# Saturation, Superfluous or Superior?

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#### **Abstract**

Of the three basic color perceptions, hue, brightness, and colorfulness, hue has no relative version, but brightness has lightness, and colorfulness has chroma and saturation. Correlates of chroma are widely used in color difference formulae, but saturation currently plays little part in color science and technology. This is perhaps because in many industries flat samples are viewed in uniform lighting for the evaluation of color differences, and in this case chroma is the appropriate contributor for samples of small angular subtense. For samples of large angular subtense, however, a correlate of saturation may be more appropriate to use. In the real world, it is common for solid objects to be seen in directional lighting; in these circumstances saturation is a more useful percept than chroma because the former remains constant in shadows. In imaging, artists and computer-graphics operators make extensive use of series of colors of constant saturation. In optical imaging, saturation can be an important percept in large dark areas. Recent experimental work has provided a much improved correlate of saturation.

#### Introduction

A recently published prestigious book on color science included the statement: 'A color's chroma is its colorfulness. A high chroma color may be described as highly saturated.' There is no further discussion of the differences between colorfulness, chroma, and saturation, and one can only conclude that these matters were something of a mystery to the author, a situation perhaps not too uncommon in the color-science community.

There are three basic color perceptions, hue, brightness, and colorfulness. Hue has no relative version, but brightness has lightness, and colorfulness has chroma and saturation. Colorfulness, saturation, and chroma are defined as follows.

**Colorfulness** is the attribute of a visual sensation according to which an area appears to exhibit more or less of its hue (*hue* being defined as: the attribute of a visual sensation according to which an area appears to be similar to one, or proportions of two, of the perceived colors, red, yellow, green, and blue).

**Saturation** is the colorfulness of an area judged in proportion to its brightness.

**Chroma** is the colorfulness of an area judged in proportion to the brightness of a similarly illuminated area that appears to be white or highly transmitting. (Chroma can be thought of as saturation allowing for the fact that as colors become darker it is harder to see saturation differences.)

The distinction between colorfulness, saturation, and chroma, can be illustrated by the following example. Consider a tomato seen out-of-doors in bright daylight, and then indoors by daylight on a dull day: the tomato appears more colorful in the former situation and less colorful in the latter; hence a change in colorfulness has occurred. But, in judging the color of the object, it would be noticed that the brightness of the surrounding objects have lower brightnesses indoors than out-of-doors, and, by judging the colorfulness of the tomato relative to those brightnesses, a judgment can be made that the object has the same color; in this case, the chroma has remained constant. Now consider the task of judging whether the tomato is the same color all over its surface (assuming that its hue and lightness are constant): in the lower areas, because of shading, the surface appears less colorful and less bright; if the colorfulness is constant in proportion to the brightness, the saturation would be constant, and this enables a judgment to be made that the tomato is the same color all over.

It is interesting that, in a recent study of color naming, neither colorfulness, saturation, or chroma was used by ordinary people. In another study in which observers had to identify differences in hue, lightness, and chroma, it was found that chroma differences were harder to identify than hue and lightness differences.

# The Use of Chroma

Conventional colorimetry provides no correlate of colorfulness, and the CIELAB system provides no correlate of saturation. These omissions often go unnoticed, because a major part of the application of colorimetry is to evaluate the color differences of flat samples seen in uniform lighting, often in a viewing booth. In this case chroma is usually the appropriate contributor to color difference rather than colorfulness or saturation. The correlates of chroma in the CIELUV and CIELAB systems¹ are evaluated, respectively, as:

$$C^*_{uv} = (u^{*2} + v^{*2})^{1/2}$$
 (1)

$$C^*_{ab} = (a^{*2} + b^{*2})^{1/2}$$
 (2)

These correlates are formulated so that a given difference in chromaticity receives progressively less weight as the sample becomes darker, because differences in chromaticity then become harder to see. In the CIELUV system this results from  $u^*$  and  $v^*$  being derived from chromaticity differences that are multiplied by  $13L^*$  (where  $L^*$  is the correlate of lightness), and in the CIELAB system by  $a^*$  and  $b^*$  being derived from tristimulus values that are divided by those for the reference white. For samples of angular subtense up to about  $10^\circ$ , this is appropriate; however if the sample is very large the perceptibility of differences in chromaticity are affected less by their lightness, and a correlate of saturation may then be more appropriate to use.

