Spherical Sampling and Color Transformations

Graham D. Finlayson[a] and Sabine Süsstrunk[a,b]
[a]*School of Information Systems, University of East Anglia, Norwich, United Kingdom*
[b]*Laboratory for Audiovisual Communications, Swiss Federal Institute of Technology (EPFL), Lausanne, Switzerland*

Abstract

In this paper, we present a spherical sampling technique that can be employed to find optimal sensors for trichromatic color applications. The advantage over other optimization techniques is that it assures a global minimum is found, and that not only one, but a set of solutions is retained if so desired. The sampling technique is used to find all possible RGB sensors that exhibit favorable chromatic adaptation transform (CAT) behavior when tested on Lam's corresponding color data set, subject to a CIE ΔE_{94} error criterion. We found that there are a number of sensors that meet the criterion, and that the Bradford, Sharp, and CMCCAT2000 sensors are not unique.

Introduction

In most trichromatic imaging applications, linear transforms from one set of RGB sensors to another set of RGB sensors are applied to color images somewhere in the processing chain. In digital photography, for example, sensor RGB code values are transformed to output-referred RGB encoding values, such as ROMM RGB or sRGB[1,2] so that the image can be processed or viewed on a monitor with the appearance that was intended. Similarly, all ICC monitor profiles contain a linear transformation to map RGB code values of a digital file to specific RGB monitor primaries, so that the image appears "correctly" on the user's monitor.

In color imaging, the transformation usually takes the form of:

$$\begin{bmatrix} \mathbf{r}_2 \\ \mathbf{g}_2 \\ \mathbf{b}_2 \end{bmatrix} = \mathbf{M} * \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{g}_1 \\ \mathbf{b}_1 \end{bmatrix} \qquad (1)$$

where $\mathbf{r}_1, \mathbf{g}_1, \mathbf{b}_1$ and $\mathbf{r}_2, \mathbf{g}_2, \mathbf{b}_2$ are vectors containing RGB or XYZ tristimulus values that describe two different sensors or encodings, and $\mathbf{M}$ is a 3x3 transformation matrix that maps vectors $\mathbf{r}_1, \mathbf{g}_1, \mathbf{b}_1$ onto $\mathbf{r}_2, \mathbf{g}_2, \mathbf{b}_2$.

It is often the case in color imaging research that the second set of RGB sensors is not known, and therefore needs to be determined using some applicable criteria. Usually, an optimization technique is employed that best minimizes (or maximizes) the parameters of these criteria. Considering the large number of computational steps necessary to find a solution, optimization software routines have been developed. For example, *Matlab*[3] has a statistical toolbox that offers a number of pre-programmed optimization techniques.

While optimization routines are helpful tools to find unique solutions to a problem, they do not, in general, allow finding a *set of solutions* that fulfill some criteria. They tend to converge to a single minimum. Depending on the optimization technique and parameters used, it is not always evident that the solution is unique, i.e. corresponds to a global and not a local minimum.

It is easier to find a set of solutions, and to be sure to find all possible solutions, if the solution space is *sampled*. Optimization through sampling implies that all possible combinations are tried, and the best solution is retained that corresponds to the global minimum. Sampling also allows retaining *all possible* solutions that fulfill the criteria if the result is not unique.

In this paper, we describe a spherical sampling technique and apply it to find all RGB sensors usable for chromatic adaptation transforms, subject to applying the von Kries chromatic adaptation model and a CIE ΔE_{94} error criterion. The algorithms are described in detail, and the results for the chromatic adaptation application are presented.

We find that there are many sensors that account for corresponding color data. It is evident therefore that the overall 'best' adaptation transform may be best chosen by looking at secondary factors such as transform plausibility.[4]

Spherical Sampling

In the case of trichromatic (RGB and XYZ) imaging applications, the basis functions span a three-dimensional space. If the lengths of the vectors are normalized to unity, then different vector combinations can be illustrated with their end-points that lie on the surface of a sphere (see Figure 1). Trying all possible combinations of three points distributed over the surface of the sphere allows us to find all possible solutions to a given problem. To determine the sample points, the surface of the sphere has therefore to be sampled at a pre-defined distance that depends on the application. That leads to the question of how to uniformly distribute a large number of points (*N*) on the surface of a sphere.

