# Halftoning and Color Noise 

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#### Abstract

A frequency modulated color halftoning algorithm is presented in this paper. Unlike the normal approach of halftoning a color image, in which the color separations of the original image are halftoned independently, the original color image is halftoned in a context dependent manner. The strategy to reduce color noise and gain control over color gamut is to prevent dot-on-dot printing as much as possible. The color shifts that might occur because of this dot-off-dot printing strategy have to be compensated before halftoning. This transformation uses some data for the printer with which the halftoned color image is supposed to be printed. The experiments verify that the color noise is notably smaller in the images that are halftoned by the proposed method compared to the images halftoned using the normal approach of halftoning color images. The method also offers the possibility of treating the color separations of the original image differently if needed. For example, the yellow separation should be treated differently from the other separations, because the yellow dots are less visible than the other color dots when they are printed on a white paper. Two criteria for objectively measuring the quality of the produced results are also discussed.


## Introduction

Monochromatic halftoning methods ([1], [2], [3], [4], [5], [6]) can directly be used to halftone color images by independently applying them to the color separations of the original color image. Since the color perception is very much dependent on how the separations behave in relation to each other, having a good structure in each separation by itself doesn't guarantee a good perception of the final color image. In this paper we present a color halftoning method that halftones color images in a context dependent manner. We also show that halftoning the separations of the original color image dependently by using the presented method can notably reduce the color noise of the final halftoned color image. The strategy to reduce color noise is to prevent dot-on-dot printing as much as possible. The color shifts that might occur are handled by transforming the original color image before halftoning. This transformation is performed by using Neugebauer's and Demichel's equations.

Since the yellow ink is less visible than cyan and magenta when printed on white paper, we will show that in the presented method it is possible to treat the yellow separation differently from the other two in order to achieve better results.
Two criteria for objectively measuring the quality of produced results are discussed and applied to several images.
In this paper a number of examples are given to show how the proposed method can increase the quality of the halftoned images. The color illustrations are shown in [7] and [8], where more examples are also given.

## Halftoning Method, Monochromatic

The monochromatic halftoning method has already been presented in previous publications ([6], [7]). In this method it is assumed that the original image is scaled between 0 and 1 . It is also assumed that 0 and 1 represent white and black respectively. The initial halftoned image is totally empty. The number of dots to be placed in the binary/halftoned image is determined in advance by the sum of the density values of the original gray scale image.
The first dot in the halftoned image is placed at the position where the original image has its largest density value, that is where the original image is darkest. In each iteration one new black dot is placed. The position of this dot depends on where the previous dots have been placed. After placing a dot, its impact is fed back to the original image in order to decrease the possibility of finding the next largest density value in a neighborhood of this position. The feed back process is performed by using a filter. This filter plays a very significant role in the appearance of the final image [7].

## Halftoning Method, Chromatic

All halftoning techniques designed for monochromatic images can directly be used for chromatic images by applying them to the color separations independently. When the separations of a chromatic image are halftoned independently, the dot placement in each separation is controlled only in that separation and doesn't affect the other separations. Since the color perception is very much dependent on how the separations behave in relation to each other, having a good structure of dot placement in each separation by itself
doesn't guarantee a very good color perception of the halftoned color image. We are going to show that a detailed control of the dot placement can reduce the color noise of the final halftoned color image (see also [7], [8], [9]). Due to the nature of the method presented in the previous section, it can be extended to a method for chromatic images that halftones the separations dependently. Recall from the previous section that in the monochromatic method the dot placement in the halftoned image was controlled by the filter utilized within the algorithm. In this section we will see how chromatic images can be halftoned in a context dependent manner by using appropriate filters.
The algorithm begins with finding the position of the largest density value, that is the largest density value in all three separations of the original color image. Then a dot is placed at the same position in the corresponding separation of the halftoned image.


Figure 1. A color image with $2 \%, 3 \%$ and $0 \%$ coverage in its $C, M$ and $Y$ separations is halftoned with the proposed methods. In $a$, the separations are halftoned independently. In $b$, the separations are halftoned dependently by the proposed color halftoning method.

