DOI: 10.2352/ISSN.2470-1173.2016.15.IPAS-195

Zonal-Alpha-Rooting Color Image Enhancement by the Two-Side 2D Quaternion Discrete Fourier Transform

Artyom M. Grigoryan, Aparna John, Sos Agaian Department of Electrical and Computer Engineering, The University of Texas at San Antonio; One UTSA Circle, San Antonio, USA TX 78249
e-mail: amgrigoryan@utsa.edu , aparna.john@utsa.edu, sos.agaian@utsa.edu

Abstract

The purpose of image enhancement is to improve the quality of a digital image, so as to support the human perception. In this paper, new methods of image enhancement are proposed for enhancement of color images, which are based on application of the concept of the two-dimensional quaternion discrete Fourier transform (QDFT) together with the well-known method of the alpha-rooting. The application of the alpha-rooting method which is based on the traditional two-dimensional discrete Fourier transform (2-D DFT) results in high quality images, when comparing with other transforms, such as Hadamard, Hartley, cosine, and heap transforms. The image enhancement technique of alpha-rooting can be used for enhancing even low contrast images. In zonal alpha-rooting method, the modification of frequency is different for different frequency zones. Optimization of alpha value is done with genetic algorithm.

Introduction

History of digital image enhancement has been for more than 50 years. It has remained one of the most active research areas in image processing and computer vision. Recently, several image enhancement algorithms and many applicable systems have been exploited. Current research in image enhancement covers such wide topics as algorithms based on human visual system, histograms with hue preservation, JPEG-based enhancement for visually impaired, and histogram modification techniques. Two major classifications of image enhancement techniques can be defined: spatial domain enhancement and transform domain enhancement [1],[25]-[30]. Digital image enhancement is a powerful tool for many image processing applications when the critical details are not seen clearly enough.

Existing methods for image enhancement focus mainly on properties of the processed image while excluding any consideration of the observer characteristics. With their specific nature, various enhancement methods are required for various types of images and applications. The Fourier transform plays important role in image processing [1]-[5],[35]-[38] and is used in different stages of processing such as filtering, coding, recognition, and restoration analysis. This transform was generalized for application of the Hartley, Hadamard, and cosine transforms[30]-[50], as well new signal induced heap transforms[51]-[58]. and elliptical Fourier transforms [5],[59]-[60]. Transform-based methods of image enhancement are based on manipulation with all or part of spectral components of the transform. We focus on the well-known method of alpha-rooting, although other methods, including the log-alpha rooting, modified unsharp masking, and methods based on wavelet transforms are also used in image enhancement. Many traditional methods of image enhancement were applied for processing the color images[37]. The Fourier transform based method is one of these methods, which in many cases is applied to each color plane-component of the color image separately. In other words, the color image is considered as a triplet of separate 2-D gray scale images and each of these images represents red, green or blue component of the color. Quaternion numbers of Hamilton's was used in Ell's works, and after that time much attention was given to the transformation of the color components to the imaginary subspaces of the quaternion numbers, "imaginary part" of which consists of three components.

Quaternion Algebra and Quaternion Discrete Fourier Transforms

Quaternion is a non-commutative, normed division algebra on real numbers [6],[9]-[19]. A quaternion is a four-dimensional hyper complex number and has the Cartesian form given by:

$$\mathbf{q} = \mathbf{a} + \mathrm{i}\mathbf{b} + \mathrm{j}\mathbf{c} + \mathrm{k}\mathbf{d} \tag{1}$$

where a, b, c, and d are real numbers. The set of quaternions with its arithmetic is denoted by $\mathbb{H}$. Each of the three imaginary units **i, j, k** with the following rules of multiplication:

$$\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = \boldsymbol{ijk} = -1 \tag{2}$$

and

$$\begin{aligned} \boldsymbol{ij} &= -\boldsymbol{ji} = \boldsymbol{k}, \\ \boldsymbol{jk} &= -\boldsymbol{kj} = \boldsymbol{i}, \\ \boldsymbol{ki} &= -\boldsymbol{ik} = \boldsymbol{j}. \end{aligned} \tag{3}$$

In general, the product of the quaternions is not commutative. In other words, for many quaternion **p**, **q** $\in \mathbb{H}$,

$$\boldsymbol{pq} \neq \boldsymbol{qp}. \tag{4}$$

The modulus, or length of **q** is denoted as |q | and is defined as

$$|q| = \sqrt{a^2 + b^2 + c^2 + d^2} \tag{5}$$

When **a** = 0, **q** is called a pure quaternion.

Quaternion Fourier transform are Fourier transforms on four dimensional hyper complex numbers [9],[2],[4],[7],[10]-[15]. This transform is especially useful for the processing of the signals that are three or four dimensional. The concepts of the 1-D and 2-D quaternion discrete Fourier transforms (QDFT) are not unique as in the complex arithmetic, because of non commutativity of the multiplication. However, many quaternion Fourier transforms are found to be an excellent tools for the processing of color image signals. The three or four channel model of the color image is represented as the **b**, **c**, **d** components of the quaternion numbers.

DOI: 10.2352/ISSN.2470-1173.2016.15.IPAS-195

Color image enhancement using QDFT and zonal alpha-rooting

The individual channel of the color models RGB, XYZ or CMYK are represented as each of the components of the quaternion numbers [6]. For the 2-D signal $f_{n,m}$, each channel of the 2–D image takes the form as

$$\begin{aligned} \boldsymbol{a} &= f_{n,m_{Channel\,1}} \\ \boldsymbol{b} &= f_{n,m_{Channel\,2}} \\ \boldsymbol{c} &= f_{n,m_{Channel\,3}} \\ \boldsymbol{d} &= f_{n,m_{Channel\,4}} \end{aligned} \qquad (6)$$

For 3 channel color models like RGB or XYZ q is chosen as a pure quaternion, i.e., **a**=0.