#### The Use of Colorfulness

When there are differences in luminance, a correlate of chroma may be very inappropriate to use. An example of this is when two self-luminous display devices are to be compared, and their luminances are significantly different. A display having lower chromas but higher luminances may be preferred because the higher luminances can give greater colorfulnesses. An example of this occurred in the application of new phosphors to display devices for broadcast television: it was found beneficial to use red and green phosphors whose chromaticities were closer together because, although this restricted the gamut of reproducible colors, there was a gain in luminance and this resulted in an in colorfulness.4 Although conventional colorimetry provides no correlate of colorfulness, the advanced colorimetry possible with the CIECAM97s model of color vision<sup>5</sup> does provide one, so that the type of situation described above in the development of broadcast television is now susceptible to quantitative analysis.

### The Use of Saturation

As already mentioned, even in evaluating color differences, a correlate of saturation may be more appropriate to use than a correlate of chroma if the angular subtense of the samples is large.

In the case of self-luminous stimuli, such as light sources and signal lights, chroma has no application. The objective measures used are then usually luminance and chromaticity; the chromaticity can be expressed as correlates of hue and saturation, by using, for example, the correlates of hue,  $h_{\rm uv}$ , and saturation,  $s_{\rm uv}$ , provided in the CIELUV system.

In the real world, object colors are not usually flat, and lighting is not usually uniform; the recognition of the colors of objects then requires that the effects of variations in illuminance be discounted, and the relevant percepts are not hue, lightness, and chroma, but hue, lightness, and saturation. In imaging, artists and computer-graphics

operators make extensive use of shadow-series colors to convey the three-dimensionality of solid objects, and these are series of constant saturation, not constant chroma; it is interesting that in the DIN color system, one of the three variables is saturation, rather than chroma, and, in the color atlas that illustrates the system, shadow series are used. In optical imaging, although chroma is useful when the objects depicted are small and uniformly lit, saturation is important in shadow areas especially if the angular subtenses involved are large. Colors seen by reflections from water usually have the same saturations as when seen directly.

The correlate of saturation in the CIELUV system is formulated as

$$S_{nv} = 13[(u' - u_n')^2 + (v' - v_n')^2]^{1/2} = C_{nv} */L*$$
 (3)

where, on the u',v' chromaticity diagram,  $u'-u_n'$  and  $v'-v_n'$  represent the distances of the sample from the reference white in the u' and v' directions, respectively. However, it is known that the u',v' chromaticity diagram, on which  $s_{uv}$  is based, has perceptual non-uniformities, and, like other measures in the CIELUV and CIELAB systems, no proper allowance is made for adaptation and other similar factors that can affect color appearance. It is preferable, therefore, to use a correlate of saturation derived from the CIECAM97s model of color vision, where allowance is made for at least some of these factors.

The correlate of saturation, *s*, incorporated in the CIECAM97s model has been criticised because it is not formulated in terms of a ratio of colorfulness to brightness, as is required to follow the CIE definition of saturation. Also, it was formulated before the availability of experimental results for the magnitude scaling of saturation, and these have now shown that it is not a good predictor.<sup>6</sup>

The quantitative evaluation of perceived saturation is not an easy task, but a recent investigation has yielded valuable data. In this investigation, observers were shown cubes of size 4.5 by 4.5 cm. They saw three sides of each cube, comprising a total angular subtense of about 6°. Cubes of 132 different colors were used and they were viewed on three different backgrounds, white, grey, and black. The instructions given to the observers were as follows:

The saturation is the attribute judged by the proportion of colorfulness to brightness. [A DIN color chart was shown to illustrate the concept.] A three-dimensional object colored with a solid color has a constant saturation but different luminance on each side. For example, if each side of a cube is painted the same color the sides could have different colorfulnesses and brightnesses but their saturations would be the same. The more the colorfulness the more the brightness, and vice versa. Please make a judgement of the saturation to give a number which is in the right relationship to the reference saturation. A neutral color has no saturation and is represented by zero on the scale. For a very dark color, if its hue can be perceived significantly, it would have high saturation. This is an open-ended scale since no top limit is set. The saturation of the reference saturation sample which is displayed beside the test sample

should always be compared so that all subsequent test colors can be related to it. The reference saturation sample has saturation 50 in the phase of the grey background.

The spectral radiances of the three sides of each cube were measured by means of a telespectroradiometer. An area providing a reference saturation was always present during the observations.

When the CIECAM97s model was adopted by the CIE, it was recognised that, although considered to be the best choice at the time, experience would most likely lead to it being modified and extended to obtain better performance, less complexity, or both. This has indeed happened. In seeking an improved predictor of saturation, it was therefore desirable to base it on a version of CIECAM97s in which modifications that were known to be beneficial were incorporated. This has been done, and the version of the model used is designated Model FC; the ways in which it differs from the original CIECAM97s model are given in the Appendix. Correlates derived from the Model FC are denoted by the subscript D.

The new correlate of saturation is evaluated as a function of the ratio of the correlate of colorfulness,  $M_{\rm D}$ , to the correlate of brightness,  $Q_{\rm D}$ , and is thus in accord with the CIE definition of saturation as colorfulness judged in proportion to brightness. To obtain good correlation with the experimental results this ratio is raised to the power of 0.5, so that the final formula is:

$$s_{\rm D} = (M_{\rm D}/Q_{\rm D})^{0.5} \tag{4}$$

It was found that changing the background from grey to either black or white made very little difference to the perceived saturation; the results given below are therefore averages for the three different backgrounds. Reducing the luminance factor of the background results, for most colors, in increases in both colorfulness and brightness; hence, saturation, being colorfulness judged in proportion to brightness, would be expected not to change very much; the only exception to this would be for very light colors for which reducing the luminance factor of the background results in some *decrease* in colorfulness with increased brightness.<sup>11</sup>

The differences, expressed as Coefficients of Variation (CVs), between the experimental results for saturation and the predictions obtained using this formula based on the FC model, and for the predictor  $s_{97}$  based on the CIECAM97s are as follows:

$$s_{\scriptscriptstyle \mathrm{D}}$$
 based on Model FC 22  $s_{\scriptscriptstyle 97}$  based on CIECAM97s 48

It is thus clear that the predictor  $s_D$  used in the FC model is far superior to the predictor  $s_{97}$  used in CIECAM97s in predicting the experimental saturation results. The CV value for observer consistency in scaling saturation was 16; the fact that the predictions have a CV of 22 suggests that it might be possible to find another predictor that would give a CV value nearer to 16. Because chroma is colorfulness

judged relative to the brightness of a white, and lightness is brightness judged relative to the brightness of a white, a correlate of saturation could perhaps be formulated as a function of  $C_D/J_D$ ; however, the CV value for  $(C_D/J_D)^{0.5}$  as a correlate of saturation (the index 0.5 being found to be optimum) is again 22, the same as the value for  $(M_D/Q_D)^{0.5}$ .

Because the formulae for the correlates of saturation and hue are evaluated in a hyperbolic cone response space with a noise factor, they show some change in values if a colour of a given chromaticity has its luminance factor changed; this applies both to the old type of formula,  $s_9$ , and to the new type of formula,  $s_9$ , and to the correlates of hue, in both the CIECAM97s and FC models, although the hue changes are minor. In the experiment with the cubes, their sides had different illuminances. To deal with the effects of illuminant variation within a scene would require greater sophistication than is provided in these models; but, because the three sides of the cubes had the same luminance factors (and chromaticities), the models generate the same saturations and hues for the three sides, as would be expected.

