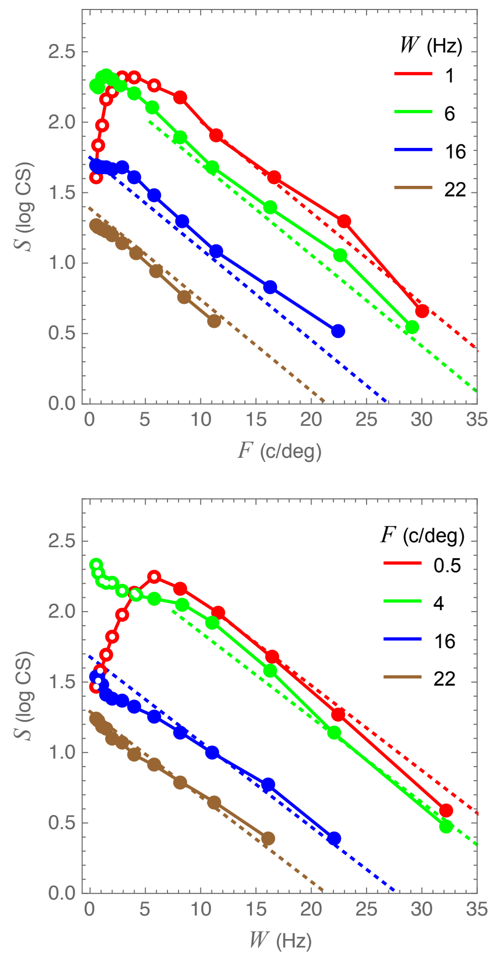

Figure 3. Linear fit to STCSF. Points are contrast sensitivities measured by Robson [3]. Top: temporal frequency was fixed and spatial frequency varied; bottom: spatial frequency was fixed and temporal frequency varied. The dashed lines are fits of the linear model to the filled points.

RMS error is 0.085 (1.7 dB). Again the fit is reasonable, considering the ranges of spatial and temporal frequency included.

The Pyramid of Visibility

We have shown that log contrast sensitivity is linearly related to temporal frequency [1, 3], spatial frequency [2, 3], and log retinal illuminance [1, 2]. Remarkably, no interaction terms between W, F, and I are required to fit the data. This means that the dependence on W is independent of F and I, the dependence on F is independent of W and I, and the dependence on I is independent of W and F. In Table 1 we summarize the estimated parameters.

Table 1. Parameters estimated from three studies.

Study	c_0	c_W	c_F	c_I	RMS
de Lange	1.60	-0.064		0.634	0.086
van Nes & Bouman	2.19		-0.042	0.329	0.091
Robson	2.71	-0.060	-0.065		0.085

It is possible to combine equations 1-3 into a linear model that includes all three variables:

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$$S_4(W,F,I) = c_0 + c_W W + c_F F + c_I I \quad (4)$$

This equation defines the "Pyramid of Visibility." It is rendered graphically in Figure 4, using parameters approximately those derived in Figures 1-3.

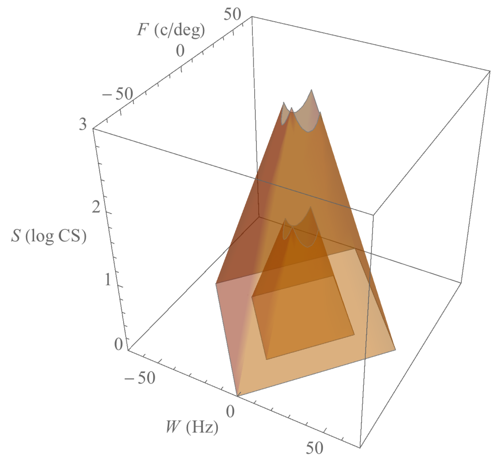

Figure 4. Pyramid of Visibility. Linear model of spatio-temporal contrast sensitivity shown as a surface. Two examples are shown, for retinal illuminances of 1 and 1000 Td. Parameters used were c_0 = 2, c_W = -0.06, c_F = -0.05, c_I = 0.5. The surface was limited to |{W,F}| < 10.

Parameters of the model

A linear model means that the variation in sensitivity with one of the variables temporal frequency W, spatial frequency F, or log retinal illuminance I is independent of the other two. This in turn suggests that in each case the response to the effective variable reflects a fundamental and likely early property of visual processing. We consider each of the parameters in turn.