There has been significant research done on this problem, and a variety of algorithms have been proposed.[5,6,7]

A popular and simple technique is that of *icosahedral dissection,* which proceeds as follows[8]: for each triangular face of the icosahedron, the midpoints of the sides are joined to form four new triangles. The centers of the triangles are then radially projected onto the sphere, yielding a total of 80 points. Continuing the dissection process produces $N = 20 \times 4^n$ points (n = 0, 1, …). One drawback of this method is the restricted number of possible *N*. Additionally, the points are not asymptotically uniformly distributed, the projection process increases the areas of the "middle triangles" more than the rest.[9]

We have therefore chosen to use the *generalized spiral set* method proposed by Rakhmanov, Saff, and Zhou,[10] which appears to perform better than the icosahedron method for large *N*. Using spherical coordinates (ϕ, θ), $0 \leq \phi \leq \pi$, $0 \leq \theta \leq 2\pi$, the coordinates of the *N* points can be calculated as follows:

$$\phi_k = \arccos(h_k), \quad h_k = -1 + \frac{2(k-1)}{(N-1)}, \quad 1 \leq k \leq N \qquad (2)$$

$$\theta_k = \left(\theta_{k-1} + \frac{3.6}{\sqrt{N}} \frac{1}{\sqrt{1-h_k^2}}\right) \quad 2 \leq k \leq N-1, \quad \theta_1 = \theta_N = 0 \quad (3)$$

Figure 1 illustrates the distribution of the points for $N = 700$. The cartesian coordinate vectors **p** (*x,y,z*) that correspond to each sample point can then be calculated as follows ($\rho = 1$):

$$\begin{aligned} x &= \rho \cos\theta \sin\phi \\ y &= \rho \sin\theta \sin\phi \\ z &= \rho \cos\phi \end{aligned} \qquad (4)$$

Each point in the sphere corresponds to some linear combination of the XYZ color matching functions. However, we must be careful and realize that the XYZ functions themselves are not orthonormal, which is to say, for example, that the X- and Y-functions are quite correlated. In order to use the points on the sphere as shown in Figure 1 to represent sensors, we need to first map the XYZ color matching functions to a new set of orthonormal functions. That is, each function will be orthogonal (at right angles) to each other function. In effect, by finding an orthonormal set of sensors, we are finding new coordinate axes, and coordinate axes generally have these orthonormal properties. We can find these orthonormal sensors using the well-known singular value decomposition (SVD). Orthonormalising sensors to decorrelate them is sometimes presented in color research as an explanation of color opponency since color opponent sensors are, depending on the study, either orthonormal or close to orthonormal

If **C** denotes the (mx3) XYZ sensor matrix, than we can write **C** as equation (5).

$$\mathbf{C} = \mathbf{Q}_1 \mathbf{D} \mathbf{Q}_2^T \qquad (5)$$

$\mathbf{Q}_1$ is an orthonormal mx3 matrix, and **D** and $\mathbf{Q}_2$ are both 3x3 matrices. It is evident then that $\mathbf{Q}_1$ is precisely the

orthonormal basis that we seek. By multiplying $\mathbf{Q}_1$ with a linear transformation matrix **L** (3x3) that consists of three sample point vectors $\mathbf{p}_i$, $\mathbf{p}_j$, $\mathbf{p}_k$, a new matrix **V** (mx3) of color values can be derived that corresponds to a new RGB sensor set:

$$\mathbf{V} = \mathbf{Q}_1 \mathbf{L}, \quad \mathbf{L} = [\mathbf{p}_i, \mathbf{p}_j, \mathbf{p}_k] \qquad (6)$$