As already mentioned, the product of the quaternion numbers are, in general, non-commutative. Based on the position of the complex co-efficient of the Fourier transform QDFT is called as right sided, left sided or sandwiched transforms. There are different algorithms for the enhancement of color image using QDFT. One of them is the column-row method. The column row method [4],[33] for the of the QDFT of the dual axis and sandwiched transform are explained in details here.

2-D QDFT Column Row Method (Dual Axis and Sandwiched):

$$\text{QDFT} = \sum_{n=0}^{N-1} W_j^{np} \left[\sum_{m=0}^{M-1} f_{n,m} W_k^{ms} \right] \qquad (7)$$

Let $f_{n,m} W_k^{ms} = q$

$$\begin{aligned} &[a + ib + jc + kd][\cos\alpha - k \sin\alpha] = \\ &\quad [a \cos\alpha + d \sin\alpha] + i[b \cos\alpha - c \sin\alpha] + \\ &\quad j[c \cos\alpha + b \sin\alpha] + k[d \cos\alpha - a \sin\alpha] \end{aligned} \qquad (8)$$

$$R_1 = \begin{pmatrix} \cos\alpha & 0 & 0 & \sin\alpha \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ -\sin\alpha & 0 & 0 & \cos\alpha \end{pmatrix} \qquad (9)$$

We see vectors (a,d)′ rotate counter clockwise direction by α and vectors (b,c)′ rotate clockwise direction by −α.

$$\begin{aligned} &[\cos\beta - j \sin\beta][q_e + iq_i + jq_j + kq_k] = \\ &\quad [q_e \cos\beta + q_j \sin\beta] + i[q_i \cos\beta - q_k \sin\beta] + \\ &\quad j[q_j \cos\beta - q_e \sin\beta] + k[q_k \cos\beta + q_i \sin\beta] \end{aligned} \qquad (10)$$

$$R_2 = \begin{pmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & \cos\beta & 0 & -\sin\beta \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & \sin\beta & 0 & \cos\beta \end{pmatrix}. \qquad (11)$$

Now, the vectors $(q_e,q_j)'$ rotate counter clockwise direction by β and vectors $(q_i,q_k)'$ rotate clockwise direction by – β.

The inverse QDFT,

$$\text{IQDFT} = \sum_{n=0}^{N-1} W_j^{-np} \left[\sum_{m=0}^{M-1} F_{p,s} W_k^{-ms} \right] \qquad (12)$$

Let $F_{p,s} = p$, i.e., $p = p_e + ip_i + jp_j + kp_k$ and let $F_{p,s} W_k^{-ms} = g$.
Then, $[p_e + ip_i + jp_j + kp_k][\cos\alpha + k \sin\alpha] =$

$$\begin{aligned} &[p_e \cos\alpha - p_k \sin\alpha] + i[p_i \cos\alpha + p_j \sin\alpha] + \\ &j[p_j \cos\alpha - p_i \sin\alpha] + k[p_k \cos\alpha + p_e \sin\alpha] \end{aligned} \qquad (13)$$

$$R_1' = \begin{pmatrix} \cos\alpha & 0 & 0 & -\sin\alpha \\ 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 \\ \sin\alpha & 0 & 0 & \cos\alpha \end{pmatrix} \qquad (14)$$

vectors $(p_e,p_k)'$ rotate clockwise direction by −α and vectors $(p_i,p_j)'$ rotate counter clockwise direction by α.

And, $[\cos\beta + j \sin\beta][g_e + ig_i + jg_j + kg_k] =$

$$\begin{aligned} &[g_e \cos\beta - g_j \sin\beta] + i[g_i \cos\beta + g_k \sin\beta] + \\ &j[g_j \cos\beta + g_e \sin\beta] + k[g_k \cos\beta - g_i \sin\beta] \end{aligned} \qquad (15)$$

$$R_2' = \begin{pmatrix} \cos\beta & 0 & -\sin\beta & 0 \\ 0 & \cos\beta & 0 & \sin\beta \\ \sin\beta & 0 & \cos\beta & 0 \\ 0 & -\sin\beta & 0 & \cos\beta \end{pmatrix} \qquad (16)$$

The vectors $(g_e,g_j)'$ rotate clockwise direction (i.e., by −β) and vectors $(g_i,g_k)'$ rotate counter clockwise direction (i.e., by β).

The image enhancement using the algorithm of the column-row method is not very fast. And, therefore, many fast algorithms for the enhancement of color image using QDFT, are also used to implement the transform.