The values of c_0 consist of the extrapolation of the linear trends to values of zero for W, F, and I, and reflect the global sensitivity of the observer. We do not expect agreement between studies, as the value will depend on extraneous variables such as duration and size of the target. In the experiments considered here, durations are unknown. Further, in Table 1, the value of c_0 estimated for Robson includes the contribution from the unknown value of retinal illuminance. However, a fixed and stable value would be expected from targets of fixed size and duration, or from an ideal observer model with specific spatial, temporal, noise, and efficiency constraints [9].

The parameter c_W, describes the linear rate of decline in log sensitivity as a function of temporal frequency. It also determines the value of the CFF. It is consistent across the two studies considered here (-0.064 vs -0.06). This suggests that this value is robust, since in one case the target was a disk, and in the other a sinusoidal grating. We speculate that this invariant rate of decline reflects some underlying physical process that limits detection of temporal change. A linear decline in log sensitivity with frequency is equivalent to an exponential modulation transfer function, which in turn is consistent with an impulse response that is a Cauchy density, but we cannot at this time associate that with any particular physical process.

The parameter c_F, which describes the linear rate of decline in log sensitivity as a function of spatial frequency, differs somewhat between the two studies considered here (-0.065 vs -0.042). Campbell and Green [10] were the first to note a linear decline in log sensitivity with linear frequency. Using their thresholds for interference fringes that bypass the eye optics, we estimate the slopes for their two observers to be -0.039 and -0.042. In a subsequent experiment, they measured sensitivities for conventional gratings, with imposed optical defocus of 1.5, 2, 2.5, and 3.5 D. All four sets are quite linear. We estimate the slopes at -0.049, -0.046, -0.058, and -0.061. As expected, the slope generally increases with increasing defocus. In short, the expected value of c_F will depend on the amount of optical blur. The values for Robson and van Nes and Bouman are in the range found by Campbell and Green, and it is worth noting that Robson used natural pupils (we estimate about 5.5 mm [6]) while van Nes and Bouman used an artificial pupil of 2 mm. In summary, steeper slopes appear to be associated with increased optical blur and larger pupils. We would also expect the value of c_F to depend on the size of the target, since that will change the relative sensitivity to high and low frequencies [11].

The parameter c_I reflects the rate of change in log sensitivity with log retinal illuminance. In the two studies considered here, the the average value is 0.48. This is close to the value of 0.5 expected from an ideal observer limited by quantum fluctuations (the deVries-Rose Law) [12, 13]. Other authors have noted this relationship [2, 14, 15], and have also noted that it does not persist at very low frequencies, or at very low or very high retinal illuminances. However, within our regime of interest, it holds quite well. The implication is that within this regime sensitivity is limited by quantal fluctuations, or by some other noise whose variance rises in proportion to retinal illumination.

From Illuminance to Luminance

One of us has recently developed a formula for the diameter of the light-adapted pupil [6]. This allows for a calculation of the expected retinal illuminance for a given adapting luminance. Using this formula, we have noted that the relationship between log luminance and log retinal illuminance is approximately linear under many conditions. An example is shown in Figure 5.

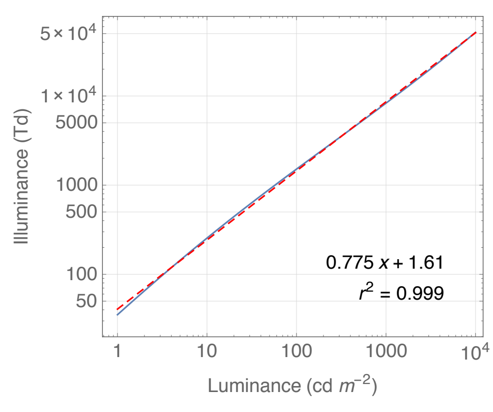

Figure 5. Relationship between log luminance and log retinal illuminance. Based on the formula of Watson and Yellott [6]. Assumed conditions are: adapting field area = 100 deg²; age = 30 years; viewing binocular. The red dashed line and text show the best fitting linear relationship and correlation.