If we take every triplet of points ($\mathbf{p}_i$, $\mathbf{p}_j$, $\mathbf{p}_k$) on the sphere and post-multiply $\mathbf{Q}_1$, then we will generate a set of sensitivities that are uniformly distributed on a sphere in sensor space. And so we have used our 3-dimensional set of points to create a corresponding set of evenly distributed sensors. With reference back to equation (1), if we start in XYZ space, then **M** will equal to:

$$\mathbf{M}^T = \left[\mathbf{D}\mathbf{Q}_2^T\right]^{-1}\mathbf{L} \qquad (7)$$

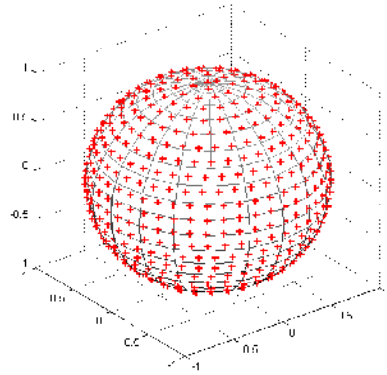*Figure 1: Evenly distributed points on a sphere (N=700), using the generalized spiral set method.*

While we have described the sampling technique starting with XYZ color matching functions, the algorithms can, of course, be adapted to using any kind of RGB color matching functions. It is important to point out that the sampling technique returns three sensors that have equal magnitude. In contrast, cone sensitivities are known to have different sensitivities: the short-wave mechanism is much less responsive than the long- and medium- wave mechanisms. Here we can avoid sensor magnitude because this variable is not important in the context of adaptation transforms (we are looking for scalars relating sensor responses across lighting conditions and these relative scalings are independent of the absolute magnitude of the sensors).

Chromatic Adaptation Transforms (CATs)

Chromatic adaptation is the ability of the human visual system to discount the color of the illumination and to approximately preserve the appearance of an object. Image capturing systems, such as scanners and digital cameras, do not have the ability to adapt to an illumination source. To

faithfully reproduce the appearance of image colors, it follows that all image processing systems need to apply a transform that converts the input colors captured under the input illuminant to the corresponding output colors under the output illuminant. This can be achieved by using a *chromatic adaptation transform* (CAT). Basically, applying a chromatic adaptation transform to the tristimulus values (X', Y', Z') of a color under one adapting light source predicts the corresponding color's tristimulus values (X", Y", Z") under another adapting light source.

There are several chromatic adaptation transforms described in the literature, most based on the von Kries model.[11] CIE tristimulus values are linearly transformed by a 3x3 matrix $\mathbf{M}_{CAT}$ to derive R'G'B' responses under the first illuminant. The resulting R'G'B' values are independently scaled to get R"G"B" responses under the second illuminant. The scaling coefficients are most often based on the illuminants' white-point R'G'B' and R"G"B" sensor values. If there are no non-linear coefficients, this transform can be expressed as a diagonal matrix. To obtain CIE tristimulus values (X"Y"Z") under the second illuminant, the R"G"B" are then multiplied by $(\mathbf{M}_{CAT})^{-1}$, the inverse of matrix $\mathbf{M}_{CAT}$. Equation (8) describes a matrix formulation of this concept:

$$\begin{bmatrix} X" \\ Y" \\ Z" \end{bmatrix} = \left[\mathbf{M}_{CAT}\right]^{-1} * \begin{bmatrix} R_w^{"}/R_w^{'} & 0 & 0 \\ 0 & G_w^{"}/G_w^{'} & 0 \\ 0 & 0 & B_w^{"}/B_w^{'} \end{bmatrix} * \left[\mathbf{M}_{CAT}\right] * \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \tag{8}$$

Quantities $R_w^{'}, G_w^{'}, B_w^{'}$ and $R_w^{"}, G_w^{"}, B_w^{"}$ are computed from the tristimulus values of the first and second illuminants, respectively, by multiplying the corresponding XYZ vectors by $\mathbf{M}_{CAT}$.