Fast Algorithm method:

The two-sided QDFT can be written as

$$F_{p.s} = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} W_j^{np} f_{n,m} W_k^{ms} = \sum_{n=0}^{N-1} W_j^{np} \left(\sum_{m=0}^{M-1} f_{n,m} W_k^{ms} \right)$$

for p=0:N-1, s = 0:M-1

(17)

For fast calculation of the $F_{p.s}$, we use DFT. Initially, 1D M-point[4] DFT is done over each row of the real sequence of $f_{n,m}$, which results in the 2D sequence

$$S_{n,s} = \sum_{m=0}^{M-1} f_{n,m} \exp(-ims\emptyset)\,, for\ s = 0{:}\,M-1 \qquad (18)$$

$$S_{n,s} = \sum_{m=0}^{M-1} f_{n,m} \cos(ms\emptyset) - i \sum_{m=0}^{M-1} f_{n,m} \sin(ms\emptyset) \qquad (19)$$

$$\sum_{m=0}^{M-1} f_{n,m}\cos(ms\emptyset) = Re(S_{n,s})\ and$$

$$\sum_{m=0}^{M-1} f_{n,m}\sin(ms\emptyset) = -Im(S_{n,s}) \qquad (20)$$

Then, we do N-point DFTs over the columns of the real and imaginary sequences obtained from the first DFT calculation.

$$C_{p,s} = \sum_{n=0}^{N-1} Re(S_{n,s})\cos(np\emptyset) - i\sum_{n=0}^{N-1} Re(S_{n,s})\sin(np\emptyset), for\ p = 0{:}N-1$$

and

$$D_{p,s} = \sum_{n=0}^{N-1} Im(S_{n,s})\cos(np\emptyset) - i\sum_{n=0}^{N-1} Im(S_{n,s})\sin(np\emptyset), \quad for\ p = 0{:}N-1 \qquad (22)$$

This algorithm uses N M-point DFTs and 2M N-point DFTs.

In quaternion space, the image is represented as $q = eE + iI + jJ + kK$

The two-side QDFT can be expressed as

$$F = (Ce - jS)q(Ce - kS) \qquad (23)$$

$$F = e(CEC + SJC + CKS + SIS) + i(CIC - SKC - CJS + SES) + j(CJC - SEC + CIS - SKS) + k(CKC + SIC - CES - SJS) \qquad (24)$$

$$F = eF_e + iF_i + jF_j + kF_k \qquad (25)$$

Therefore, the N x M point QDFT is obtained by calculating F_e, F_i, F_j, F_k.

In alpha-rooting method of image enhancement [6] [20] –[24], for each frequency point (p,s), the magnitude of the discrete quaternion transform are transformed as

$$F_{p,s} \to M[F_{p,s}]^{\alpha} \qquad (26)$$

The value of α is taken from the interval (0,1) and can be adjusted interactively by the user or can be found automatically [6],[20]-[24],[37].

In zonal alpha-rooting method [9],[24]-[27], the alpha-rooting is done with different alpha for different frequency zones. The spectrum is center-shifted, and the frequency values of each zone are modified by different alpha values.

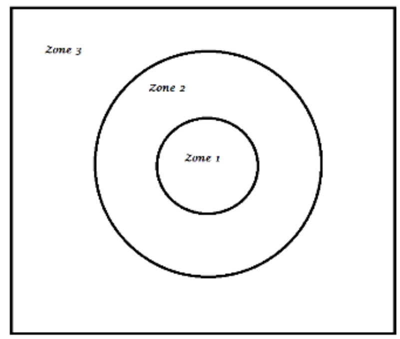


FIGURE 1: *Schematic diagram of three zonal division of frequency values of an image.*

The enhancement measure [6]-[10] which is called the color enhancement measure estimation (CEME), is based on the contrast of the images. In this enhancement measurement method, the 2-D discrete image N×M is divided by k_1k_2 blocks of size $L_1 \times L_2$ blocks each, where $k_n = \lfloor N_n/L_n \rfloor$, for $n = 1{,}2$. For a RGB color model, when an image is transformed by DQFT,

$$f = (f_R, f_G, f_B) \to \hat{f} = (\hat{f_e}, \hat{f_R}, \hat{f_G}, \hat{f_B})$$

the image enhancement measure is given as

$$E_q(\alpha) = CEME_\alpha(\hat{f}) = \frac{1}{k_1k_2}\sum_{k=1}^{k_1}\sum_{k=1}^{k_2} 20\ log_{10}\left[\frac{\max_{k,l}(\hat{f_e}, \hat{f_R}, \hat{f_G}, \hat{f_B})}{\min_{k,l}(\hat{f_e}, \hat{f_R}, \hat{f_G}, \hat{f_B})}\right] \qquad (27)$$

Optimization of Alpha: Genetic Algorithm

The value chosen for alpha is the alpha-value that gives maximum CEME. Optimization of alpha for the maximum CEME is done with genetic algorithm.

Genetic algorithm (GA) is an efficient tool[61] to solve the optimization problems. GA is an artificial intelligence approach for optimization. And the method adopted for GA optimization is similar to the method of successive evolution of genes from generations to generations. The concepts of generations, mutations and cross-over in gene-evolution is used to configure and train the neural networks to achieve the best optimum value of the parameter, for a given fitness function. The steps involved in GA is as follows:

1. Generate a random population, with the specified constraints.
2. Find the fitness functions for the generated population.
3. If the original population does not satisfy the fitness function, then we should proceed to get the new population.

DOI: 10.2352/ISSN.2470-1173.2016.15.IPAS-195

4. For generating the new population, select two members each of the original population for cross-over and mutations.
5. The two parent selected for cross-over and mutation, is based on members of the population which give high value for the fitness function.
6. Now the parents in original population are replaced by the new offspring generations, obtained after crossing-over and mutations.
7. Fitness function of the newly generated population is done.
8. The process is repeated till we get optimum value for the parameter that satisfies the fitness function.

In finding the optimum value of alpha, a random population of alpha with the given constraints, is generated. Then, the CEME of generated population of alpha values are calculated. The number of populations to be generated is restricted based on the generation that consistently gives maximum CEME value. The alpha corresponding to the maximum CEME is chosen as the optimum value.