This allows us to construct a formula for log spatio-temporal contrast sensitivity that is a linear function of adapting luminance, rather than retinal illuminance. Under specified viewing conditions, the pyramid of visibility can thus be described by the following equation:

$$S_5(W,F,L) = c_{L,0} + c_W W + c_F F + c_L L \quad (5)$$

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where L is the log of adapting luminance. The parameters $c_{L,0}$ and c_L can be easily derived from the parameter c_I, once the linear relationship between log luminance and log retinal illuminance is known. This relationship is readily available from an online calculator [6]. For example, given the relationship shown in Figure 5,

$$\begin{aligned} c_{L,0} &= c_0 + 1.61 \\ c_L &= 0.775\, c_I \end{aligned} \tag{6}$$

While sensitivity is likely to be more robustly associated with retinal illuminance, for the reasons discussed above, knowing the relationship to luminance is of greater practical value. Displays are specified in terms of luminance, and consequently knowing the form of the pyramid of visibility for a particular adapting luminance is needed when specifying rendering limits in space and time.

Estimating Parameters

Robson (1966)

Because no one of the three sets of data considered covers all three variables of interest, it it is difficult to estimate a consistent set of the four model parameters c_0, c_W, c_F, and c_I. However we can obtain a preliminary estimate by way of a few assumptions. First we note that Robson states that "The grating pattern subtended 2.5° x 2.5° in the center of a 10° x 10° screen illuminated to the same mean luminance of 20 cd/m2" [3]. Making use of the formula of Watson and Yellott [6], we estimate the pupil diameter at this luminance and field size to be 5.32 mm. This in turn leads to an estimate of retinal illuminance of 2.65 log Td.

If we now fit Equation 4 to Robson's data, setting $I = 2.65$ and assuming $c_I = 0.5$, then we obtain estimated parameters shown in Table 2. Using Equation 6, we can then compute the parameters for the luminance model, which are also shown in Table 2.

Table 2. Parameters estimated from Robson [3].

S_4	c_0	c_W	c_F	c_I
	1.39	-0.060	-0.065	0.500
S_5	$c_{L,0}$	c_W	c_F	c_L
	2.19	-0.060	-0.065	0.388

ModelFest (2005)

Another data set of interest, due to its wide utilization as a benchmark, are the so-called ModelFest data [16]. Here we analyze the mean data for 16 observers, for Gabor functions of fixed 2 deg size, with a Gaussian time course and fixed duration of 500 msec (standard deviation of 125 msec). The data are shown in Figure 6.

Elsewhere we have estimated the pupil diameter in the ModelFest experiments to be 5.15 mm [9]. The mean luminance was 30 cd/m^2 for an estimated retinal illuminance of $I = 2.8$ log Td. Using this value, and setting $W = 0$ and $c_I = 0.5$, we have fit Equation 4 to these data, for $F > 3$ cycles/deg. The fit is shown by the red line in Figure 6. The fit is remarkably good (RMS = 0.65 dB). The assumed and estimated parameters are shown in Table 3. Once again, we also transform the retinal illuminance parameters to luminance parameters, as shown in Table 3.

The parameter c_0 is lower than that estimated from Robson's data, but in the ModelFest case we have a smaller stimulus size and a specific briefer duration.

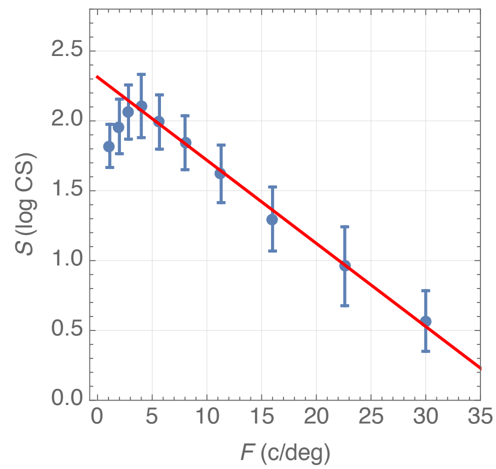

Figure 6. Fit of pyramid model to ModelFest data. Points are contrast sensitivities for Gabor functions averaged over 16 observers [16].

Table 3. Parameters estimated from ModelFest [16].

S_4	c_0	c_W	c_F	c_I	*RMS (dB)*
	0.92		-0.060	0.500	0.650
S_5	$c_{L,0}$	c_W	c_F	c_L	
	1.74		-0.060	0.391	

Window of Visibility

The Window of Visibility is the region of the spatiotemporal frequency domain that contains visible frequencies [4, 5]. It is shown graphically by the diamond-shaped intersections in Figure 4 between the pyramid and the plane at $S = 0$. More generally, we can consider the window formed by any particular value of $S > 0$, that is, the frequencies visible at a particular log contrast $-S$. We illustrate the shape of the window in Figure 7. For any particular values of S and I, the shape of the window is fixed, and there exist particular spatial and temporal resolution limits, F_0 and W_0, that mark the corners of the diamond.