It is possible to derive a model in which the hyperbolic function with a noise factor is replaced with a power function without a noise factor. It is then possible to derive predictors of hue and saturation that remain constant when a colour of a given chromaticity has its luminance factor changed. By re-optimising the parameters of such a model, predictors of hue, lightness, colourfulness, brightness, and saturation can be obtained whose performance is similar to that of CIECAM97s.<sup>12</sup> It seems inevitable that there is noise in the visual system, and that at very high stimulus intensities a maximum response is gradually reached; the hyperbolic function with noise is therefore more physiologically plausible than the power function without noise. This suggests that there probably are in reality some changes in perceived hue and saturation when, for a color of constant chromaticity, the luminance factor is changed to very low or extremely high values. But if, for color engineering purposes, it is desirable that hue and saturation remain constant for a color of constant chromaticity when the luminance factor changes, then the power function model could be used.

#### **Conclusions**

A feeling that chroma is what matters, and that saturation is superfluous, may be engendered by concentration on the evaluation of flat surfaces in uniform lighting. Although this is a very important part of color science and technology, it is only a part. Even in this application, if the samples are very large, correlates of chroma may be less appropriate to use than correlates of saturation. When self-luminous stimuli, such as light sources or signal lights are considered, chroma has no meaning, and saturation or colorfulness are what matters. Object colors in the real world are nearly always not flat and not viewed in uniform lighting; the percepts used for recognising such objects are then, not hue, lightness, and chroma, but hue, lightness, and saturation. In imaging, artists and computer-graphics operators, make

much use of shadow-series of colors in which the saturation, not the chroma, is constant. The use of saturation in these areas is now facilitated by the availability of a satisfactory predictor of perceived saturation in a model of color appearance. So, is saturation superfluous? Certainly not in many applications. Is saturation superior? The answer must be yes for a number of applications; saturation is superior to chroma: for self-luminous stimuli, such as light sources and signal lights; for the recognition of the colors of three-dimensional objects seen in the non-uniform lighting common in the real world; for representing shadows in art work and in computer graphics; and for evaluating color differences in very large areas that fill most of the field of view.

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# **Appendix**

The FC model differs from the original CIECAM97s model in the following ways.

(1) In the Chromatic Adaptation Transform, the Index in the Blue Channel is Set to 1.0, and the Following Matrix is Used:

$$\begin{pmatrix}
0.7982 & 0.3389 & -0.1371 \\
-0.5918 & 1.5512 & 0.0406 \\
0.0008 & 0.0239 & 0.9753
\end{pmatrix}$$
 instead of
$$\begin{pmatrix}
0.8951 & 0.2664 & -0.1614 \\
-0.7502 & 1.7135 & 0.0367 \\
0.0389 & -0.0685 & 1.0296
\end{pmatrix}$$
 (5)

The reverse matrix being

$$\begin{pmatrix} 1.076450 & -0.237662 & 0.161212 \\ 0.410964 & 0.554342 & 0.034694 \\ -0.010954 & -0.013389 & 1.024343 \end{pmatrix} \text{ instead of }$$

$$\begin{pmatrix}
0.98699 & -0.14705 & 0.15996 \\
0.43231 & 0.51836 & 0.04929 \\
-0.00853 & 0.04004 & 0.96849
\end{pmatrix} (5a)$$

(2) The D Factor in the Chromatic Adaptation Transform is Changed to  $D_{00S}$ 

$$D = F - F/(1 + 2L_{\Delta}^{1/4} + L_{\Delta}^{2}/300)$$
 (6)

$$D_{00S} = F[0.08 \log_{10}(L_{A}) + 0.76]$$
 (6a)

