The Estimation of Spectral Reflectances Using the Smoothness Constraint Condition

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Abstract

A spectral match ensures a colour match of two object colours for all observers across different illuminants. Developing a spectral based colour reproduction system requires a spectral analysis in which the spectral reflectance of each pixel must be known.

This paper describes a method for estimating the surface reflectance values of objects in a scene (or image) under a given illuminant and sensors of a digital camera. This work is motivated by the work of van Trigt [1,2] using the smoothness condition for recovering reflectance of a given set of CIE tristimulus values X,Y,Z corresponding to a particular illuminant. He used integration of the square of the first derivative of the reflectance function as the smoothness restriction, and the functional analysis approach to develop a method for recovering the reflectance values. In this work, by using numerical integration techniques, the smoothness constraint is replaced by the square of the two norm of a vector, which is the multiplication of the smooth operator matrix and the reflectance vector. The proposed method is to solve a constrained-least-square problem. The testing results showed that the current method performed consistently better than those from the basis functions were compared with those from the basis-functions.

Introduction

In many colour applications such as colour measurement, computer vision, computer graphics, and colour image reproduction, it is required to obtain the surface spectral reflectance of objects in a scene. In this paper we consider the recovery of the reflectance values of objects in an image. Let p be the camera data for a pixel in the image, $R(\lambda)$ be the reflectance function of the corresponding object, $E(\lambda)$ be the illuminant used by the camera, and $S(\lambda)^T = (S_1(\lambda), S_2(\lambda), \ldots, S_n(\lambda))$ be camera sensors vector. Thus they have the following relationship:

$$p = \int_a^b E(\lambda)S(\lambda)R(\lambda)d\lambda \qquad (1)$$

Since $S(\lambda)$ is a n-component column vector, therefore p must be a n-component column vector. Here the range (a, b) is the visible spectrum (a=400 nm and b=700 nm for typical industry applications). For a trichromatic digital camera, there are only three spectrally broad and overlapping sensors corresponding to the red, green and blue channels. While for multispectral imaging systems n can be quite large (say 10). By uniformly sampling the spectra at N-1 wavelength intervals equation (1) can be rewritten as the following in the matrix vector form:

$$p = W^T r. \qquad (2)$$

Here W is a N by n matrix derived from the illuminant function and the sensors' vector, and r is the N-component vector defined by $r^T = (R(\lambda_1), R(\lambda_2), \ldots, R(\lambda_N))$. Thus, the problem is to determine r for given W and p. If the number of the sensors (n) equals to N, then the spectral vector r can be uniquely determined by equation (2). However, this is not the case. N is normally greater than n since the larger the n, the more expensive the camera. Therefore, how to accurately recover the spectral vector r from the given W and p becomes a problem.

One approach for solving the problem is to use the basis functions [3-7] to reduce the dimensionality of the reflectance vector. The main opportunity to improve spectral estimation performance is to use statistical information about the set of spectral reflectance functions, which one is likely to find in the input materials. Suppose we have collected a representative set of reflectance values. Then we can use the singular value decomposition (SVD) [8] or the principle component analysis (PCA) [9] to obtain the orthogonal basis vectors: $r^{(1)}, r^{(2)}, \ldots, r^{(N)}$. The first basis vector corresponds to the largest singular value and the second basis vector corresponds to the second largest singular value, and so on. Any spectral reflectance vector r can be a linear combination of the basis vectors, i.e.

$$r = \sum_{i=1}^{N} c_i r^{(i)} .$$

Here the coefficients, c_is, are uniquely determined by vector r. Several researchers suggested that the naturally occurring spectral reflectance curves are highly constrained, smoother, and can be well represented by only a few basis vectors, say the first k basis vectors. i.e.