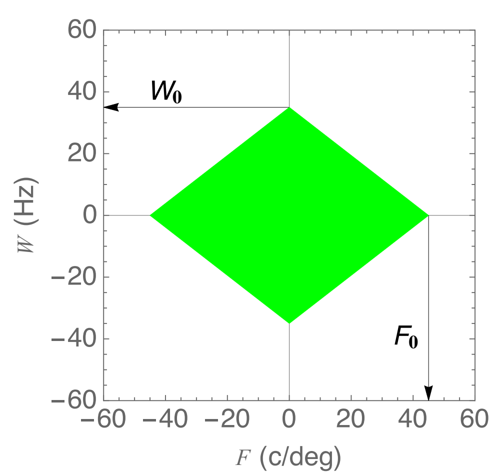

Figure 7. Window of Visibility. The green region marks the window of visibility. The spatial and temporal limits of the window F_0 and W_0, are also shown.

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Since Equation 4 is linear, it is possible to rearrange terms and provide expressions for the boundaries of the window. These describe the spatial or temporal "fusion frequencies" as a function of contrast, retinal illuminance, and the value temporal or spatial frequency, respectively:

$$F = \frac{S - c_0 - c_W W - c_I I}{c_F} \tag{7}$$

$$W = \frac{S - c_0 - c_F F - c_I I}{c_W} \tag{8}$$

With S and I fixed, it is evident that W is a linear function of F, and vice-versa. These are the straight lines that form the sides of the diamond-shaped window of visibility. This reciprocity between W and F was noted by Kulikowski [7]. To our knowledge, this relationship has not previously been given a name, but we propose that it be called "Kulikowski's Law."

We note that Equation 8 is a generalized version of the Ferry-Porter Law, which states that the critical fusion frequency is a linear function of log retinal illuminance [17]. It is generalized in that it applies to any contrast (not just $S = 0$) and any spatial frequency. The generalization with respect to contrast was also noted by Tyler and Hamer [17].

Interestingly, Equation 7 states that the spatial resolution limit ("fusion frequency") is also a linear function of retinal illuminance. To our knowledge, this relationship has also not previously been given a title.

The spatial and temporal resolution limits, F_0 and W_0 (see Figure 7), are also easily expressed by setting W or F to zero in Equations 7 and 8 respectively,

$$F_0 = \frac{S - c_0 - c_I I}{c_F} \tag{9}$$

$$W_0 = \frac{S - c_0 - c_I I}{c_W} \quad . \tag{10}$$

These equations make explicit the fact that these limits are linear functions of both contrast and retinal illuminance. Elsewhere we have shown how these quantities may be used to compute the required frame rate for apparently smooth stroboscopic motion [4, 5], so it is of some utility to have simple formulas to compute their values under arbitrary conditions of contrast and retinal illuminance.

Because the log contrast sensitivity can be written as a linear function of either retinal illuminance (Equation 4) or luminance (Equation 5), Equations 7-10 can also be written as functions of luminance. For completeness, we provide them here:

$$F = \frac{S - c_{L,0} - c_W W - c_L L}{c_F} \tag{11}$$

$$W = \frac{S - c_{L,0} - c_F F - c_L L}{c_W} \tag{12}$$

$$F_0 = \frac{S - c_{L,0} - c_L L}{c_F} \tag{13}$$

$$W_0 = \frac{S - c_{L,0} - c_L L}{c_W} \tag{14}$$

Limitations

Mark Twain observed that "There is something fascinating about science. One gets such wholesale returns of conjecture out of such a trifling investment of fact" [18]. We have constructed a comprehensive description of spatial and temporal contrast sensitivity, and their dependence on retinal illuminance, and derived therefrom a series of powerful relationships, all from the modest results of three very old studies. Certainly our model needs to be confirmed by additional modern data. De Lange's data are for a single spatial target of uncertain duration, van Nes and Bouman's data are for static presentation of unspecified duration, and Robson's data are for unspecified duration, a single luminance, and unspecified pupil diameter (and thus retinal illuminance). A more comprehensive survey of the relevant parameter space, both from the literature and from new data, is desirable. These data should control spatial frequency, temporal frequency, size, duration, and retinal illuminance.