(3) The Correlate of Lightness is  $J_{\rm D}$  Instead of J:

$$J_{\rm D} = 100(A_{\rm D}'/A_{\rm wD}')^{cz'},$$
  
where  $z' = 0.85 + (Y_{\rm b}/Y_{\rm w})^{0.5}$  (7)

$$J = 100(A/A_{w})^{cz},$$
where  $z = 1.0 + (F_{LL})(Y_{b}/Y_{w})^{0.5}$  (7a)

where

$$A_{\rm D}' = [2\rho_{\rm a}' + \gamma_{\rm a}' + (1/20)\beta_{\rm a}' - 3.05]N_{\rm bb}$$
 (8)

$$A = [2\rho_a + \gamma_a + (1/20)\beta_a - 2.05]N_{bb}$$
 (8a)

$$A_{\rm wD}' = [2\rho_{\rm aw}' + \gamma_{\rm aw}' + (1/20)\beta_{\rm aw}' - 3.05]N_{\rm bb}$$
 (9)

$$A_{\rm w} = [2\rho_{\rm aw} + \gamma_{\rm aw} + (1/20)\beta_{\rm aw} - 2.05]N_{\rm bb}$$
 (9a)

 $A_{_{
m D}}{'}$  and  $A_{_{
m wD}}{'}$  are the same as A and  $A_{_{
m w}}$  but use the constant 3.05 instead of 2.05, and use different cone responses  $ho_{_{
m a}}{'}$ ,  $\gamma_{_{
m a}}{'}$ ,  $\beta_{_{
m a}}{'}$  and  $\rho_{_{
m aw}}{'}$ ,  $\gamma_{_{
m aw}}{'}$ ,  $\beta_{_{
m aw}}{'}$  because the

different matrix in the chromatic adaptation transform generates different tristimulus values for the corresponding colors. The subscript w denotes that the value is for the reference white. The value of the parameter, c, depends on the nature of the surround.

## (4) The Correlate of Chroma is $C_D$ Instead of C:

$$C_D = 2.44 (s_{97}')^{0.69} (J_D/100)^{0.67n} (1.64 - 0.29^n)$$
 (10)

$$C = 2.44 \left( s_{07} \right)^{0.69} \left( J/100 \right)^{0.67n} (1.64 - 0.29^{\text{n}}) \tag{10a}$$

where

$$s_{97}$$
 =  $N_c$  (5000 $e$  ( $a^2+b^2$ )<sup>0.5</sup>(10/13)  $N_{cb}$ /[ $\rho_a'+\gamma_a'+(21/20)\beta_a'$ ] (11) and is the same as in CIECAM97s except that the new chromatic adaptation transform matrix results in the use of  $\rho_a'$ ,  $\gamma_a'$ , and  $\beta_a'$  instead of  $\rho_a$ ,  $\gamma_a$ , and  $\beta_a$ 

#### (5) The Correlate of Colorfulness is $M_p$ Instead of M:

$$M_{\rm D} = C_{\rm D} \cdot F_{\rm L}^{0.15}$$
 (12)

$$M = C \cdot F_L^{0.15}$$
 (12a)

## (6) The Correlate of Brightness is $Q_p$ Instead of Q:

$$Q_{\rm D} = (1.24/c)(J_{\rm D}/100)^{0.67}(A_{\rm wD}' + 3)^{0.9}$$
 (13)

$$Q = (1.24/c)(J/100)^{0.67}(A_w + 3)^{0.9}$$
 (13a)

# (7) The Correlate of Saturation is $s_{\rm D}$ Instead of $s_{97}$ :

$$s_{\rm D} = (M_{\rm D}/Q_{\rm D})^{0.5} \tag{14}$$

$$s_{97} = N_c (5000e (a^2+b^2)^{0.5} (10/13) N_{cb})/[\rho_a + \gamma_a + (21/20) \beta_a]$$
 (14a)