$$r \approx \sum_{i=1}^{k} c_i r^{(i)} .$$

Set $B_k = (r^{(1)}, r^{(2)}, \ldots, r^{(k)})$ be the N by k matrix and the coefficient vector $c^T = (c_1, c_2, \ldots, c_k)$, then the problem is to find vector c so that the desired reflectance vector r is given by:

$$r = B_k c\,. \quad (3)$$

If vector r in equation (2) is replaced by equation (3), then equation (4) can be obtained.

$$p = W^T B_k c. \quad (4)$$

Thus if the number of sensors (n) equals to the number of the basis vectors (k) used, then vector $c = (W^T B_k)^{-1} p$. However, this method has some shortcomings. Firstly, the basis vectors depend on the collection of a particular data set. As mentioned in reference 10 and reference 6, there is a strong dependence of the spectral reconstruction performance on the data-base used for the principal component analysis. Secondly, the reflectance vector given by equation (3) may exceed the boundaries, i.e., some components of it may be greater than 1 or less than 0. The simplest remedy method is to force those components to be the nearest boundaries. In this case, the exact matches under the illuminants used for recovering the reflectance values cannot be met. Finally, it was found that the recovery accuracy does not necessarily become better with the increase of the number of sensors. Figure 1 illustrates this phenomenon. Suppose there is a need to recover a neutral colour with the reflectance values of 0.5 across visible spectrum, a perfect smooth curve. We use multi-illuminant plus colour matching functions to simulate the multi-sensors. Figure 1 shows the simulated spectral reflectances using one illuminant to simulate three sensors and 2 illuminants to simulate 6 sensors, and so on. It is clear that no one case gives the right reflectance values.

Another approach for finding the spectral reflectance $R(\lambda)$ is to add the smoothness restriction [1,2] :

$$\frac{Min}{0 \le R(\lambda) \le 1} \int_a^b \left(\frac{dR}{d\lambda}\right)^2 d\lambda \quad (5)$$

on $R(\lambda)$. When the number of sensors (n) is 3, and the sensors are the colour matching functions, the p vector in equation (2) is the CIE tristimulus values, i.e., $p^T = (X,Y,Z)$ with

$$X = \int_a^b E(\lambda)\bar{x}(\lambda)R(\lambda)d\lambda$$
$$Y = \int_a^b E(\lambda)\bar{y}(\lambda)R(\lambda)d\lambda \quad (6)$$
$$Z = \int_a^b E(\lambda)\bar{z}(\lambda)R(\lambda)d\lambda$$

van Trigt gave an algorithm for estimating the reflectance functions satisfying equations (5) & (6). Troost and de Weert[11] compared the van Trigt's method and the method using 3 basis functions for estimating the reflectance values from a given set of *X,Y,Z* under one illuminant . They found that the two methods gave the same performance. Lucassen [12] investigated the van Trigt method by visualizing spectral changes due to the change of illuminant. The advantage of van Trigt's method is that it does not depend on the basis functions.

One of the possible drawback of the van Trigt's algorithm as mentioned by Lucassen[12] is that the recovered smoothness reflectance for a given *X,Y,Z* under one illuminant has 16 types of solutions, each type being valid in a certain domain of the colour space. In order to guarantee that the smoothness reflectance values are all within 0 and 1 across visible wavelengths, there are some restrictions on the values of *X,Y,Z.* Since the domain of the reflectance function types depends on the property of colour matching functions, it seems also complicated to generalize the smoothest method to multi-sensors. On the other hand, the smoothest reflectance function may not be close to the actual reflectance function under a trichromatic system. Figure 2 demonstrates this phenomena.