The model fits in Figures 1-3 are not perfect. There are suggestions of small but possibly systematic departures. Additional data may clarify how well the model fits in the general case. But we argue that the linear model accounts for an extraordinary amount of the variance over a very large part of the photopic visual domain.

The model, by design, only describes sensitivity at high spatial or temporal frequencies. The region of excluded low frequencies is tentatively identified as

$$\left(\frac{F}{F_1}\right)^2 + \left(\frac{W}{W_1}\right)^2 < 1 \tag{15}$$

where F_1 = 6 cycles/deg, and W_1 = 6 Hz. Further research is required to better determine these two parameters.

However, it is the limits at high spatial and temporal frequencies that usually determine the practical limits of human vision. We also note that the widespread practice of plotting contrast sensitivity against a log frequency abscissa, while valuable for many purposes, may exaggerate the importance of low frequencies. For example, in linear coordinates, the three-dimensional volume enclosing all visible frequencies at one retinal illuminance is a pair of cones (the rotation of the diamond-shaped window of visibility as spatial orientation is varied), as shown in Figure 8. The volume of this cone of visibility is

$$V = F_0^2 W_0 \frac{2\pi}{3} \quad . \tag{16}$$

The volume enclosing the frequencies defined by Equation 15 is an oblate spheroid, as shown in Figure 8, with volume

$$V_{\text{low}} = F_1^2 W_1 \frac{4\pi}{3} \quad . \tag{17}$$

At I = 2, F_1 = 6 cycles/deg, and W_1 = 6 Hz, the low frequencies occupy only V_{low}/V = 0.002 of the total visible volume. In other words, in linear coordinates, our model covers 99.8 % of all visible frequencies.

Another limitation of our model is that we deal only with photopic retinal illuminances, and only up to about 1000 Td. Above this level, contrast sensitivity asymptotes, and the linear relationship with log retinal illuminance no longer holds. But our model again covers a very large region of great practical importance.

Our model describes foveal contrast sensitivity. It is unknown how well the linear model describes peripheral contrast sensitivity. Further, even foveal contrast sensitivity is dependent on the size of

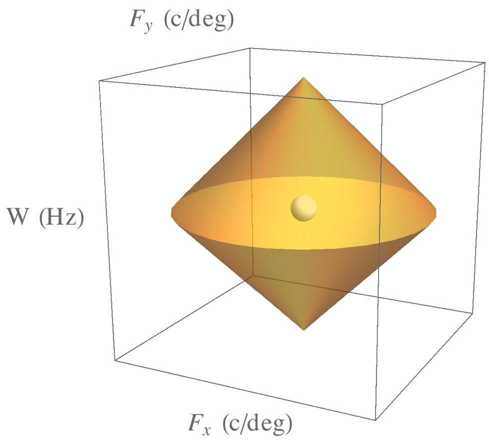

Figure 8. The conical solid enclosing all visible spatial and temporal frequencies at one retinal illuminance and contrast and the oblate spheroid enclosing only low frequencies. Dimensions are approximately correct for $S = 0$, $I = 2$, and $F_1 = 6$ cycles/deg and $W_1 = 6$ Hz.

the target. The model appears to work well for the several sizes we have considered, but different sizes will certainly affect the estimate of c_0, and probably c_F, and possibly whether the model fits at all. Answers await further data.

As noted, the value of c_0 the will depend on target size, but it will also depend on duration. None of the three primary studies considered specified duration, although ModelFest did. Thus our estimates of c_0 are at this time not well defined.

The luminance model that we have described, as an extension of the model for retinal illuminance, requires knowledge of the pupil diameter, or at least of the adapting luminance and field size.

Summary

The shape of the the spatio-temporal contrast sensitivity function, and its dependence on adapting light level, has been of great interest for almost a century [1, 3, 19-22]. The practical implications of the surface to imaging technology have been detailed [4, 5]. Elaborate formulas have been developed to characterize the shape of the surface [19]. Following Kulikowski, [7], we have found that over a broad and important range of luminance, spatial frequency, and temporal frequency the surface can be described by a linear model. The model has important theoretical and practical implications. Because of its shape as a surface, and because of its close relation to the "window of visibility," we have called this model the "pyramid of visibility."

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