In the monochromatic case a filter was used to control the dot placement and the filtered version of the binary image was subtracted from the filtered version of the original one. Now besides that we do the same in this particular separation, we use another (or the same) filter to feedback the impact of this dot to the other separations. That means, when a dot is placed for example in the M separation, some neighborhood of this position in the M separation is affected and the same (another) neighborhood of this position in C \& Y is also affected using the same (another) filter.
In order to show how the proposed method can lead to more homogeneous results let us give an example. Suppose that we have a color image that has $2 \%, 3 \%$ and $0 \%$ coverage in its C, M and Y separations, respectively. This image is first halftoned by applying the monochromatic halftoning method to the separations independently, see Fig. 1a. This image is also halftoned by the proposed color halftoning method, see Fig. 1b. Since in the first case the separations are halftoned independently, the filter used for each separation is designed
only for that separation. For having a good structure of dot placement the filter used for the C and M separations should be $15 \times 15$ and $13 \times 13$ respectively [7]. The size of the filter used for each separation is only dependent upon the coverage of that separation. But in the dependent case, we want the dots in both separations to be placed homogeneously over the entire halftoned color image. The filter size should therefore correspond to a coverage of $2+3=5 \%$ [7]. The filter used for both separations is therefore $9 \times 9$ in this case.

## Color Rendition

In the previous section we saw how we can halftone the color separations of a color image dependently and prevent the dots in different separations from being placed on top of each other as much as possible. The question we ask now is: If the resulting halftoned image is printed, will all the colors in the image be produced correctly? The answer is no. Let us give an example. In Fig. 2 the results of halftoning a color image with $50 \%$ cyan and magenta by the proposed methods, i. e. independently and dependently, are shown. Obviously they are not the same color. This color shift should be compensated before halftoning. First of all by using Demichel's equations we can calculate the fractional areas covered by each primary and secondary color. These colors' tristimulus values are also measured when they are printed using the printer with which the color images are supposed to be printed. Then by using Neugebauer's equations the tristimulus values of the color we want the method to produce, i. e. the target color, are calculated.


Figure 2. A color image with $50 \%$ coverage in its Cyan and Magenta separations is halftoned. a) by the monochromatic halftoning method being applied to the separations independently. b) by the proposed color halftoning method.

Suppose that we want our algorithm to produce a certain color $\left(\mathrm{X}_{\mathrm{t}}, \mathrm{Y}_{\mathrm{t}}, \mathrm{Z}_{\mathrm{t}}\right)$. We assume that no dots in different separations will be placed on top of each other. The proposed method can fulfill this demand if the sum of the newly calculated coverages of the separations does not exceed $100 \%$.


Figure 3. The original image is a color image with $50 \%$ coverage in its Cyan and Magenta separations. a) This image is halftoned by the monochromatic halftoning method being applied to the separations independently. b) The original image is first transformed to an image with $50.82 \%$, $36.51 \%$ and $0.54 \%$ coverage in its $C, M$ and $Y$ separations and the result is then halftoned with the proposed color halftoning method. The transformation is performed in order to achieve the same color as it would have been obtained if a semi-stochastic overlap behavior was assumed.

Of course a negative coverage is not accepted either [7]. Therefore we will only have the primary colors and the bare paper. Now in order to find the coverage in the $\mathrm{C}, \mathrm{M}$ and Y separations the following equation system should be solved for the target color $\left(X_{t}, Y_{t}, Z_{t}\right)$, i.e. the color we want our algorithm to produce.

$$
\left\{\begin{array}{l}
X_{t}=c X_{c}+m X_{m}+y X_{y}+(1-c-m-y) X_{\text {paper }}  \tag{1}\\
Y_{t}=c Y_{c}+m Y_{m}+y Y_{y}+(1-c-m-y) Y_{\text {paper }} \\
Z_{t}=c Z_{c}+m Z_{m}+y Z_{y}+(1-c-m-y) Z_{\text {paper }}
\end{array}\right.
$$

where $\mathrm{X}_{\mathrm{t}}, \mathrm{Y}_{\mathrm{t}}$ and $\mathrm{Z}_{\mathrm{t}}$ denote the tristimulus values for the target color and $c, m, y$ denote the coverage in the $C, M$ and $Y$ separations, respectively. $X_{c}, X_{m}, X_{y}, X_{\text {paper }}, Y_{c}, Y_{m} \ldots$, as their names suggest, denote the measured $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ values for cyan, magenta, yellow and the white paper when they are printed using the printer that is supposed to be used.
As long as Eq. 1 has acceptable solutions for any required color's X, Y and Z values the color can be produced by dot-off-dot printing strategy and by using the inkjet printer (Deskjet 970CXi), i. e. the printer we have used in our experiments.
Let us now make our algorithm produce an image whose color matches the one of the image in Fig. 2a. For finding the tristimulus values for this color Demichel's equations are used. For this example we will have $a_{c m y}=0, a_{c}=0.25, a_{m}=$
$0.25, a_{y}=0, a_{c m}=0.25, a_{c y}=0, a_{m y}=0$, and $a_{\text {paper }}=0.25$. $a_{x y}$ denotes the fractional area covered with only $x$ and $y$ color inks. Note that according to Demichel's equation, for instance the fractional area covered with cyan and magenta is calculated by $a_{c m}=c m(1-y)$. Now we calculate the color of the image in Fig. 2a by Neugebauer's equations and use this color as the target color. Then Eq. 1 is solved for this color and the new coverages are calculated. The calculated coverages of $\mathrm{C}, \mathrm{M}$ and Y separations are $50.82 \%, 36.51 \%$ and $0.54 \%$, respectively. That means, the image that should be halftoned by the proposed color halftoning method in order to produce this color has $50.82 \%, 36.51 \%$ and $0.54 \%$ in its $\mathrm{C}, \mathrm{M}$ and Y separations respectively.
Fig. 3a shows again this image being halftoned by applying the monochromatic halftoning method to the color separations independently. Fig. 3b shows the transformed image being halftoned by the proposed color halftoning method. The transformation was performed in order for the algorithm to produce the same color as the one in Fig. 3a.