Experimental Results

The color images were enhanced by different QDFT algorithms. In QDFT, enhancement of images were such that the all the channels, three or four channels, based on the color models, were considered as a single entity. The results obtained are shown below.

Figure 2 shows the original color image "tree" in the RGB color model and its enhancement using alpha-rooting, 7-zone alpha-rooting 2D-QDFT. The optimum value obtained from genetic algorithm for the zonal radii are [7, 21, 54, 55, 88, 102] and the alpha values for the corresponding zones are [0.9602, 0.9591, 0.9579, 0.8270, 0.9122, 0.8252, 0.6586]

(a)

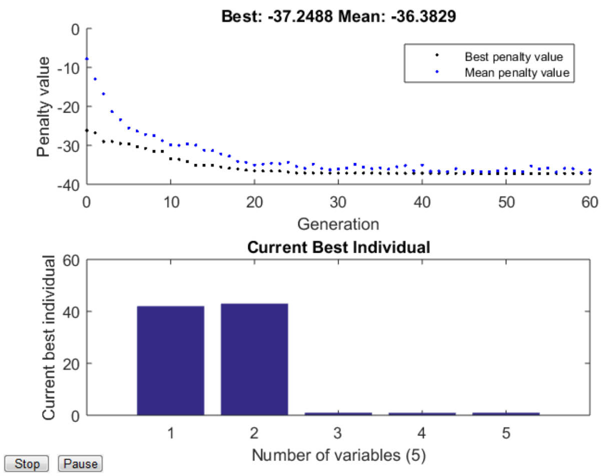
FIGURE 2: *(a) Original "tree" Image*

(b)

(c)

FIGURE 2: *(continuation) (b) Alpha-rooting on "tree" image with alpha equals 0.96706 (c) 3-zone Alpha-rooting on "tree" image with alpha equals [0.97466 0.92718 0.99572] and zonal boundary radii equals [42 43]*

©2016 Society for Imaging Science and Technology
DOI: 10.2352/ISSN.2470-1173.2016.15.IPAS-195



(d)

FIGURE 2: *(continuation) (d) Genetic algorithm best-plot value of 3-zone alpha-rooting.*

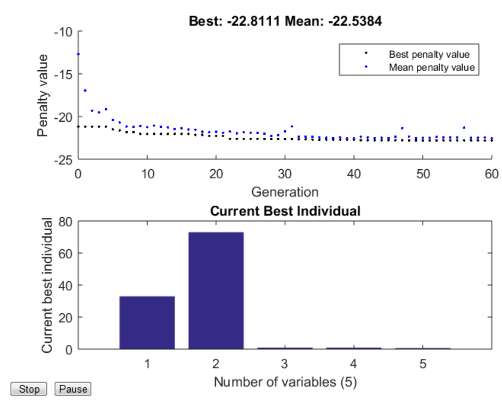
In Figure 3, the color image "girl" is enhanced by alpha-rooting, 3-zone alpha-rooting 2D-QDFT. The optimum value obtained from genetic algorithm for the zonal radii are [33, 73] and the alpha values for the corresponding zones are [0.97101 0.97696 0.66507]

(a)

FIGURE 3: *(a) Original "girl" image*

(b)

(c)



(d)

FIGURE 3: *(b) Alpha-rooting on "girl" image with alpha equals 0.98836 (c) 3-zone Alpha-rooting on "girl" image with alpha equals [0.97101 0.97696 0.66507] and zonal boundary radii equals [33, 73] (d) Genetic algorithm best-plot value of 3-zone alpha-rooting.*

In Figure 4, the color image "FingersAzusa" is enhanced by alpha-rooting, 7-zone alpha-rooting 2D-QDFT. For the dark images, the

©2016 Society for Imaging Science and Technology
DOI: 10.2352/ISSN.2470-1173.2016.15.IPAS-195

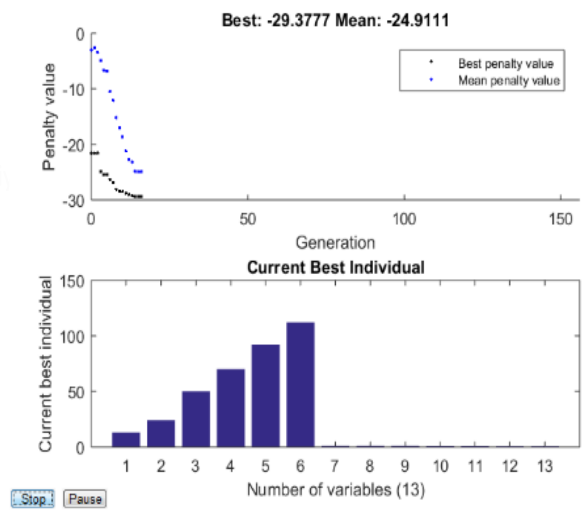
enhancement gives better results when applied on the negative of the image. The optimum value obtained from genetic algorithm for the zonal radii are [13 24 50 70 92 112] and for the alpha values for the corresponding zones are [1.0000 0.9960 0.9943 0.9424 0.8380 0.8006 0.7717]

(a)

(b)

FIGURE 4: *(a) Original "FingersAzusa" image (b) Alpha-rooting on "FingersAzusa" image with alpha equals [0.90275]*

(c)