The currently most popular chromatic adaptation transforms are the von Kries CAT,[12] the linearized Bradford CAT,[13,14] the Sharp CAT,[15] and the CMCCAT2000 transform.[16] All are based on the von Kries model as described in equation (8), but they apply the white-point scaling to different RGB sensors (see Figure 6), i.e. they use different transformation matrices $\mathbf{M}_{CAT}$. Recent studies[17] have shown that for given sets of corresponding colors,[18] the performance of Bradford, Sharp, and CMCAT2000 transforms is approximately the same when using perceptual error criteria of CIE ΔE_{Lab}, CIE ΔE_{94}, or CIE $\Delta E_{CMC(1:1)}$, even though the transformation matrices were derived differently.

That leads to the question if there are other RGB sensors that perform just as well as the Bradford, Sharp and CMCCAT2000 sensors and that have not yet been considered.

Experiment and Result

The experiment was designed to find other chromatic adaptation transform matrices besides Bradford, Sharp and CMCCAT2000 that performed as well if not better, using Lam's corresponding color data set and an error criterion based on CIE ΔE_{94}. Using the spherical sampling method described in Section 2, all possible RGB sensor combinations can be tried to find other transformation matrices $\mathbf{M}_{CAT}$.
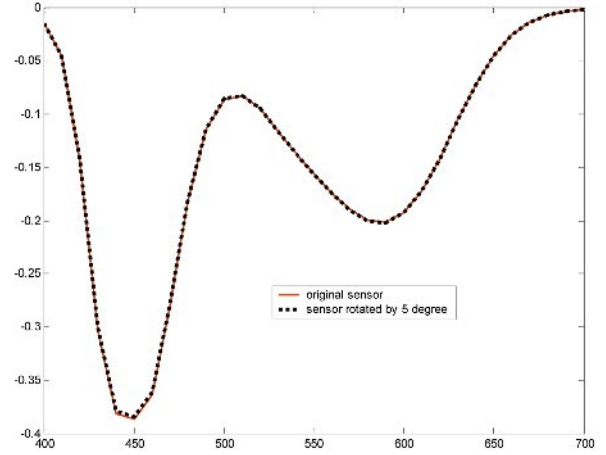

Figure 2: The variation in spectral sensitivity of a sensor rotated (x,y) by 5 degrees.

First, we determined the number of surface points necessary to uniformly sample the sphere. By visually comparing the difference between a sensor and a rotated sensor, we determined that the maximum distance between sample points should be no larger than 5 degrees (see Figure 2). We found for $N = 5,000$, the angle between two neighboring vectors varies between ~3-5 degrees. Therefore, the necessary combinations to be checked are equal to:

$$\binom{N}{k} = \frac{N!}{k!(N-k)!} = 2.08 \times 10^{10} \quad N = 5,000, k = 3 \tag{9}$$