From the above discussions, it seems that the smoothness constraint under 3 sensors is insufficient to recovering the real reflectance more accurately (close to the original). In the next section, the smoothness constraint condition (5) is replaced by minimizing the square of the 2-norm of the vector Gr, by using a numerical integration technique. Here G is a N-by-N matrix, and is called the smooth operator or smooth matrix, and the vector r is the reflectance vector defined in equation (2). Thus, the smoothness condition can be easily combined with multi-sensors in equation (1)

The Proposed Method

As in the last section, by uniformly sampling the visible spectra at $N-1$ wavelength intervals with the length of the intervals being $\Delta\lambda$ the integration in the equation (5) can be approximated by the following:

$$\int_a^b \left(\frac{dR}{d\lambda}\right)^2 d\lambda \approx \Delta\lambda\{0.5[(\frac{dR(\lambda_1)}{d\lambda})^2 + (\frac{dR(\lambda_1)}{d\lambda})^2] + \sum_{k=2}^{N-1}(\frac{dR(\lambda_k)}{d\lambda})^2\} \quad (7)$$

The details were described by Kockler.[13] Here

$$\frac{dR(\lambda_k)}{d\lambda}$$

is the derivative of the spectral function $R(\lambda)$ at point λ_k. If we replace the derivatives by

$$\frac{dR(\lambda_k)}{d\lambda} = [R(\lambda_{k+1}) - R(\lambda_k)]/\Delta\lambda\,, \; k = 1,2,\ldots,N-1.$$

$$\frac{dR(\lambda_k)}{d\lambda} = [R(\lambda_k) - R(\lambda_{k-1})] / \Delta\lambda \text{ , } k = N$$

then equation (7) becomes:

$$\int_a^b (\frac{dR}{d\lambda})^2 d\lambda \approx \| Gr \|^2 / \Delta\lambda \qquad (8)$$

Here, the N by N matrix G and the vector r are given by

$$G = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & & & \\ & -1.0 & 1.0 & & \\ & & \ddots & \ddots & \\ & & & -1.0 & 1.0 \\ & & & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \; r = \begin{pmatrix} R(\lambda_1) \\ R(\lambda_2) \\ \vdots \\ R(\lambda_N) \end{pmatrix} \qquad (9)$$

and here for any N-component vector y,

$$\| y \| = \sqrt{\sum_{k=1}^{N} y_k^2} \text{ ,}$$

which is called the 2-norm of the vector y. Since $\Delta\lambda$ is a constant, by equation (8), the smoothness constraint (5) can be replaced by the following:

$$\underset{r}{Min} \| Gr \|^2 , \text{ s.t. } 0 \le r_k = R(\lambda_k) \le 1, \text{ for } k = 1,2,...,N \qquad (10)$$

The above equation is called the smooth equation and the matrix G is called the smooth operator or smooth matrix. The main advantage of the present approach is that the smooth equation (10) can be freely combined with either equation (1) or (2) in the multi-sensors or equation (6) in CIE space. Thus the following method is proposed for recovering the reflectance vector r from given camera response vector p.

The Proposed Method

1. Given camera data p, sensors $S_i(\lambda)$, $i = 1,\ldots,n$ and the illuminant $E(\lambda)$
2. Solve the following constrained least squares problem for the reflectance vector r:

$$\underset{r}{Min} \| Gr \|^2$$
$$\text{s.t. } p = W^T r \text{ and } 0 \le r_k \le 1, \; k = 1, 2,..., N$$

Here W is n-by-N matrix and is consisted of sensors $S_i(\lambda_k)$, and the illuminant $E(\lambda_k)$

Performance of the Proposed Method

a) Simulation Design

Since there is a lack of the digital sensors' spectral sensitivity data, CIE colour matching functions and multi-illuminants approach are used to simulate the digital camera response vector p in equation (2). The CIE 1964 colour matching functions, and some CIE illuminants (D65, A, F11, D50, F2, and F7) are used to form the weighting tables or equivalently the matrix W in equation (2). In order to simulate the camera response vector p, reflectance vector r is required from equation (2). 1560 Munsell glossy samples were measured by a spectrophotometer. These reflectance vectors were used as inputs to obtain the 1560 simulated response vectors p. These spectral reflectances are also used to derive the basis functions.

b) Performance Measures

Two measures were used to indicate the performance of each method: RMS and metamerism Index (MI). For the former, let r be the reflectance values from the Munsell data set and $\bar{r}$ be the estimated reflectance vector. The measure of fit used was the root mean square (RMS) defined below.