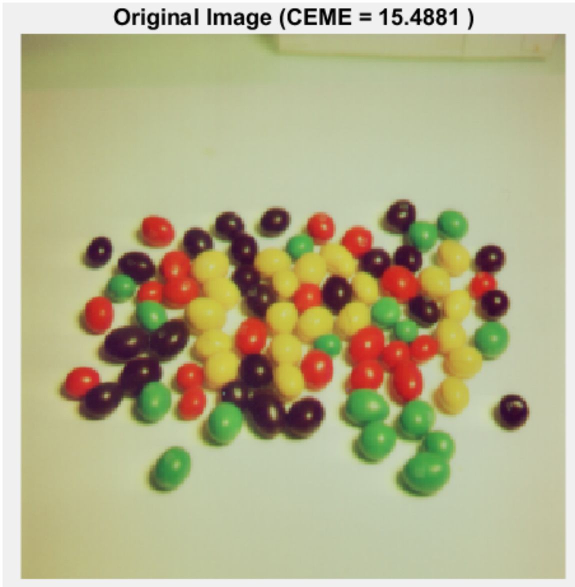


(d)

FIGURE 4: *(contination) (c) 7-zone Alpha-rooting on negative of "FingersAzusa" image with alpha equals [1.0000, 0.9960, 0.9943, 0.9424, 0.8380, 0.8006, 0.7717] and zonal boundary radii equals [13, 24, 50, 70, 92, 112] (d) Genetic algorithm best-plot value of 7-zone alpha-rooting.*

In Figure 5, the color image "JellyBeans" is enhanced by alpha-rooting, 5-zone alpha-rooting 2D-QDFT. The optimum value obtained from genetic algorithm for the zonal radii are [26 51 54 81] and the alpha values for the corresponding zones are [0.97984 0.95853 0.72803 0.89065 0.61928]

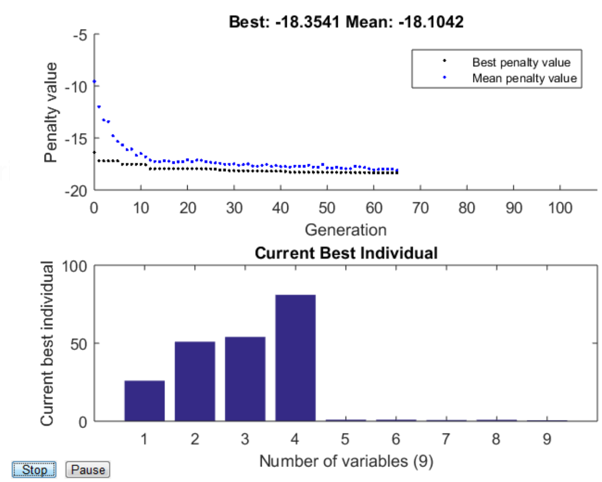
©2016 Society for Imaging Science and Technology
DOI: 10.2352/ISSN.2470-1173.2016.15.IPAS-195



(a)

(b)

FIGURE 5: *(a) Original "JellyBeans" image (b) Alpha-rooting on "JellyBeans" image with alpha equals 0.98032*



(d)

FIGURE 5: *(contination) (c) 5-zone Alpha-rooting on "JellyBeans" image with alpha equals [0.97984 0.95853 0.72803 0.89065 0.61928] and zonal boundary radii equals [26 51 54 81] (d) Genetic algorithm best-plot value of 5-zone alpha-rooting.*

In figure 6, the color image "Sail Boat on Lake" is enhanced by alpha-rooting, 7-zone alpha-rooting 2D-QDFT. The optimum value obtained from genetic algorithm for the zonal radii are [4, 54, 79, 138, 178, 189] and the alpha values for the corresponding zones are [0.9737, 0.9568, 0.9541, 0.9481, 0.7676, 0.8508, 0.9876]

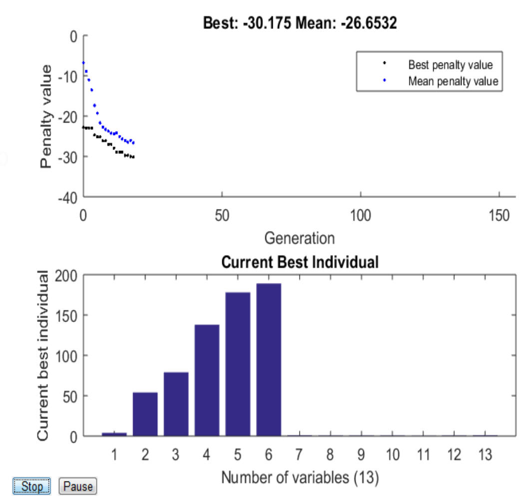
©2016 Society for Imaging Science and Technology
DOI: 10.2352/ISSN.2470-1173.2016.15.IPAS-195

(a)

(b)

FIGURE 6: *(a) Original "Sail Boat on Lake" image (b) Alpha-rooting on "Sail Boat on Lake" image with alpha 0.9941*

(c)



(d)

FIGURE 6: *(continuation) (c) 7-zone Alpha-rooting on "Sail Boat on Lake" image with alpha equals [0.9737 0.9568 0.9541 0.9481 0.7676 0.8508 0.9876] and zonal boundary radii equals [4, 54, 79, 138, 178, 189] (d) Genetic algorithm best-plot value of 7-zone alpha-rooting.*

In figure 7, the color image "house" is enhanced by alpha-rooting, 5-zone alpha-rooting 2D-QDFT. The optimum value obtained from genetic algorithm for the zonal radii are [49, 82, 135, 202] and the alpha values for the corresponding zones are [0.9849, 0.9931, 0.9935, 0.9954, 0.8905]