## Treating Different Colors Differently

Since the yellow ink printed on a white paper is not as visible as the other two one can discuss that the color separations shouldn't be treated equally. This problem can be solved by simply ignoring the Y separation and only halftoning the C and M separations dependently. Another way, which we use in our example below, is only to prevent the dots in the Y separation from being placed on top of the dots in the other two separations and try to place the dots in the C and M separations as homogeneously as possible. This is done by using a $1 x l$ filter for the C and M separations when the dot is placed in the Y separation. The same $1 \times 1$ filter is also used for the Y separation when the dot is placed in either C or M separation. Let us give an example. A color image with $2 \%$, $3 \%$ and $4 \%$ coverage in its C, M and Y separation is halftoned.


Figure 4. An image with $2 \%, 3 \%$ and $4 \%$ coverage in its C, $M$ and $Y$ separations is halftoned by the proposed color halftoning method. In a, the color separations are treated equally. In $b$, the $Y$ separation is treated differently.

First, this image is halftoned by the proposed method with the separations being treated equally, see Fig. 4a. Fig. 4b shows the same image being halftoned by the same method while the Y separation is treated differently. In this example we use a $9 \times 9$ filter for the C and M separations and a $1 \times 1$ for the Y separation when the dot is placed in either C or M separation. Note that we found the size of the filter for C and M separations using the coverage $p=2+3=5 \%$. When the dot is placed in the Y separation we use an $11 \times 11$ filter for this separation and a $1 x 1$ filter for the other two [7]. Observe also that the filter size for the Y separation has been calculated for $p=4 \%$, i.e. the coverage of this separation. We can clearly see that the image in Fig. 4b is perceived better than that in $a$.

## Objective Quality Measures

One of the demands on a well formed halftone pattern is that its dots should be placed homogeneously over the entire image. By halftone pattern we mean the result of halftoning a constant image, i. e. an image with constant pixel values. The set of distances from each dot to its closest dot gives a good picture of how close/far the dots in the halftone pattern are placed. The couple mean value and standard deviation of the data in this set can be used as a measure for homogeneousness of the halftone pattern. A big mean value indicates that the dots are placed far from each other while a small standard deviation indicates that the dots are placed homogeneously. This measure can also be used for halftone color patterns. For being able to use this measure we perform a logical OR between the separations to obtain a black and white binary image. Images in Fig. 5 show the result of performing a logical OR between the C and M separations of the corresponding images in Fig. 1.


Figure 5. The result of performing a logical OR between the color separations of corresponding images in Fig. 1.

By applying the presented measure above to the images in Fig. 5 we get ( mean value, standard deviation $)=(2.69,1.06)$ and ( $3.97,0.37$ ). This result also indicates that the image in Fig. 1b is more homogeneous than that in $a$.

Doing the same for the images in Fig. 4 we will have (mean value, standard deviation $)=(2.88,0.36)$ and $(2.00,0.69)$ for images in Fig. 4a and 4b, respectively. This result does not correspond to what we expected from images in Fig. 4.
Since the yellow separation is treated differently in order to achieve better result, the presented measure should also be changed accordingly. Therefore it is more realistic and correct if the logical OR is performed between only the cyan and magenta separations of the color image. If we do so, we get $($ mean value, standard deviation $)=(3.20,0.61)$ and ( $3.90,0.34$ ). This result shows that the cyan and magenta dots are distributed more homogeneously in Fig. 4b than Fig. 4a and it is actually what we also can notice when viewing these images.
Another measure that can be used is to find the standard deviation of the ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ) values over the entire halftone color pattern. The smaller the standard deviations the better the final halftone color pattern. In the following we will give a short description on how to calculate these standard deviations.
Suppose that we have a $M x N$ color halftoned image called $b$. For simplicity, like images in Fig. 3, we assume that this image consists of two color separations cyan and magenta, which are denoted by $b_{c}$ and $b_{m}$, respectively. The X value of the image $b$ at position $(i, j)$ when the image is printed can be calculated by:

$$
\begin{equation*}
X(i, j)=c \cdot X_{c}+m \cdot X_{m}+c m \cdot X_{c m}+p \cdot X_{p} \tag{2}
\end{equation*}
$$

where,
$c=\left[b_{c}(i, j)\right.$ and $\left.\left(n o t b_{m}(i, j)\right)\right]$,
$m=\left[b_{m}(i, j)\right.$ and $\left(\right.$ not $\left.\left.b_{c}(i, j)\right)\right], c m=\left[b_{c}(i, j)\right.$ and $\left.b_{m}(i, j)\right]$ and $p=\left[\left(\operatorname{not} b_{c}(i, j)\right)\right.$ and (not $\left.\left.b_{m}(i, j)\right)\right]$.
and, or and not denote the logical "and", "or" and "not". $X_{c}$, $X_{m}, X_{c m}$ and $X_{p}$ are the measured data for cyan, magenta, blue and the white paper, respectively. Eq. 2 can be generalized to be used for the entire image as,

$$
\begin{equation*}
X=C \cdot X_{c}+M \cdot X_{m}+C M \cdot X_{c m}+P \cdot X_{p} \tag{3}
\end{equation*}
$$

where,
$C=\left[b_{c}\right.$ AND (NOT $\left.\left.b_{m}\right)\right], M=\left[b_{m}\right.$ AND (NOT $\left.\left.b_{c}\right)\right]$,
$C M=\left[b_{c}\right.$ AND $\left.b_{m}\right]$ and $P=\left[\left(\operatorname{NOT} b_{c}\right)\right.$ AND (NOT $\left.\left.b_{m}\right)\right]$.
$A N D, O R$ and NOT denote the pixel-wise logical "and", "or" and "not" between matrices. In Eq. 3, $X$ is a matrix and at each position it holds the X value of the corresponding position in the binary color image $b$ when it is printed using the printer the measurements were done for. The Y and Z values are calculated correspondingly. Now we define the standard deviation for the X values as,

$$
\begin{equation*}
\sigma_{x}=\sqrt{\left(\sum_{i, j}\left(X(i, j)-X_{\text {mean }}\right)^{2}\right) /(N \cdot M)} \tag{4}
\end{equation*}
$$

where $X_{\text {mean }}$, the mean of matrix $X$, is:

$$
\begin{equation*}
X_{\text {mean }}=\left(\sum_{i, j} X(i, j)\right) /(N \cdot M) \tag{5}
\end{equation*}
$$

The standard deviations for Y and Z values are calculated correspondingly. We calculated these standard deviations for images in Fig. 3. For the image in Fig. 3a, that means the result of halftoning the separations independently, we have, $\sigma_{x}=21.43, \sigma_{y}=25.97$ and $\sigma_{z}=4.28$. For the image in Fig. 3 b , i. e. the image that was halftoned by the proposed color halftoning method, we have, $\sigma_{x}=13.73, \sigma_{y}=23.85$ and $\sigma_{z}=$ 3.57. The sum of these values are 51.68 and 41.15 for the images in Fig. 3a and 3b respectively. The result is exactly what we expected, i. e. the variation in color over the entire image is less for the image that is halftoned dependently. Note that $X_{\text {mean }}$ is the same for both of the images shown in Fig. 3 because the compensation was actually performed in order to make the image to the right have the same mean as the one to the left.
This measure can in general be applied to homogeneous parts of ordinary halftoned color images in order to compare different halftoned images or halftoning methods with each other.

## Conclusions

This paper very briefly describes a color halftoning method that halftones the color separations of the original image dependently. This method is described in detail in [7]. Two objective measures for judging the quality of halftoned color images are also discussed in this paper. They are also applied to a number of images shown in this paper.

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## Biography

Sasan Gooran received his M.Sc. degree in Computer Science and Engineering and his Ph. D. in Computer Engineering, Media technology, from Linköping University, Sweden, in 1994 and 2001, respectively. He successfully defended his dissertation titled "High Quality Frequency Modulated Halftoning" on March 2001. His dissertation presents two novel halftoning methods, one for monochromatic and the other one for color images.