Out of computational considerations, we reduced the number of combinations by assuming that the sample points giving a positive result are located around the points that describe the Bradford, Sharp, and CMCAT2000 transform. We then determined the location of the vectors for the three transforms, and retained only the sample points that fell within 20 degrees of those points (see Figure 3). As a result, 222 "red", 180 "green", and 165 "blue" vectors were retained, resulting in 6.68×10^6 different transforms to be tested.
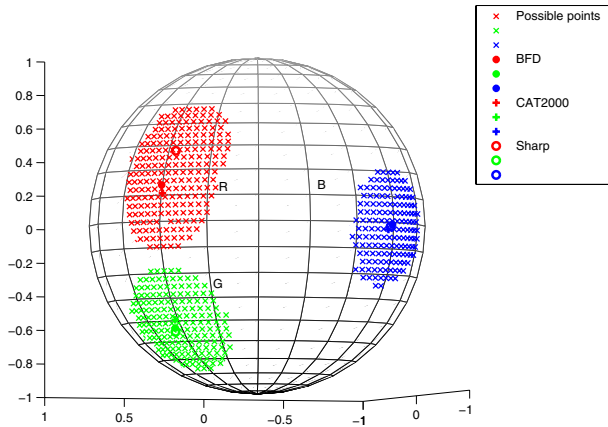
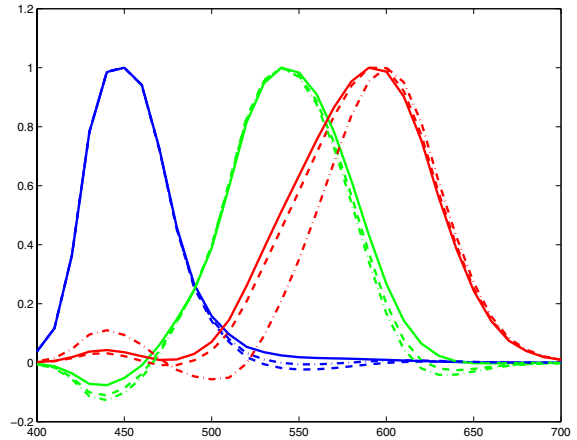*Figure 3: All sample points within a 20 degree radius of Bradford, Sharp and CMCCAT2000 (for N = 5000).*

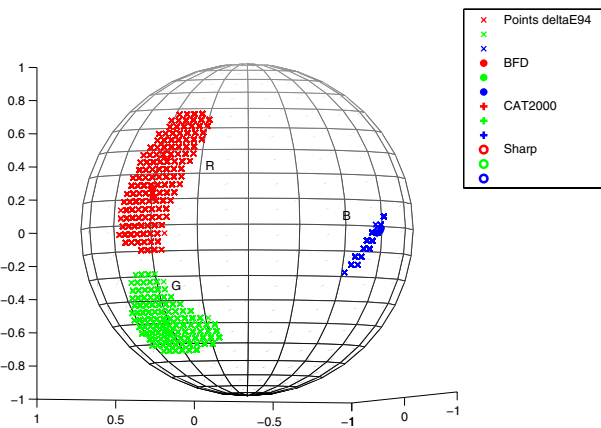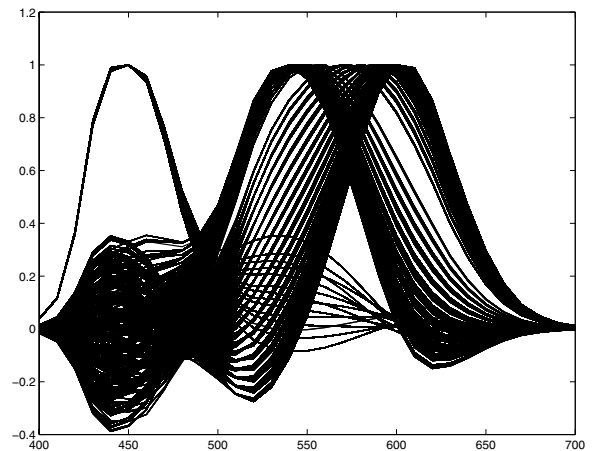*Figure 4: All sample points that result in sensor combinations with a RMS CIE $\Delta E_{94} \leq 4$ prediction error.*
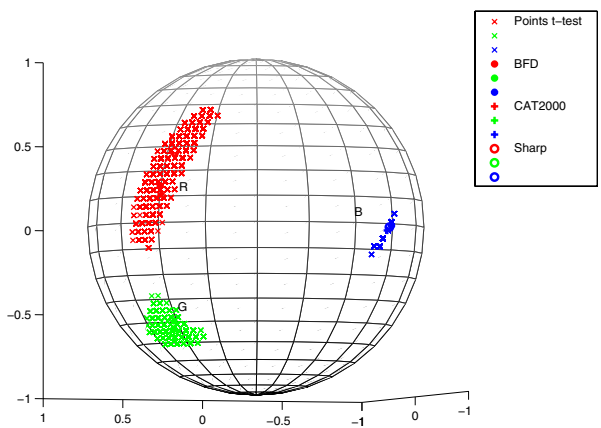

Figure 5: All sample points that result in sensor combinations that are not statistically significantly different from CMCCAT2000 at 95 percent confidence.