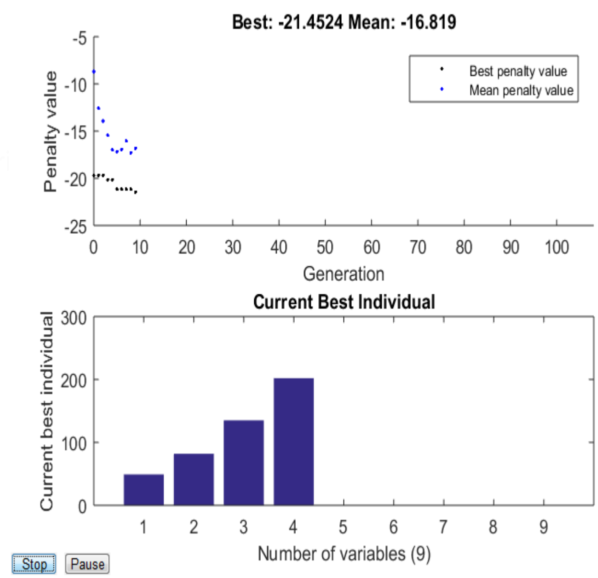
DOI: 10.2352/ISSN.2470-1173.2016.15.IPAS-195

(a)

(b)

FIGURE 7: (a) Original “house” image (b) Alpha-rooting on “House” image with alpha equals 0.98596

(c)



(d)

FIGURE 7 *(continuation): (c) 5-zone Alpha-rooting on “House” image with alpha equals [0.9849, 0.9931, 0.9935, 0.9954, 0.8905] and zonal boundary radii equals [49, 82, 135, 202] (d) Genetic algorithm best-plot value of 5-zone alpha-rooting.*

References

[1] R.C. Gonzalez and R.E. Woods, Digital Image Processing, 2nd Edition, Prentice Hall, 2002.

[2] A.M. Grigoryan, S.S. Again, A.M. Gonzales, “Fast Fourier transform-based retinex and alpha-rooting color image enhancement,” [9497-29],

Proc. SPIE Conf., Mobile Multimedia/Image Processing, Security, and Applications, 20 - 24 April 2015, Baltimore, Maryland United States.

[3] A. M. Grigoryan, S.S. Agaian, “2-D Left-Side Quaternion Discrete Fourier Transform: Fast Algorithm,” Proceedings, Electronic Imaging 2016, IS&T, February 14-18, San Francisco, 2016.

[4] A.M. Grigoryan, S.S. Again, “Tensor representation of color images and fast 2-D quaternion discrete Fourier transform,” [9399-16], Proceedings of SPIE vol. 9399, 2015 Electronic Imaging: Image Processing: Algorithms and Systems XIII, February 10-11, San Francisco, California, 2015.

[5] A.M. Grigoryan and M.M. Grigoryan, Brief Notes in Advanced DSP: Fourier Analysis With MATLAB, CRC Press Taylor and Francis Group, 2009.

[6] A.M. Grigoryan and S.S. Agaian, "Retooling of color imaging in the quaternion algebra," Applied Mathematics and Sciences: An International Journal, vol. 1, no. 3, pp. 23-39, December 2014

[7] A.M. Grigoryan and S.S. Agaian, “Transform-based image enhancement algorithms with performance measure,” Advances in Imaging and Electron Physics, Academic Press, vol. 130, pp. 165–242, May 2004.

[8] A.M. Grigoryan and S.S. Agaian, “Optimal color image restoration: Wiener filter and quaternion Fourier transform,” [9411-24], Proceedings of SPIE vol. 9411, 2015 Electronic Imaging: Mobile Devices and Multimedia: Enabling Technologies, Algorithms, and Applications 2015, February 10-11, San Francisco, California, 2015.

[9] A.M. Grigoryan, J. Jenkinson and S.S. Agaian, “Quaternion Fourier transform based alpha-rooting method for color image measurement and enhancement,” SIGPRO-D-14-01083R1, Signal Processing, vol. 109, pp. 269-289, April 2015, (doi: 10.1016/j.sigpro.2014.11.019).

[10] S.S. Agaian, K. Panetta, and A.M. Grigoryan, “A new measure of image enhancement, in Proc. of the IASTED Int. Conf. Signal Processing Communication, Marbella, Spain, Sep. 1922, 2000.

[11] T.A. Ell, N. Bihan and S.J. Sangwine, “Quaternion Fourier transforms for signal and image processing,” Wiley NJ and ISTE UK, 2014

[12] W.R. Hamilton, *Elements of Quaternions*, Logmans, Green and Co., London, 1866.

[13] T.A. Ell, “Quaternion-Fourier transforms for analysis of 2-dimensional linear time-invariant partial differential systems,” In Proceedings of the 3nd IEEE Conference on Decision and Control, vol. 1-4, pp. 1830–1841, San Antonio, Texas, USA, December 1993.

[14] S.J. Sangwine, “Fourier transforms of colour images using quaternion, or hypercomplex, numbers,” Electronics Letters, vol. 32, no. 21, pp. 1979–1980, October 1996.

[15] S.J. Sangwine, “The discrete quaternion-Fourier transform,” IPA97, Conference Publication no. 443, pp. 790–793, July 1997.

[16] S.J. Sangwine, T.A. Ell, J.M. Blackledge, and M.J. Turner, “The discrete Fourier transform of a color image,” in Proc. Image Processing II Mathematical Methods, Algorithms and Applications, pp. 430–441, 2000.