$$RMS = \| r - \bar{r} \| / \sqrt{N} = \sqrt{\sum_{i=1}^{N} (r_i - \bar{r}_i)^2 / N}$$

The MI is the colour difference between the original measured reflectance vector (r) and the estimated reflectance vector ($\bar{r}$) under a specified illuminant. Here, the CIE 1994 colour difference equation[14] is used.

c) Results

The 1560 Munsell colours were also used to test the current method and the method based upon basis functions. The results are listed in Tables 1 to 10. Note that the 6 illuminants used are in the following order: D65, A, F11, D50, F2 and F7. For example, 'one illuminant' in Table 1 means that the first illuminant D65 was used for estimating the reflectance vectors and the other five illuminants A, F11, D50, F2 and F7 were used to compute the MI. If ' two illuminants' are used, this means that the first two illuminants D65 and A were used for estimating the reflectance vectors and the other four illuminants F11, D50, F2 and F7 were used for computing the MI, and so on. 'Ave' in the tables means that the average (or mean) of either RMS measurements or MI under each illuminant. While, 'Max' means that the worst case for the 1560 Munsell colours either in RMS or MI measures.

Tables 1 to 5 showed that the current method performs better with the increase of the number of illuminants. Considering the RMS measure, not much difference was found when more than three illuminants (9 sensors) are used. From the MI measure, the current method always ensures exact matches under the illuminants used for the recovering the reflectance values.

Considering the MI measure under illuminants D50, F2 and F7, the results indicate that two illuminants (6 sensors) are sufficient for estimating the reflectance vectors in many applications.

Tables 6 to 10 list the testing results for the basis function method. The results from the averaged measures showed that the basis function method and the current method gave roughly same performance. However, the basis function method seems not stable with the increase of the number of illuminants (sensors) used for the recovering the real reflectance vectors, i.e., the increase of the illuminants used does not correspond to the decrease of the maximum ΔE values. Besides, the basis function method cannot ensure the exact matches under the illuminants used for estimating the spectral information. The reason for causing this is that after obtaining the coefficient vector *c* form equation (4), the reflectance values computed from equation (3) exceed the lower or upper boundaries. Whenever the reflectance values are beyond the boundaries, they are forced to be the nearer boundary. This will violate the equation (4).

Conclusion

A method for recovering the real reflectance functions form the digital image under the knowledge of illuminant and the multi-sensors for capturing the image is developed. This work is based upon the smoothness measure derived by van Trigt.[1,2] However, the use of the smoothness constraint condition between the current and the van Tright is different. The numerical integration approach is used to replace the smoothness condition (5) by minimizing the square of the two norm of the vector obtained by multiplying the reflectance vector *r* by a smooth operator *G*. In this way, the smoothness condition can be freely combined with the multi-sensors. The current method and the basis function method were compared. The results show that:

- The current method is quite stable and performing better with the increase of the number of illuminants (sensors) used.
- Two illuminants (6 sensors) are sufficient for the current method for many applications.
- The basis function method is not stable. This may be limited to the way of our simulation. Further investigation is needed.
- Finally, the current method gives the exact reflectance for the neutral colour under one or more illuminants, while the basis functions approach cannot gives the exact reflectance for the neutral colours no matter how many illuminants are used.