Figure 6: Bradford (--), Sharp (-.) and CMCCAT2000 (–) RGB Sensors.

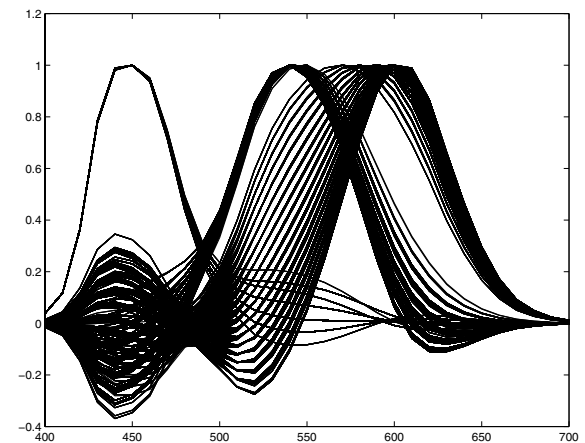*Figure 7: All RGB sensor sets (13,801) with a RMS CIE $\Delta E_{94} \leq 4$ prediction error.*

Figure 8: All RGB sensor sets (2,491) that are not statistically significantly different from CMCCAT2000 at 95 percent confidence.

To evaluate if the resulting RGB sensors have good chromatic adaptation transform behavior, Lam's corresponding color data set[18] was used, which was also employed to derive both the Bradford and the Sharp CATs[13,15] Lam had observers predict the appearance of 58 wool samples under illuminants A and D65. The resulting corresponding color data set has been used extensively to test chromatic adaptation transforms and has been found to be quite stable.[16]

The $\mathbf{M}_{CAT}$ transforms found through the spherical sampling technique were retained if the RMS prediction error was CIE $\Delta E_{94} \leq 4$. For comparison, the Sharp CAT, Bradford CAT, and CMCCAT2000 prediction errors for the same data set are 3.4, 3.5, and 3.6, respectively.[17] The assumption is that any chromatic adaptation transform with a perceptual error of CIE $\Delta E_{94} \leq 4$ for Lam's data set is adequate. For example, RMS CIE ΔE_{94} is equal to 5.0[17] for von Kries operating on cone responses, a chromatic adaptation that is still widely used.[12]

Of the ~6.7 million possible chromatic adaptation transforms evaluated, 13,801 fulfilled the error criterion. See Figure 4 for the corresponding sampling points and Figure 7 for the corresponding RGB sensors.

In addition, one-tail student t-tests for matched pairs[17,19] were calculated to evaluate how many of the 13,801 RGB sensors resulted in a chromatic adaptation transform that was not statistically different from the CMCCAT2000 transform. At the 95 percent confidence level, 2,491 RGB sensor sets remained. The resulting sampling points are shown in Figure 5, and the corresponding sensor sets in Figure 8.

The corresponding color data set clearly does not uniquely support a single von Kries type chromatic adaptation transform. That is particular relevant as the CIE and other standards bodies are proposing a single standard chromatic adaptation transform. Looking at Lam's data alone, there are probably at least 2,491 sensor sets to consider.

Conclusion

We present a spherical sampling technique that can be used to evaluate and/or find linear color transformations. It has the advantage over other optimization techniques that it not only can easily find a global minimum, it can also return a set of solutions if so required.

We also show that the Bradford, Sharp, and CMCCAT-2000 sensors are not unique. There is a number of other RGB sensors that exhibit the same favorable chromatic adaptation behavior. If the sampling distance is further decreased, the number would increase even more. This leads to the conclusion that there is either too much noise in the corresponding data set used to evaluate CATs, that the von Kries model used to implement chromatic adaptation transforms is too much of a simplification, or that there are many possible solutions to chromatic adaptation transforms and it is not critical which one is used.