[18] S.J. Sangwine and T.A. Ell, “Hypercomplex Fourier transforms of color images,” in Proc. IEEE Intl. Conf. Image Process., vol. 1, pp. 137-140, 2001

[19] S.S. Agaian, K. Panetta, and A.M. Grigoryan, (2001) “Transform-based image enhancement algorithms,” IEEE Trans. on Image Processing, vol. 10, pp. 367-382.

[20] A.M. Grigoryan and S.S. Agaian, (2014) “Alpha-rooting method of color image enhancement by discrete quaternion Fourier transform,” [9019-3], in Proc. SPIE 9019, Image Processing: Algorithms and Systems XII, 901904; 12 p., doi: 10.1117/12.2040596

[21] J.H. McClellan, (1980) “Artifacts in alpha-rooting of images,” Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing, pp. 449-452.

[22] F.T. Arslan and A.M. Grigoryan, (2006) “Fast splitting alpha-rooting method of image enhancement: Tensor representation,” IEEE Trans. on Image Processing, vol. 15, no. 11, pp. 3375-3384.

[23] A.M. Grigoryan and F.T. Arslan, "Image enhancement by the tensor transform," Proceedings, IEEE International Symposium on Biomedical Imaging: from Macro to Nano, ISBI 2004, pp. 816-819, April 2004, Arlington, VA.

[24] A.M. Grigoryan and S.S. Agaian, Multidimensional Discrete Unitary Transforms: Representation, Partitioning and Algorithms, Marcel Dekker Inc., New York, 2003.

[25] A.M. Grigoryan and S.S. Agaian, “Cell Phone Cameral Color Medical Imaging via Fast Fourier Transform,” in Electronic Imaging Applications in Mobile Healthcare, J. Tang, S.S. Agaian, and J. Tan, Eds., SPIE Press, Bellingham, Washington, pp. 33-76 (2016).

[26] A.M. Grigoryan and S. Dursun, "Multiresolution of the Fourier transform," in the Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing, 2005 (ICASSP '05), vol. 4, pp. 577–580, March 18-23, 2005, Philadelphia, PA

27] A.M. Grigoryan, Multidimensional Discrete Unitary Transforms. In the Third Edition Transforms and Applications Handbook (Editor in Chief, Prof. Alexander Poularikas). CRC Press Taylor and Francis Group, 2010

28] A.M. Grigoryan, “Decomposition by series direction images: Image reconstruction and enhancement" [7245A-25], International Conference: Electronic Imaging, Science and technology, IS&T/SPIE 21st Annual Symposium, San Diego, CA, 18-22 January, 2009.

[29] A.M. Grigoryan and S.S. Agaian, “Tensor form of image representation: Enhancement by image-signals," [5014-26] (Image Processing: Algorithms and Systems II, Electronic Imaging, Science and Technology 2003, IS&T/SPIE 15 Annual Symposium, Santa Clara, CA, January 20-24, 2003)

[30] A.M. Grigoryan, S.S. Agaian, “Monotonic sequences for image enhancement and segmentation," *Digital Signal Processing* (2015), vol. 41, pp. 70–89, June 2015,

[31] A.M. Grigoryan, “2-D and 1-D multi-paired transforms: Frequency-time type wavelets,” IEEE Trans. on Signal Processing, vol. 49, no. 2, 344–353 (2001)

[32] A.M. Grigoryan, K. Naghdali, “On a method of paired representation: Enhancement and decomposition by series direction images,” Journal of Mathematical Imaging and Vision, vol. 34, no. 2, 185–199 (2009)

[33] F.T. Arslan and A.M. Grigoryan, "Enhancement of medical images by the paired transform," in Proceedings of the 14th IEEE International Conference on Image Processing, vol. 1, pp. 537-540, San Antonio, Texas, September 16-19, 2007.

[34] F.T. Arslan, J.M. Moreno, and A.M. Grigoryan, "Paired directional transform-based methods of image enhancement," in Proceedings of SPIE – vol. 6246, Visual Information Processing XV, April 18-19, 2006.

[35] A.M. Grigoryan and S.S. Agaian, “Split manageable efficient algorithm for Fourier and Hadamard transforms,” IEEE Trans. on Signal Processing, vol. 48, no. 1, 172–183 (2000)

[36] A.M. Grigoryan, E.R. Dougherty, S.S. Agaian, "Optimal Wiener and Homomorphic Filtration: Review," *Signal Processing,* p. 28, vol. 121, pp. 111-138, April 2016, (doi:10.1016/j.sigpro.2015.11.006)

[37] A.M. Grigoryan and S.S. Agaian, “Color Enhancement and Correction for Camera Cell Phone Medical Images Using Quaternion Tools,” in Electronic Imaging Applications in Mobile Healthcare, J. Tang, S.S. Agaian, and J. Tan, Eds., SPIE Press, Bellingham, Washington, pp. 77-117 (2016).

DOI: 10.2352/ISSN.2470-1173.2016.15.IPAS-195

[38] A.M. Grigoryan and S.S. Agaian, “Efficient algorithm for computing the 2-D discrete Hadamard transform,” IEEE Transactions on Circuits and Systems II, vol. 47, no. 10, pp. 1098–1103, October 2000.

[39] A.M. Grigoryan and S.S. Agaian, “Shifted Fourier transform-based tensor algorithms for the 2-D DCT,” IEEE Trans. Signal Processing, vol. 49, no. 9, pp. 2113-2126, Sep. 2001.

[40] A.M. Grigoryan, “Algorithm of computing the 1-D discrete Hadamard transform,” Izvestia VUZov SSSR, Radioelectronica, vol. 34, no 8, 100–103 (1991)

[41] A.M. Grigoryan, "Paired algorithm of the 1-D discrete Hartley transform," (The International Signal Processing Conference (ISPC) and Global Signal Processing Expo (GSPx), Dallas, March 31-April 3, (2003).