References

1. C. van Trigt, Smoothest reflectance functions. I: Definition and main results, *J. Opt. Soc. Am. A* **7**, 1891-1904(1990).
2. C. van Trigt, Smoothest reflectance functions. II: Complete results, *J. Opt. Soc. Am. A* **7,** 2208-2222(1990).
3. Laurence T. Maloney, Evaluation of linear models of surface spectral reflectance with small numbers of parameters, *J. Opt. Soc. Am. A* **3**(10), 1673-1683(1986).
4. M. J. Vrhel and H. J. Trussell, Color correction using pricipal components, *Color Res. Appl.* **17**, 328-338(1992).
5. J. E. Farrell, J. Cupitt, D. Saunders, and B. A. Wandell, Estimating spectral reflectance of digital image of Art, in *Proceedings of International Symposium on Multispectal Imaging and Color Reproduction for Digital Archives*, pp 58-64, Chiba University, Chiba, Japan, 1999.
6. L. Macdonald, S. Westland and J. Shaw, Colour image reproduction: Spectral vs Spatial, in *Proceedings of International Symposium on Multispectal Imaging and Color Reproduction for Digital Archives*, pp 81-91, Chiba University, Chiba, Japan, 1999.
7. L. T. Maloney and B. A. Wandell, Color constancy: a method for recovering surface spectral reflectance, *J. Opt. Soc. Am. A* **3**(1), 29-33 (1986).
8. G. H. Golub and C. van Loan, *Matrix Computations*, The Johns Hopkins University Press, Baltimore, 1983.
9. R. A. Johnson and D. W. Wichern, *Applied multivariate statistical analysis*, third edition.
10. F. H. Imai and Roy S. Berns, Spectral estimation using trichromatic digital cameras, in *Proceedings of International Symposium on Multispectal Imaging and Color Reproduction for Digital Archives,* pp 42-49, Chiba University, Chiba, Japan, 1999.
11. J. M. Troost, C. M. M. de Weert, Techniques for stimulating object color under changing illuminant conditions on electronic displays, *CR&A* **17**, 316-327 (1992).
12. M. Lucassen, Application of smoothest reflectance function for the visulization of spectral changes due to the illuminant*, International Conference on Color in Graphics and Image Processing- CGIP'2000*, France, 2000.
13. N. Kockler, *Numerical methods and scientific computing*, Oxford: Clarendon; 1994.
14. CIE (1995), Industrial colour difference equation, CIE Publ. No. 116, *Central Bureau of the CIE,* Vienna, Austria.

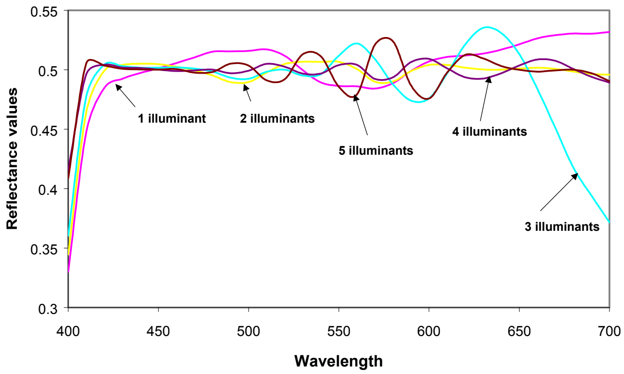

Figure 1. The basis functions based method is used to recovering the reflectance values of the object. The original reflectance values are all equal to 0.5. Five estimated reflectance functions are obtained using one, two, …, and five illuminants respectively, and shown in the diagram.

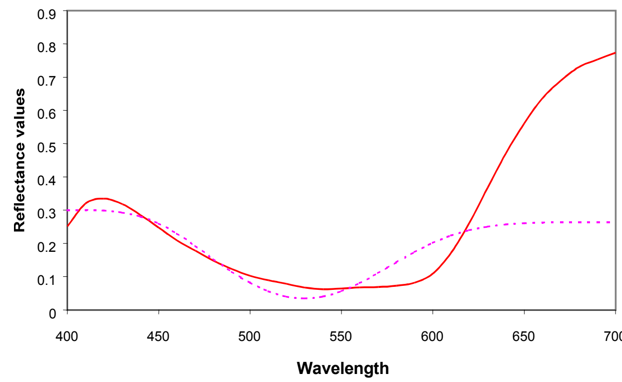

Figure 2. The smoothness approach is used to estimate the reflectance function for given a colour and one illuminant. The full line is the original one, and the dashed line is the estimated one.