We speculate that in order to make a final choice on a single color chromatic adaptation transform, other second-ary factors should be examined. For example, if one color space or one set of RGB sensors fits better with other color imaging workflows. It has been shown that sharp sensors are fairly close to the sRGB sensors.[20]

References

1. PIMA 7666: 2001 Photography – *Electronic Still Picture Imaging – Reference Output Medium Metric RGB Color encoding: ROMM-RGB.*
2. IEC 61966-2-1 (1999): *Multimedia systems and equipment – Colour measurement and management – Part 2-1: Colour management – Default RGB colour space – sRGB.*
3. Matlab, Version 6.0, Statistical Toolbox, The MathWorks Inc.
4. G.D. Finalyson and P. Morovic, "Is the sharp adaptation transform more plausible than CMCCAT2000?" *Proc. IS&T/SID 9th Color Imaging Conference*, November 2001.
5. R. Alexander, "On the sum of distances between *N* points on a sphere," *Acta Math. Acad. Sci. Hungar.* 23, 443-448, 1972.
6. J.H. Conway and N.J.A Sloane, *Sphere Packings, Lattices and Groups,* 2nd ed., Springer-Verlag, New York (1993).
7. E.L. Altschuler, T.J. Williams, E.R. Ratner, F. Dowla, and F. Wooten, "Method of constrained global optimization," *Phys. Rev. Lett.,* vol. 72, pp. 2671-2674, 1994.
8. E.B. Saff and A.B.J. Kuijlaars, "Distributing Many Points on a Sphere," *Math. Intelligencer*, vol. 19, pp. 5-11, 1997.
9. J. Cui and W. Freedan, "Equidistribution on the sphere," *SIAM J. Sci. Comp.,* vol. 18, pp. 595-609, 1997.
10. E.A. Rahkmanov, E.B. Saff, and Y.M. Zhou, "Minimal discrete energy on the sphere," *Math. Res. Lett.,* **1**, 647-662, 1994.
11. J. von Kries, "Theoretische Studien über die Umstimmung des Sehorgans," *Festschrift der Albrecht-Ludwig Universität*, pp. 145-158, 1902.
12. M.D. Fairchild, *Color Appearance Models,* Addison-Wesley, Reading, MA, (1998).
13. K.M. Lam, "Metamerism and Colour Constancy," *Ph.D. Thesis,* University of Bradford, 1985.
14. S. Swen and L. Wallis, "Chromatic Adaptation Tag Proposal," ICC Votable Proposal Submission, No. 8.2, June 9, 2000.
15. G.D. Finalyson and S. Süsstrunk, "Performance of a chromatic adaptation Transform based on spectral sharpening," *Proc. IS&T/SID 8th Color Imaging Conf.,* pp. 49-55, 2000.
16. C. Li, M.R. Luo, B.Rigg, "Simplification of the CMCCAT-97," *Proc. IS&T/SID 8th Color Imaging Conf.,* 56-60, 2000.
17. S. Süsstrunk, J. Holm, and G.D. Finalyson, "Chromatic adaptation behavior of different RGB sensors," *Proc. SPIE,* vol. 4300, pp. 172-183, 2001.
18. M.R. Luo and P.A. Rhodes, *Corresponding Colour Data Sets,* University of Derby, <http://colour.derby.ac.uk/colour/info/catweb/>.
19. R.E. Walpole, R.H. Myers, S.L. Myers, *Probability and Statistics for Engineers and Scientists,* 6th ed., Prentice Hall International, Upper Saddle River, NJ, 1998.
20. P.M. Hubel, J. Holm, G. Finlayson, "Illumination estimation and colour correction," *Colour Imaging: Vision and Technology,* ed. L.W. MacDonald and M.R. Luo, pp. 72-95, John Wiley, 1999.