[42] A.M. Grigoryan and S.S. Agaian, “Method of fast 1-D paired transforms for computing the 2-D discrete Hadamard transform,” IEEE Transactions on Circuits and Systems II, vol. 48, no. 1, January 2001.

[43] N. Ranganadh, A.M. Grigoryan, P.A. Patel, and B. Tushara, "Implementation and performance evaluation of paired transform based Faster FFT: Grigoryan FFT on Xilinx FPGAs and TMS DSPs using MATLAB: SIMULINK and CC Studio," International Journal of Scientific and Engineering Research, IJSER NASA Astrophysics Database Systems French international journal, vol. 4, no. 8, pp. 373-383, August 2013

[44] A.M. Grigoryan, “New algorithms for calculating discrete Fourier transforms,” Journal Vichislitelnoi Matematiki i Matematicheskoi Fiziki, AS USSR, vol. 26, no. 9, 1407–1412 (1986)

[45] A.M. Grigoryan and M.M. Grigoryan, “Two-dimensional Fourier transform in the tensor presentation and new orthogonal functions,” Avtometria, AS USSR Siberian section. no. 1, pp. 21–27, Novosibirsk 1986.

[46] A.M. Grigoryan, “Efficient algorithms for computing the 2-D hexagonal Fourier transforms,” IEEE Transaction on Signal Processing, vol. 50, no. 6, pp. 1438–1448, June 2002.

[47] A.M. Grigoryan, “Comments on “The discrete periodic Radon transform”,” IEEE Trans. on Signal Processing, vol. 58, no. 11, pp. 5962 5963, Nov. 2010.

[48] A.M. Grigoryan and Nan Du, “Comments on “Generalised finite Radon transform for NN images”,” IMAVIS, vol. 29, no. 11, pp. 797–801, November 2011.

[49] A.M. Grigoryan and S.P.K. Devieni, “New method of signal denoising by the paired transform,” Applied Mathematics and Sciences: An International Journal (MathSJ), vol. 1, no. 2, 1–20 (2014)

[50] A.M. Grigoryan and M.M. Grigoryan, “Nonlinear approach of construction of fast unitary transforms,” Proc. the 40th Annual Conference on Information Sciences and Systems (CISS 2006), Princeton Univ., March 22-24, 2006, Princeton,1073–1078, 2006.

[51] A.M. Grigoryan and M.M. Grigoryan, “Discrete unitary transforms generated by moving waves” [6701-78], Proc. the International Conference: Wavelets XII, SPIE: Optics+Photonics 2007, San Diego, CA, (2007)

[52] A.M. Grigoryan and M.M. Grigoryan, “Discrete signal induced unitary transforms,” (paper number: 577-827), in ISI Book (Thomson) published by American Conference on Applied Mathematics, WSEAS 2008, Cambridge, MA, (2008)

[53] A.M. Grigoryan and M.M. Grigoryan, “New discrete unitary Haar-type heap transforms” [6701-79], Proc. the International Conference: Wavelets XII, SPIE: Optics+Photonics 2007, San Diego, CA, (2007)

[54] A.M. Grigoryan and M. Hajinoroozi, “A novel method of filtration by the discrete heap transforms, [9019-2], SPIE proceedings, 2014 Electronic Imaging: Image Processing: Algorithms and Systems XII, February 2-6,San Francisco, California (2014)

[55] A.M. Grigoryan and M. Hajinoroozi, “Image and audio signal filtration with discrete Heap transforms,” Applied Mathematics and Sciences: An International Journal (MathSJ), vol. 1, no. 1, 1–18 (2014)

[56] Naghdali, K., Ranjith, R., and Grigoryan, A., “Fast signal-induced transforms in image enhancement,” Proc. Systems, Man and Cybernetics, SMC 2009. IEEE Int. Conference on, 565-570 (2009)

[57] A.M. Grigoryan, “New method of Givens rotations for triangularization of square matrices," *Journal of Advances in Linear Algebra & Matrix Theory (ALAMT) ,* vol. 4, no. 2, pp. 65-78, June 2014

[58] A.M. Grigoryan and M.M. Grigoryan, “Discrete integer Fourier transform in real space: Elliptic Fourier transform,” [7245A-1], International Conference: Electronic Imaging, Science and technology, IS&T/SPIE 21st Annual Symposium, San Diego, CA, 18-22 January, (2009)

[59] A.M. Grigoryan, “Elliptic discrete Fourier transforms of Type II,” Proc. 2009 IEEE International Conference on Systems, Man, and Cybernetics, San Antonio, Texas, USA, 954–959 (2009)

[60] A.M. Grigoryan, “Two classes of elliptic discrete Fourier transforms: Properties and examples,” Journal of Mathematical Imaging and Vision (0235), vol. 39, 210-229 (2011)

[61] V. A. Kostenko and A. V. Frolov, “Self learning genetic algorithm”, Journal of Computer And Systems Sciences International, Vol. 54 No. 4 2015

Author Biography

Artyom M. Grigoryan received the PhD degree in Mathematics and Physics from Yerevan State University (1990). He is an associate Professor of the Department of Electrical Engineering in the College of Engineering, University of Texas at San Antonio. The author of three books, 9 book-chapters, 3 patents and more than 120 papers and specializing in the design of robust filters, fast transforms, tensor and paired transforms, discrete tomography, quaternion imaging, image encryption, processing biomedical images.