Table 1: One illuminant (3 sensors) for the proposed method

	RMS	MI/D65	MI/A	MI/F11	MI/D50	MI/F2	MI/F7
Ave	0.04	0.00	1.22	1.53	0.37	1.08	0.28
Max	0.20	0.00	5.92	7.33	2.04	6.62	1.08

Table 2: Two illuminants (6 sensors) for the proposed method

	RMS	MI/D65	MI/A	MI/F11	MI/D50	MI/F2	MI/F7
Ave	0.02	0.00	0.00	0.99	0.02	0.24	0.16
Max	0.08	0.00	0.00	5.52	0.13	1.19	0.79

Table 3: Three illuminants(9 sensors) for the proposed method

	RMS	MI/D65	MI/A	MI/F11	MI/D50	MI/F2	MI/F7
Ave	0.01	0.00	0.00	0.00	0.01	0.13	0.04
Max	0.07	0.00	0.00	0.00	0.09	1.17	0.19

Table 4: Four illuminants (12 sensors) for the proposed method

	RMS	MI/D65	MI/A	MI/F11	MI/D50	MI/F2	MI/F7
Ave	0.01	0.00	0.00	0.00	0.00	0.06	0.03
Max	0.07	0.00	0.00	0.00	0.00	0.53	0.13

Table 5: Five illuminants (15 sensors) for the proposed method

	RMS	MI/D65	MI/A	MI/F11	MI/D50	MI/F2	MI/F7
Ave	0.01	0.00	0.00	0.00	0.00	0.00	0.02
Max	0.07	0.00	0.00	0.00	0.00	0.00	0.17

Table 6: One illuminant (3 sensors) for the basis functions based method

	RMS	MI/D65	MI/A	MI/F11	MI/D50	MI/F2	MI/F7
Ave	0.03	0.11	1.10	1.57	0.37	1.14	0.33
Max	0.19	4.31	5.22	7.10	4.07	7.70	4.68

Table 7: Two illuminants (6 sensors) for the basis functions based method

	RMS	MI/D65	MI/A	MI/F11	MI/D50	MI/F2	MI/F7
Ave	0.01	0.00	0.00	0.91	0.01	0.24	0.13
Max	0.06	0.88	1.50	10.79	0.89	3.63	1.89

Table 8: Three illuminants (9 sensors) for the basis functions based method

	RMS	MI/D65	MI/A	MI/F11	MI/D50	MI/F2	MI/F7
Ave	0.05	0.18	0.25	0.13	0.21	0.73	0.22
Max	0.28	12.85	14.19	10.73	13.21	18.50	13.81

Table 9: Four illuminants (12 sensors) for the basis functions based method

	RMS	MI/D65	MI/A	MI/F11	MI/D50	MI/F2	MI/F7
Ave	0.00	0.00	0.00	0.00	0.00	0.04	0.02
Max	0.03	0.07	0.15	0.06	0.08	0.39	0.16

Table 10: Five illuminants (15 sensors) for the basis functions based method

	RMS	MI/D65	MI/A	MI/F11	MI/D50	MI/F2	MI/F7
Ave	0.01	0.01	0.01	0.01	0.01	0.02	0.06
Max	0.09	4.14	5.97	3.72	4.62	6.20	4.53

Biography

Dr. Changjun Li is a Research Fellow at the Colour & Imaging Institute, University of Derby, UK. He received his B.Sc. in computational mathematics from Peking University, China in 1979, M.Sc. in numerical analysis from the Department of Technical Sciences, Chinese Academy of Science in 1982 and Ph.D. at the Department of Computer Studies of Loughborough University, UK in 1989. He was Professor of computational mathematics in the Northeastern University of China before joining the Colour & Imaging Institute. He has published over 40 papers in international journal in numerical linear algebra, parallel computing, and colour imaging science. He received the Scientific Progress First Prize awarded by the Educational Committee of Liaoning Province of China in 1998.

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